2 Elements of Dual Polarization Radar Systems

A radar system is the central element in the study of radar meteorology. In order to better interpret weather radar observations of precipitation, it is useful to understand the basics of the radar system itself. The fundamentals of the radar system, its performance characteristics, and its operations are introduced in this chapter. We introduce the subsystems that commonly make up a weather radar system, and we discuss the features that determine radar performance, in particular, those relating to its sensitivity for detecting clouds and precipitation. Precipitation radars and cloud radars are generally delineated by their intended use. The microphysics that define clouds and precipitation are covered in Chapter 3. Although it is convenient to partition weather as clouds and precipitation, as a radar observable, these exist as a continuum with overlap. Cloud reflectivities can be well below −50 dBZ, whereas precipitation reflectivities can exceed 70 dBZ with large hail (reflectivity is defined in Section 2.3.1).

The exact choices of components and subsystems and the details of weather radar system designs continue to evolve with advances in technology. Even so, the basic subsystem concepts have not changed significantly since the inception of radar itself. For a radar to operate, it needs a source of power (a transmitter); a means to focus and direct the power into the target of interest and, similarly, to receive the echoes signals (an antenna); and finally, a receiver and data system to convert and visualize the echo power in a way that is meaningful to the radar's users. Arguably, the biggest difference between the early radars and the radars in use today is in how we capture, store, and utilize/visualize the received echoes. The first radar observations were a time trace of voltage versus range on a cathode-ray tube, with photography as the only means of recording the observations. Today, the radar's received echoes can be digitally recorded without any loss of fidelity. This allows the data to be processed in real time or archived for the future for processing using advanced algorithms.

The design and development of a radar system is ultimately a multifaceted tradeoff between system size, radar coverage, frequency allocation, application, sensitivity, and importantly, radar cost, to name a few. In this chapter, we introduce these elements of the radar system and provide an overview of the radar hardware. We also discuss how to estimate the performance parameters of the radar system.

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2.1 Polarization of an Electromagnetic Wave

The radar transmits and receives electromagnetic waves. The wave's electric field is generated (for transmitting) or measured (for receiving) by the radar. Time-varying electric and magnetic fields are linked, and these coupled oscillating fields form the electromagnetic waves. In the 1870s, James Clark Maxwell developed the theory to explain the electromagnetic waves, which is summarized by a set of equations called *Maxwell's equations*. Although the topic of electromagnetic waves and wave polarization is an in-depth field in itself, we will attempt to provide a simple explanation to introduce it here. (Additional details are provided in Chapter 4.)

Physicist Heinrich Hertz showed that electromagnetic waves can leave the wire and propagate in free space. The wave's polarization is related to the orientation of the electromagnetic wave's electric field. When transmitting, the wave propagates radially outward from the antenna in the form of an expanding sphere, as shown in Figure 2.1. As the range increases, the curvature of the sphere becomes negligible for small regions on the spherical surface, and the wave can be approximated locally as a plane wave (consider, e.g. a region around a raindrop positioned many kilometers from the radar). Following this, the plane-wave approximation is frequently used.

The polarization of electromagnetic waves is the topic of interest here. The electromagnetic wave propagates in three-dimensional space. One dimension is the direction of propagation, and the other two dimensions form the polarization plane. Polarization is a basic property of an electromagnetic wave. The polarization describes the

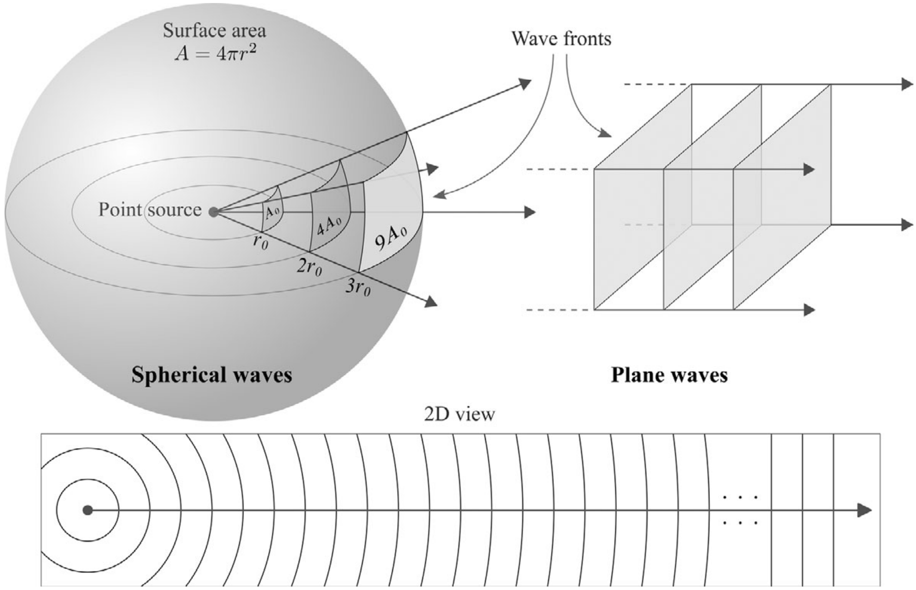

Figure 2.1 Spherical and plane waves. At large distances, spherical waves can be locally approximated by plane waves.

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direction of the wave's electric field, and the polarization is defined in a plane that is perpendicular to the direction in which the wave is propagating. For dual polarization radar, we precisely control and observe the polarizations of the wave. For weather radar, linear polarizations are the most commonly used, and the electric-field vectors are the horizontal and vertical polarizations. The horizontal and vertical orientations are determined by the antenna, as illustrated in Figure 2.2, where the polarization bases are shown together with the reference frame of a raindrop represented by an oblate spheroid. The unit vectors $\hat{h}_i$ and $\hat{v}_i$ denote the directions of the incident wave's

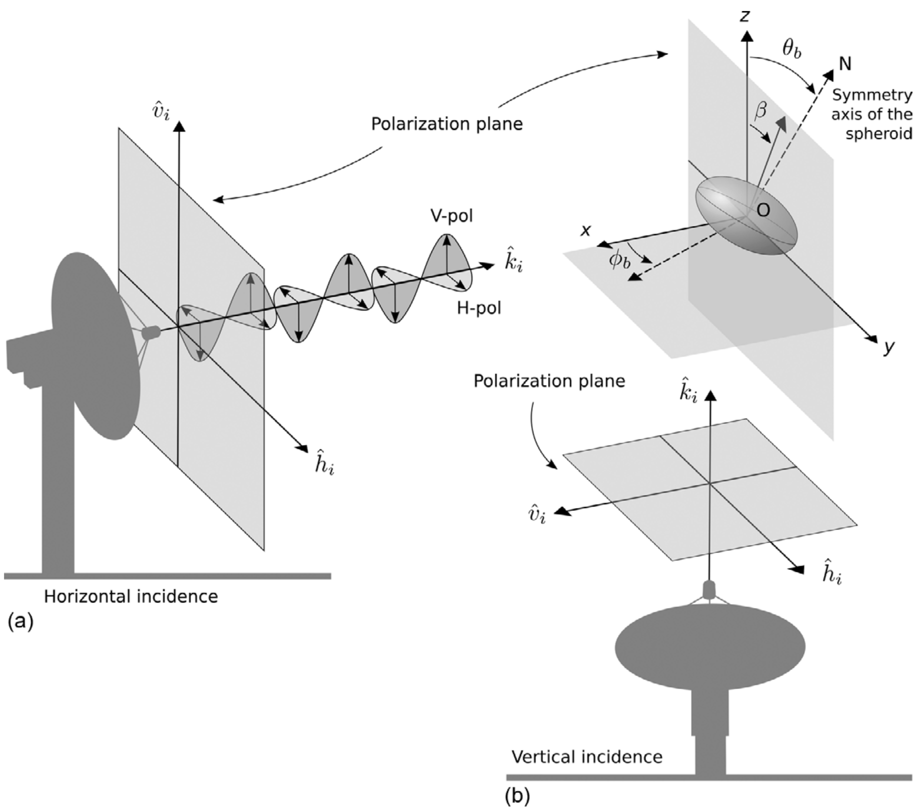

Figure 2.2 The antenna transmitting at horizontal incidence in panel (a) illustrates the parallel and perpendicular polarizations with respect to Earth's surface. For linear polarization systems, these are typically referred to as the *horizontal* and *vertical* polarizations (unit vectors $\hat{h}_i$ and $\hat{v}_i$). Note that the orthogonal vectors $\hat{h}_i$ and $\hat{v}_i$ are aligned with the true horizontal and vertical directions only for the case of horizontal incidence. The unit vector $\hat{k}_i$ represents the propagation direction of the incident wave and is given by $\hat{k}_i = \hat{v}_i \times \hat{h}_i$. Because of the circular symmetry of the spheroid in panel (b), its orientation can be described by only two Euler angles (θ_b and ϕ_b). The canting angle β is defined by the angle between the projection of the spheroid's symmetry axis (oriented along a line from O to N) on the polarization plane and the direction of the vertical electric field $\hat{v}_i$. Note that changing the canting angle of the particle is equivalent to rotating the polarization plane around the propagation direction.

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"horizontal" and "vertical" components. As illustrated by the two different antenna positions (horizontal and vertical incidence), it is worth remarking that the terms *horizontal* and *vertical* keep their intuitive meaning (with respect to Earth's surface) only when the antenna is pointing at the 0° elevation angle.

When discussing polarimetric radar, it is implied that the polarizations of the radar's signals are controlled so that the precipitation's polarization characteristics can be measured consistently. For weather radar applications, linear horizontal and vertical polarizations are the most commonly used (although in the early days, there were circularly polarized weather radars).

There are two interesting properties of electromagnetic waves that can be combined to describe the concept of polarization. First, the electric field is a vector and therefore has a magnitude and direction. Second, the oscillatory fields that produce waves can be considered as phasors, such as a sinusoid. A sinusoid voltage (or field) at any given time is described not only by its amplitude but also by its phase. The electric-field component that is the horizontal polarization can be described by a time-varying sinusoid as

$$E_h = |E_h| \cos(\omega t + \phi_h), \tag{2.1}$$

where ω is the angular frequency ($\omega = 2\pi c/\lambda$). Similarly, a vertically oriented electric-field component can also be described by a sinusoid as

$$E_v = |E_v| \cos(\omega t + \phi_v), \tag{2.2}$$

where the relative phasing between the two related sinusoids is $\phi = \phi_h - \phi_v$. The peak amplitudes of the horizontal and vertically oriented electric fields are $|E_h|$ and $|E_v|$, respectively.

A wave is a propagating disturbance that varies with time. Consider a ripple on a pond or a note on a string instrument; the amplitude of the wave depends on the time and location at which it is observed. The instantaneous field of a propagating plane wave is similarly dependent on the time and range at which it is observed. In eq. (2.1), the amplitude of the sinusoidal electric field is considered as a function of time for a fixed position (i.e., a range of zero). For a wave that propagates at the speed of light, c, we can calculate the time it takes to travel a range r as $\tau = r/c$. With a propagating wave, because it takes additional time to travel a distance r, the plane wave's electric field as a function of time and range are captured together:

$$E = E_0 \cos\big(\omega(t - \tau)\big) = E_0 \cos\left(\omega t - \frac{\omega}{c} r\right) = E_0 \cos\left(\omega t - k_0 r\right), \tag{2.3}$$

where k_0 is the free-space wave number ($k_0 = \omega/c = 2\pi/\lambda$).

If the range is held constant, the signal oscillates with time. Similarly, if the time is held fixed, the signal varies sinusoidally with range. This principle of eq. (2.3) is shown in panel (a) of Figure 2.3, where a vertically oriented electric field oscillates as the range from the radar changes. This also holds true for the horizontal polarization. In each case, the direction of the electric field is the polarization vector. If the

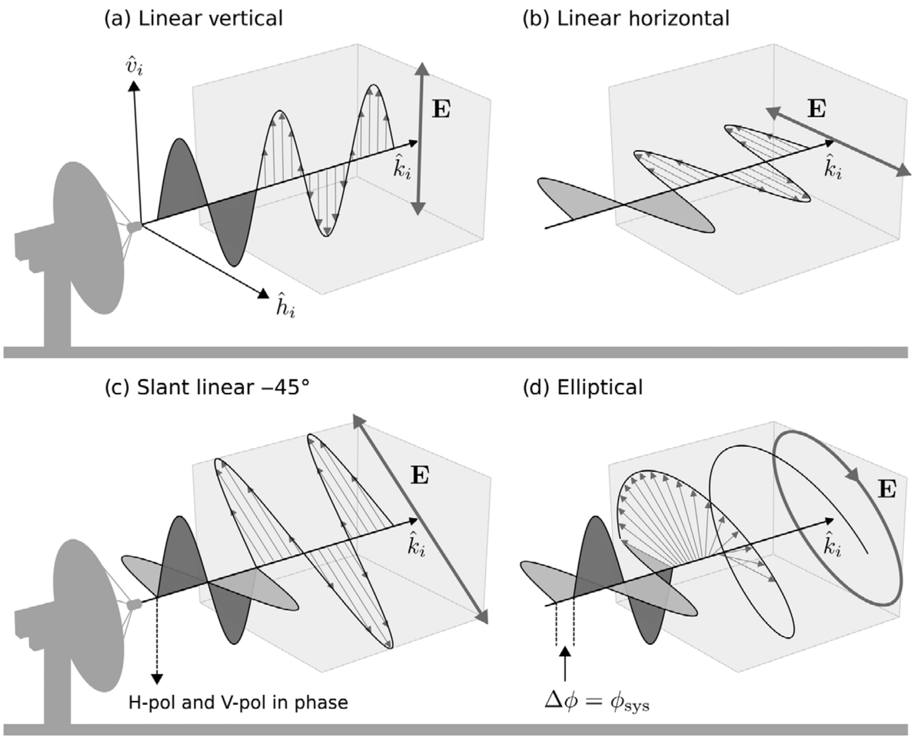

Figure 2.3 Propagation of the electric field for vertical (a), horizontal (b), slant linear -45° (c), and elliptical polarization (d).

electromagnetic wave has only a vertically oriented electric field, it is called a *vertically polarized wave*, whereas if it has only a horizontally oriented electric field, it is called a *horizontally polarized wave*.

When we combine these two sinusoids, which represent the horizontal and vertical polarization of the plane wave, and vary their amplitudes $|E_h|$ and $|E_v|$ and their relative phase (ϕ), we can produce widely varying patterns in the polarization planes as a function of time. If the amplitudes are the same for the horizontal and vertical polarizations ($|E_h| = |E_v|$) and the phases are the same ($\phi = 0^\circ$) or opposite ($\phi = 180^\circ$), the resulting electric field is polarized at a either $+45^\circ$ or -45° angle, as shown in panel (c) of Figure 2.3 ($\phi = 180^\circ$) (Note, Slant -45° can also be referred to as Slant 135°.) If there is amplitude imbalance or a nonzero phase shift (i.e., $|E_h| \neq |E_v|$ or $\phi \neq 0$), the polarization will trace out an elliptical shape with a range and time like that shown in panel (d) of Figure 2.3. The general vector electric field of the wave (which includes the components of both polarizations) can be written as

$$\mathbf{E} = (|E_h|\hat{h} + |E_v|e^{j\phi}\hat{v})e^{j(\omega t - k_0 r)}, \tag{2.4}$$

where $\hat{h}$ and $\hat{v}$ are the horizontal and vertical unit vectors, and ϕ is the phase of the vertical polarization relative to the horizontal polarization. (Note the change to a

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complex phase notation, rather than the cosine, which is adopted here to represent the forward-propagating wave and is consistent with the discussions in Chapters 4 and 5.) The term $\exp(j\omega t)$ describes the time-harmonic (or oscillatory) nature of the electric field. The combination $\exp[j(\omega t - k_0 r)]$ describes a traveling electromagnetic wave that propagates in the $\hat{k}$ direction (where $\hat{k} = \hat{v} \times \hat{h}$). The sinusoidal nature of $E_h = \mathbf{E} \cdot \hat{h}$ and $E_v = \mathbf{E} \cdot \hat{v}$, with corresponding arbitrary phases, can be described as

$$\mathbf{E} = (|E_h| e^{j\phi_h} \hat{h} + |E_v| e^{j\phi_v} \hat{v})\, e^{j(\omega t - k_0 r)}. \tag{2.5}$$

Consider the different resulting shapes of the transmitted dual polarization signal in Figure 2.4, where we plot the instantaneous polarization in the plane defined by the $\hat{h}$ and $\hat{v}$ unit vectors. As time advances, the curve will trace out the polarization's behavior, whether it be linear, circular, or something in between. Any variation of these shapes can be created with control over both polarizations. Special cases include the following:

- $|E_h| = |E_v|$, and $\phi_h = \phi_v$: The wave is linearly polarized in the "slant 45°" configuration (panel [b] of Fig. 2.4).

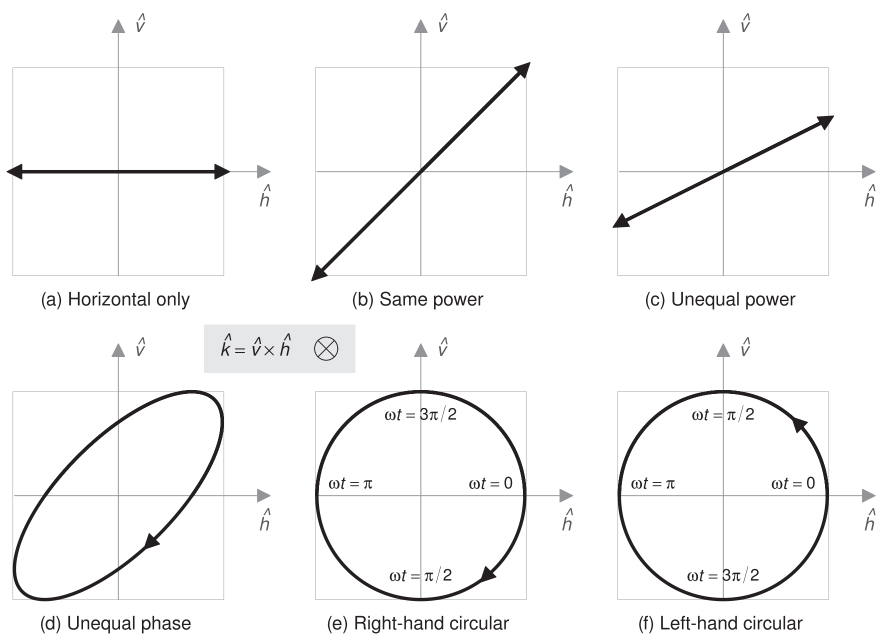

Figure 2.4 The relationship between the dual polarization signals in the linear polarization coordinates (horizontal and vertical). The amplitude and phase of the linear dual polarization signals are modified, giving the curves shown here (see eq. [2.5]). The direction of propagation is perpendicular and into the page ($\hat{k} = \hat{v} \times \hat{h}$). The arrow indicates the direction in which the signals vary with time. (a) Horizontal only: $E_v = 0$, and ϕ is any value. (b) Slant 45°: $|E_h| = |E_v|$, and $\phi = 0°$. (c) Unequal power: $|E_v| = |E_h|/2$, and $\phi = 0°$. (d) Unequal phase: $|E_h| = |E_v|$, and $\phi = 45°$. (e) Right-hand circular: $|E_h| = |E_v|$ and $\phi = +90°$. (f) Left-hand circular: $|E_h| = |E_v|$, and $\phi = -90°$.

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- $|E_h| = |E_v|$, and $\phi_h - \phi_v = \pm 90^\circ$: The wave is circularly polarized. For $\phi_h - \phi_v = +90^\circ$, it is right-hand circularly polarized (panel [e] of Fig. 2.4), and for $\phi_h - \phi_v = -90^\circ$, it is left-hand circularly polarized (panel [f] of Fig. 2.4).
- $E_h = 0$: The wave is vertically polarized.
- $E_v = 0$: The wave is horizontally polarized (panel [a] of Fig. 2.4).

An arbitrary combination of $|E_h|$, $|E_v|$, ϕ_h, and ϕ_v results in elliptical polarization. Thus, depending on the type of polarization state needed to probe the precipitation, the radar antenna transmits the corresponding type of polarization wave. The antenna is the primary device that determines the type of polarization transmitted and received by the radar. The antenna is the subsystem that transmits the electromagnetic waves and also receives the waves coming back into the radar. The antenna determines the polarization state. Because the polarization is essentially the direction of the electric field, one can think of a unit polarization vector that represents the polarization state. Any polarization state can be written as the composition of two orthogonal components. The most common orthogonal components are the horizontal and vertical polarization states, where the term *horizontal* refers to the horizontal direction when the antenna is pointing at a 0° elevation.

2.2 Polarimetric Radar Architectures

Polarimetric radars are defined by how they implement transmitting and receiving the two polarizations. Three operating modes are used for dual polarization radar. If a polarimetric radar transmits two polarization states simultaneously (e.g., horizontal and vertical) and receives the same two states, that radar is operating in *simultaneous transmit and simultaneous receive* (STSR) mode. The STSR mode also has a variety of other names, including simultaneous mode or SIM mode, simultaneous horizontal and vertical (SHV) mode, simultaneous transmit and receive (STAR), and hybrid mode. Similarly, two other modes of operation are radars that selectively transmit only one polarization at a time and either alternate transmit and alternate receive (ATAR) or alternate transmit and simultaneously receive (ATSR) both polarization states. The rationale for operating in the different modes is discussed in Chapter 4. (A simultaneous-transmit, alternate-receive radar is not considered because it does not have advantages over the other modes.)

For the STSR dual polarization mode, the ideal polarization state is slant 45° or 135°. The STSR mode is sometimes referred to as a *slant 45°* polarization. In practice, these systems typically transmit elliptical polarization because the phase between the horizontal and vertical polarizations are not exactly 0° (for 45° slant polarization) or not exactly 180° (for 135° slant polarization), and the amplitude between the horizontal and vertical polarizations is not exactly equal.

STSR mode requires a high degree of polarimetric isolation to minimize bias in the dual polarization variables. This also implies that the scatterers themselves must have negligible cross-polarization scattering, or else the scatterer's cross-polar signals will bias the measurements. The cross-polarization bias can be significantly reduced in ATSR mode, and the scatterer's cross-polarization characteristics can be estimated. The downside is that the horizontal and vertical polarizations' copolar measurements are not made at the same time. As we'll discuss in Chapter 5, the motions of the scatterers result in a loss of correlation between the horizontal and vertical measurements, which can increase the estimation error for the polarimetric variables in some instances (ATSR estimators are discussed in Chapter 6).

To implement the three modes, various radar architectures have been developed. The selection of transmit/receive polarizations is either fixed by the radar manufacturer or, for some radar designs, it is an option that can be configured during operation. Based on the polarization states that the radar can transmit and receive, dual polarization weather radars are broadly divided into four system architectures. As with most engineering endeavors, linear dual polarization radar systems can be implemented in a number of ways. Not all radar systems support all modes of operation, and in fact, the majority of operational dual polarization weather radars only operate in STSR mode. (As an example, the WSR-88D operates in STSR mode.) Some research radar systems allow the selection of either the ATSR and STSR sampling modes depending on the science application (e.g., the Colorado State University–Chicago Illinois radar [CSU-CHILL] or the National Aeronautics and Space Administration [NASA] D3R radars).

Figure 2.5 illustrates four common radar architectures for dual-polarization weather radar systems. The architecture in panel (a) of Figure 2.5 uses a single transmitter and receiver with a switch to select the antenna's polarization. Before the introduction of polarimetric radars, systems used one transmitter and one receiver. Dual polarization could be adapted to these systems with a radio-frequency (RF) switch and a suitable antenna. The early dual polarization observations used a mechanical switch to change polarizations. Today, advanced dual transmitter systems are available that can enable full polarimetric mode to sample the four elements of the scattering matrix.

The most common weather radar system architecture that implements STSR mode is shown in panel (b) of Figure 2.5. This architecture uses a signal transmitter whose power is evenly split between the two polarizations using a power divider (e.g., a "Magic T"). Both polarizations have their own duplexer (shown as a circulator) and their own dedicated receiver. Panel (c) of Figure 2.5 shows another single-transmitter, dual receiver architecture. This architecture only implements ATSR mode (in contrast to the architecture in panel [b]). The architecture in panel (b) splits the transmitted power between the two polarizations. Because the architecture in panel (c) doesn't split the power between the two polarizations, it has 3-dB-higher transmit power for the selected polarization and therefore has 3-dB-better sensitivity.

The final architecture that is more commonly found in advanced research radar is shown in panel (d) of Figure 2.5 and has two transmitters and two receivers. This architecture provides flexibility for the radar operators, allowing STSR, ATSR, or variants of these operating modes to be used depending on the application.

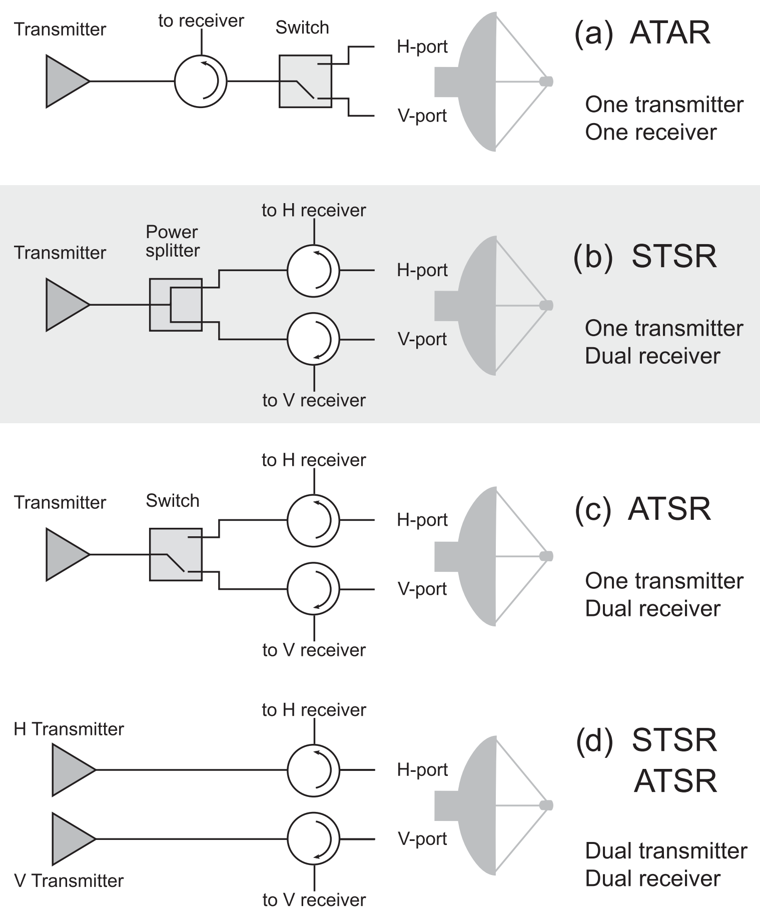

Figure 2.5 Example architectures for linear dual polarization. (Variations on these architectures can be found.) The majority of operational dual polarization weather radar systems operate in STSR using the mode option shown in panel (b). Note that STSR could be implemented with the option shown in panel (d) by using the same transmitted signal and timing for both the H and V transmitters. Similarly, the option shown in panel (d) can also implement ATSR mode with proper selection of the H and V transmitter for each pulse.

2.3 Polarimetric Doppler Weather Radar Measurements

A Doppler weather radar measures the amplitude and phase of echoes for one or more polarizations (i.e., the horizontal and vertical polarization for a dual polarization radar). These measurements are sampled at different ranges in the direction in which the antenna is pointing. The amplitude is proportional to the size, composition, and number of scatterers. The phase is related to the distance to the scatterers, their velocity, and scattering properties. Three common Doppler radar observations are estimated: equivalent reflectivity factor (Z_e), mean Doppler velocity ($\bar{v}$), and Doppler

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spectrum width (σ_v). With dual polarization (using ATSR and STSR modes), three additional variables are estimated: differential reflectivity (Z_{dr}), differential phase shift (Φ_{dp}), and copolar correlation (ρ_{hv}). In ATSR mode, the linear depolarization ratio (LDR) and cross-polar correlation are also available.

2.3.1 Reflectivity (η), Reflectivity Factor (Z), and Equivalent Reflectivity Factor (Z_e)

Radar reflectivity has entered everyday language, just like other weather-related terms. This term (or simply *reflectivity*) is commonly used to designate the intensity of the precipitation in weather radar maps, and it will also be largely used in this book to indicate the reflectivity observed by the radar. However, the term *reflectivity* can be used to reference several different quantities, all of which are related but have subtle differences. The following discussion highlights the various "reflectivities" and, more importantly, the convention that will be followed through the remainder of this book.

There are commonly used terms that are quite distinct, namely, *radar reflectivity*, *radar reflectivity factor*, and *equivalent reflectivity factor*. There is also a difference between the radar's *observed reflectivity* and the precipitation's *intrinsic reflectivity*. The radar reflectivity factor is an attribute of the precipitation within the radar's observation volume. The radar's actual observation is only an estimate of the radar reflectivity factor of this volume, whose value may vary because of system calibration, finite observation times, and attenuation effects resulting from the radar's signal propagating through the atmosphere and precipitation.

Observations of clouds may produce values below −50 dBZ, whereas −20 dBZ provides a lower bound for the lightest drizzle [6], and 10 dBZ is typically associated with very light precipitation (approximately 0.1 mm h^{-1} or less). Heavy rain in convective systems is in general less than 55–60 dBZ. Higher reflectivities such as 60–70 dBZ could be from hail, whereas values well above 70 or 75 dBZ could be from nonmeteorological objects, such as buildings or mountains, typically termed *clutter*.

The radar reflectivity factor (Z) for raindrops can be expressed as

$$\boxed{Z = \frac{\sum_n D_n^6}{V},} \tag{2.6}$$

where D is the raindrop's equivolume diameter, and the summation is taken over all drops within a volume V. To express reflectivity in the common units of mm^6 m^{-3}, we can rewrite eq. (2.6) as

$$Z = 10^{18} \frac{\sum_n D_n^6}{V}, \tag{2.7}$$

where D is in mm, V is in m^3, and the factor 10^{18} is used to convert from m^6 m^{-3} to mm^6 m^{-3}. The basic concept of the D^6 relationship with the reflectivity is shown in Chapter 4.

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The reflectivity factor has units of $\mathrm{mm}^6\,\mathrm{m}^{-3}$ but is typically referred to using a logarithmic scale with units of dBZ:

$$Z\,(\mathrm{dBZ}) = 10\log_{10}(Z). \tag{2.8}$$

Note that there are no radar-dependent parameters used to calculate the reflectivity factor. The radar cross-section (σ) and its wavelength (λ) are not found in the calculation. The reflectivity factor has only two assumptions: the particles are spherical, and the particle sizes are small with respect to a radar's wavelength (the Rayleigh-scattering assumption). The radar reflectivity factor is a measurable property of rain within a volume and can be calculated just like the liquid water content:

$$W \propto \frac{\sum_n^N D^3}{V}, \tag{2.9}$$

which is a measure of the amount of water in a unit volume. Compare this to the radar reflectivity factor, which is proportional to the sixth power of the diameter. This is to say that the radar reflectivity is generally related (although not uniquely; see, e.g., the discussion of Fig. 3.3) to the liquid water content (i.e., the amount of rain).

The radar reflectivity, $\bar{\eta}$, is defined as the radar cross-section (RCS), σ, per unit volume. Intuitively, the concept of the RCS of an object refers to how big the object appears, "electromagnetically" (the concept of the RCS is introduced formally in Chapter 4). The radar reflectivity is defined as the mean RCS per unit volume for all of the scatterers within the volume, V:

$$\boxed{\bar{\eta} = \frac{\sum_n^N \sigma_n}{V}.} \tag{2.10}$$

For a weather radar, when the precipitation particles are much smaller than the wavelength, λ, the RCS of a spherical particle with diameter D is as follows:

$$\sigma(D) = \frac{\pi^5}{\lambda^4}|K|^2 D^6, \tag{2.11}$$

where $|K|^2$ is the dielectric factor, which is a function of the complex-valued relative permittivity (or complex-valued refractive index) of the scatterer (see Section 4.2.1).

The equivalent reflectivity factor (Z_e) is a (nearly) frequency-independent measurement (as opposed to $\bar{\eta}$), which makes it suitable for comparing observations at different frequencies and also for relating the physical properties of the precipitation. For raindrops that are small compared with the radar wavelength (Rayleigh-scattering regime), the radar's equivalent reflectivity factor is the same as the reflectivity factor (for Rayleigh scattering, $Z = Z_e$).

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It is important to note that the value measured by weather radar systems, neglecting attenuation effects, is by convention named the *equivalent reflectivity factor* Z_e, which is related to the radar reflectivity $\bar{\eta}$ following

$$\boxed{Z_e = \frac{\lambda^4}{|K_w|^2 \pi^5} \bar{\eta},} \tag{2.12}$$

where $|K_w|^2$ is the dielectric factor of water. Typically, $|K_w|^2 = 0.93$ for precipitation radars (refer to Section 4.2.1 for details on the dielectric factor for ice and water). Equation (2.12) reduces to eq. (2.6) for Rayleigh scatterers (use eq. [2.10] with the RCS from eq. [2.11]).

In a typical radar, the received power is proportional to $\bar{\eta}$, and therefore we need to make an assumption to convert that to reflectivity, and the convention is to use the dielectric factor of water (independent of what type of scatters are present; e.g., the signals could be coming from ice crystals, buildings, or even insects). The intrinsic reflectivity of a precipitation particle with arbitrary composition (i.e., water, ice, or mixtures) using the corresponding dielectric factor is

$$Z_{\text{intrinsic}} = \frac{\lambda^4}{|K|^2 \pi^5} \bar{\eta}, \tag{2.13}$$

where $|K|^2$ varies according to the hydrometeors, composition.

From the foregoing discussion, the following should be clear:

- The radar reflectivity factor Z is a quantity defined in terms of the particle size distribution, which is the fundamental microphysical property of precipitation (Section 3.1).
- Radar reflectivity $\bar{\eta}$, the radar cross-section per unit volume, is directly related to what the radar measure via the electromagnetic response of the scatterers (Chapter 4).
- The equivalent radar reflectivity factor Z_e is directly related to the radar reflectivity $\bar{\eta}$, but it is converted to a microphysically significant quantity similar to Z using the dielectric factor of water. It is an estimate of the radar reflectivity factor. It is the "reflectivity" variable ordinarily shown on radar displays.

All of the three different "reflectivities" are often used in the literature related to weather radar, and all can be referred to as just "reflectivity." Most frequently when discussing radar observations, we use *reflectivity* to specifically discuss the equivalent reflectivity factor (because this is what the radar is actually measuring, neglecting attenuation effects), but care should be taken to interpret the context of the discussion.

For the remainder of this book, when referring to reflectivity observations, we are specifically talking about the radar's measured estimate of the equivalent reflectivity factor for the horizontal polarization. This estimate often includes the effects of path-integration signal attenuation. This may also be referred to as *observed reflectivity* or *measured reflectivity*. If techniques to estimate and remove the effects of signal attenuation are used for the reported estimates, they are referred to as *corrected reflectivity*.

The corrected reflectivity is the estimate of the equivalent reflectivity factor for the horizontal polarization.

2.3.2 Doppler Velocity ($\bar{v}$) and Spectrum Width (σ_v)

For coherent radars (systems for which there is a known relationship between the phase of the transmitted and received pulses; see Section 2.6), the mean Doppler velocity and the Doppler spectrum width can also be measured. These additional measurements complement the microphysics information provided by the radar reflectivity with the capability of sensing the atmospheric dynamics.

The mean Doppler velocity ($\bar{v}$) is determined by measuring the Doppler frequency shift of the scatterers (Section 5.3.2) that results from a weighted average of the scatterers' velocity within the radar resolution volume. It is important to note that the average is weighted by the reflectivity and the observation volume, which takes into account both the antenna's illumination pattern and the transmitted pulse's "shape" (Section 2.4). Therefore, the resulting measured Doppler velocity is primarily influenced by the velocity of the larger particles within the sampled volume. For radars scanning at low-elevation angles, the Doppler velocity gives the radial component of the wind velocity vector. Observations at the vertical incidence provide information about the falling velocity of particles and can be useful, for example, to discriminate between rain and snow (Section 3.6.1).

The Doppler spectrum width (σ_v or sometimes w) is a measure of the dispersion of velocities within the resolution volume and can provide information about wind shear and turbulence. This is used in particular to help in the decision process for severe thunderstorms and tornado warnings.

2.3.3 Polarimetric Radar Measurements: Z_{dr}, Φ_{dp}, ρ_{hv}

Similar to Doppler radar observations, polarimetric radar observations rely on measurements of the amplitude and phase of the signal, although in this case from two orthogonal (horizontal and vertical) polarizations. The terms *observations* and *measurements* are used interchangeably. Both the propagating and the scattered waves can be described by their complex amplitudes. Several radar variables can be derived based on the magnitude of the backscattered wave at the horizontal and vertical polarizations, their phase, or both.

The most straightforward additional measurement that a polarimetric radar can provide is the differential reflectivity, Z_{dr}. It is the ratio of the equivalent reflectivities at horizontal and vertical polarizations (Z_h and Z_v, respectively). Note that Z_{dr}, just like reflectivity, is often expressed in logarithmic scale (dB):

$$\boxed{\mathrm{Z_{dr}}\,(\mathrm{dB}) = 10\log_{10}(\mathrm{Z_{dr}}) = Z_h\,(\mathrm{dBZ}) - Z_v\,(\mathrm{dBZ}).} \tag{2.14}$$

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Throughout this book, the symbol Z_{dr}, just like Z_h, represents the variable in linear units, whereas when we refer to, for example, Z_{dr} in dB, this will be explicitly noted in the text, or the corresponding log transform (dB scale) will be added in the figure axis.

Z_{dr} reveals information about the shape, composition, and density of the hydrometeors. For any given particle shape, Z_{dr} may vary depending on the particular composition (i.e., water, ice, or mixed phase), as a consequence of the different relative permittivity of the material (Section 4.2). For example, oblate raindrops will have a larger Z_{dr} than solid ice particles with the same shape (see, e.g., Fig. 4.10). Low-density aggregates, such as snowflakes, will have even smaller Z_{dr} values (typically close to 0 dB) because of the small effective relative permittivity of diluted ice–air mixtures (Fig. 4.21).

Z_{dr} reveals properties of the precipitation medium based on the ratio of the backscattered signal strength at the two orthogonal polarization channels. Analogously, the differential phase shift Φ_{dp} (i.e., the phase difference between the received signals at horizontal and vertical polarization) can be highly informative about the composition of the particles and concentration along the propagation path.

The copolar correlation coefficient between the horizontal and vertical polarization signals (ρ_{hv}) provides a measure of how similar the horizontal and vertical scattering cross-sections of the objects are. A higher value of ρ_{hv} (close to 1) is typically observed in rain, as a result of the consistency of the raindrops' shape and fall behavior. Lower values generally indicate a greater variability in the scatterers' population; for example, lower values are normally observed in the melting layer (coexistence of liquid, solid, and mixed-phase particles with varying shape) or with non-meteorological echoes, such as tornado debris, insects, and birds (Section 7.3.5).

Finally, for radars that transmit one polarization at a time and receive both the copolar and cross-polar signals (e.g., ATSR systems; panel [c] of 2.5), an additional variable can be estimated. Similar to Z_{dr}, the magnitude of two received polarization signals are compared, although in this case the ratio is between the cross-polar and the copolar power. For example, if the radar transmits a horizontal polarization, the LDR is given by the ratio of the power received on the vertical polarization (cross-polar signal) to the power received on the horizontal polarization (copolar signal). Although the measurement is realized in the same way as for Z_{dr} (two power levels are measured by means of two different receivers connected to the antenna), the significance of the LDR observations is different. Similar to ρ_{hv}, LDR can be a good indicator of regions where a mixture of precipitation types occurs (typically when the radar resolution volume is populated with randomly oriented particles). In other instances, LDR can be used to detect a preferential orientation of the hydrometeors, such as in the presence of strong electric fields in thunderstorms (see discussion in Sections 4.9 and 8.5.3).

2.4 Observation Geometry

The radar's observation geometry is most naturally described using a modified form of spherical coordinates: range (r), azimuth (ϕ), and elevation (θ). The radar is located

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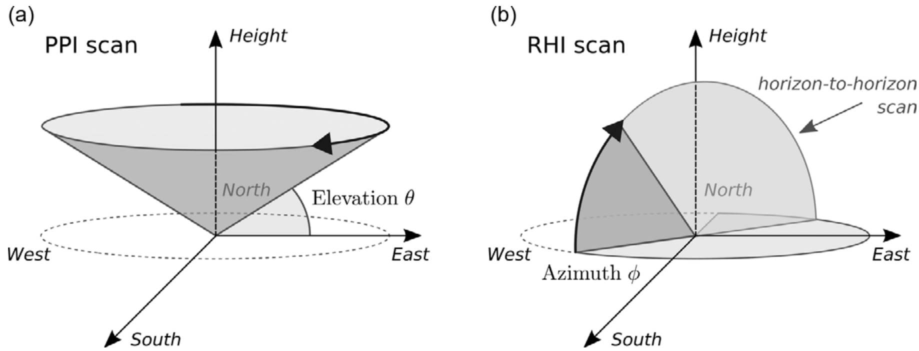

Figure 2.6 Illustration of weather radar elemental scan modes. The elevation angle, θ, is measured above the horizontal plane. The azimuth, ϕ, is measured clockwise from north (north is $\phi = 0°$; east is $\phi = 90°$). In PPI mode (a), the antenna completes a full rotation in the azimuth, ϕ, at a fixed elevation angle, θ. The PPI scan makes observations on a conical surface. In RHI mode (b), the antenna moves in elevation, θ, with the azimuth angle, ϕ, fixed. The RHI collects data on a vertical plane that may extend across the zenith to obtain a "horizon-to-horizon" scan.

at the center of the sphere and observes outward. For scanning radar systems, the radar can steer the radar's beam in both the azimuth and the elevation. The radar's transmitted power travels radially away from the radar's antenna, and the echoes from precipitation are received along the same radials. For a fixed-pointing radar system, there are no scanning capabilities; the radar is pointed along a single radial, typically pointing vertically, in the zenith direction.

For scanning radar systems, different scanning strategies have been adopted, but all are typically composed of the range-height indicator (RHI) and plan position indicator (PPI) scans, shown in Figure 2.6. Multiple RHI or PPI scans can be combined to scan a volume. Similarly, the extent of the RHI or PPI scan can be limited to cover only a limited sector of interest.

For radar observations, the cardinal directions and Cartesian coordinates are related as follows:

- The positive x-axis is east, and the negative x-axis is west.
- The positive y-axis is north, and the negative y-axis is south.
- The positive z-axis is the zenith, and the negative z-axis is the nadir (which is not encountered for ground-based radar).

The radar's azimuth angle starts pointing north ($\phi = 0°$) and increases as it rotates clockwise with east ($\phi = 90°$), south ($\phi = 180°$), and west ($\phi = 270°$). The radar's elevation angle starts pointing parallel to the ground ($\theta = 0°$) and increases in angle toward its zenith ($\theta = 90°$). For radars with the capability of scanning from horizon to horizon, the elevation angles are sometimes represented as $0° < \theta \leq 180°$, where elevation angles $\theta > 90°$ correspond to azimuth angles $\phi + 180°$. Section 2.4.5 discusses the conversion between the radar and Cartesian coordinates in more detail.

2.4.1 Radar Ranging, *r*

In free space, the speed of light is $c = 299,792,458\ \mathrm{m\,s^{-1}}$. When determining the distance that a radar's signal travels, the speed of light in Earth's atmosphere is assumed as the speed of light in a vacuum. At this speed, it takes light 33.356 μ s for the radar's energy to travel 1 km.

Consider a monostatic radar detecting a target at a distance of $r = 1$ km. The transmitted pulse takes 33.356 μ s to travel from the radar to the target. A portion of the transmitted pulse's energy is scattered by the target, and some of this scattered energy is directed back at the radar (the energy is said to be *backscattered*). The backscattered energy then takes another 33.356 μ s to travel from the target back to the radar, where it is received. The radar's signal travels $2r$. The total round-trip travel time of the radar's signal is

$$\Delta t = \frac{2r}{c}. \tag{2.15}$$

In practice, the radar system measures the time delay between the transmitted signal and the received signal, Δt, and therefore the radar's range to the target can be calculated:

$$r = \frac{\Delta t \cdot c}{2}. \tag{2.16}$$

A simple relationship to remember is that every 1 μ s of elapsed time is equivalent to 150 m of radar range. Since the introduction of the radar, the technology for accurate time measurements (and therefore accurate range measurements) has improved significantly and is an important technical aspect of modern radars.

2.4.2 Range Resolution, Δr

In pulsed radar (the most common weather radar), the radar signal is transmitted for a short period of time. The ranges covered by the pulse can be calculated from the relative difference in the starting and ending times of the transmitted pulse (see eq. [2.16]). For a pulsed radar whose pulse duration is T_{tx}, the range resolution is calculated as

$$\Delta r = \frac{cT_{tx}}{2}. \tag{2.17}$$

For pulsed radar systems, the pulse width and bandwidth are inversely related as $T_{tx} = 1/B$.

The radar's range resolution can be rewritten using the radar signal's bandwidth as

$$\boxed{\Delta r = \frac{c}{2B}.} \tag{2.18}$$

For radars that utilize pulse compression techniques, the range resolution is still determined by the radar signal's total bandwidth (but the pulse length T_{tx} is no longer related to range resolution because $B \neq 1/T_{tx}$). Pulse compression is an advanced

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topic and is discussed in Section 6.5. Equation (2.18) is generally more appropriate for relating the radar's operating parameters to the range resolution.

When determining the range resolution, the bandwidth, B, is the effective bandwidth of the transmitted signal convolution with the receiver's filter. The pulse-shaping effects of the transmitter and receiver set the resolution of the radar's signal. After convolution, the -6 dB power threshold determines the range resolution for estimation of the V_6 observation volume. This may be approximated as the half-power level (i.e., -3 dB) of the transmitter's pulse (which assumes a receiver filter with similar bandwidth is used).

2.4.3 Observation Volume, V_6

Although the first radars were introduced to detect hard targets such as aircraft, the applications for radar have vastly extended beyond detecting single targets, which are often referred as *point targets*. The applications for radars have expanded to look at surfaces from airborne and spaceborne radars, as well as volumes of scatterers that include our particular application: precipitation and weather. Weather radars observe a class of scatterers broadly referred to as *volume targets*. For weather radar, the formulation of the radar equation must be updated to accommodate a volume of scatterers.

Atmospheric systems of clouds and precipitation can extend over large areas and heights, whereas at the smallest microphysical scales, these are individual ice crystals or water drops that exist among a vast number of other particles and fill the volume. The radar's antenna pattern and the range resolution of the radar's signal, Δr, and the range from the radar, r, all determine the shape and size of the radar's observation volume, V_6 (often referred to simply as V), that connects to the received signal at a given time. Figure 2.7 illustrates the range resolution volume and its dependencies.

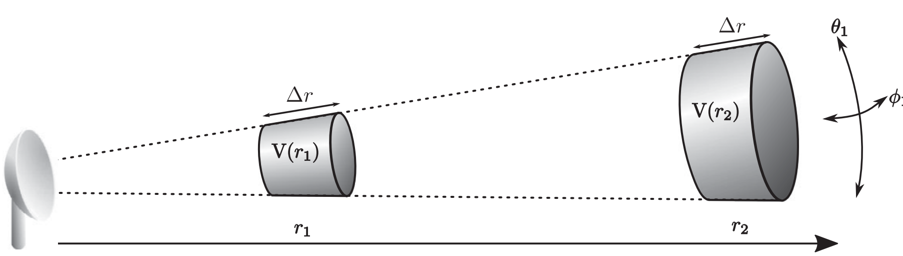

Figure 2.7 The radar's observation volume is a function of the range resolution, Δr; the radar range, r; and the antenna pattern. The one-way antenna pattern's half-power beamwidth for elevation, θ_1, and azimuth, ϕ_1, are used. The radar range is from the radar to the center of the range resolution volume. The range resolution remains constant for all ranges but, as the range from the radar increases, so does the distance covered in the azimuth and elevation directions.

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The transmitted pulse's amplitude with respect to time determines how the radar volume is illuminated in the range dimension. The orthogonal dimensions of the volume (in the spherical coordinate system) are the azimuth and elevation angles, which are determined by the transmitting antenna's beam pattern. Within the weather radar equation for a monostatic system, the transmitting and receiving antennas' 3-dB, one-way azimuth and elevation beamwidths are used along with the radar's range to determine the observation volume [7]:

$$V_6 = (\Delta r)\frac{\pi\theta_1\phi_1}{8\ln 2}r^2, \tag{2.19}$$

where θ_1 and ϕ_1 are the one-way, -3-dB antenna beamwidths in the elevation and azimuth, respectively. The observation volume, denoted V_6, is a disk, assuming a circularly symmetric beam, whose angular and range extents represent the -6-dB two-way beamwidth of the monostatic antenna (the -3-dB beamwidth of the transmitting antenna and the -3-dB beamwidth of the receiving antenna). The V_6 volume represents the volume where the majority of the radar signal's power is focused at an instance in time, using the traditional 3-dB convention commonly used in electrical engineering to represent beamwidths and filter bandwidths. As shown in Figure 2.7, as the range from the radar increases, the size of the radar volume increases as a result of the increasing width and height of the disk in the azimuth and elevation dimensions. The range extent of the volume is determined by the range resolution, which remains fixed as a function of range.

2.4.4 The Beamwidth

It is clear from Figure 2.7 that as the range increases, the width and height of the observation volume increase. The angular beamwidths are determined by the antenna and therefore constant (for most mechanical antennas), but the arc distance spanned by the radar volume scales with the radar's range. In the elevation direction, the distance spanned by the beam is

$$b_\theta = 2r\sin\left(\frac{\theta_1}{2}\right) \approx r\theta_1, \tag{2.20a}$$

and the beam's width in the azimuth direction is

$$b_\phi = 2r\sin\left(\frac{\phi_1}{2}\right) \approx r\phi_1. \tag{2.20b}$$

The approximations shown adopt the small-angle assumption, $\sin x \approx x$. The beamwidths b_θ and b_ϕ both have the same units as r (typically meters). The beamwidths only consider the size of the beam as a result of the antenna and do not take into consideration the broadening effects when the radar is scanning.

For a Gaussian approximation of the antenna pattern, which is common for weather radars with reflector antennas, the cross-sectional area can be approximated as an ellipse:

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$$A = \pi \frac{b_\theta b_\phi}{4} = \pi \frac{r^2 \theta_1 \phi_1}{4}. \tag{2.21a}$$

From eq. (2.19), the pattern-weighted antenna beam area, after removing the range resolution contribution from V_6, is

$$A_{\text{beam}}^{(\text{weighted})} = \frac{\pi \theta_1 \phi_1}{8 \ln 2} r^2. \tag{2.21b}$$

Comparing the weighted pattern's area to the uniform cross-section area reveals that:

$$A_{\text{beam}}^{(\text{weighted})} = \frac{A}{2 \ln 2}. \tag{2.21c}$$

The Gaussian antenna pattern's weighting scales the total echo power from the observation volume by a factor of $1/(2\ln 2)$, which is found in the formulation of the weather radar equation.

2.4.5 The Beam Height and Ground Distance

The weather radar observations are given in the spherical coordinates of azimuth (ϕ), elevation (θ), and range (r). Although the spherical coordinates are natural from the perspective of how the radar observes, it is not typically intuitive for users when trying to place the location of the observation relative to different towns or cities. (Rectilinear grids are also natural for comparing multiple sensors, numerical weather models, and some processing algorithms.) Conversion to Cartesian coordinates for other applications or users is done through geometric relations:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \cos\left(\pi/2 - \phi\right) \cos(\theta) \\ r \sin\left(\pi/2 - \phi\right) \cos(\theta) \\ r \sin(\theta) \end{bmatrix}. \tag{2.22}$$

The zenith direction is $+z$, west is $-x$, and east is $+x$; south is $-y$, and north is $+y$ (refer to Fig. 2.6). For observations close to the radar, the surface can initially be assumed as a flat surface with little error. As the range from the radar increases, however, the error in the estimated height above the surface can become significant if Earth's curvature and atmospheric refraction are neglected.

In the troposphere, the index of refraction (n) decreases with increasing altitude (h) [8]. Gradients in temperature, pressure, and water vapor within the atmosphere all result in variations in the index of refraction. From Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{2.23}$$

the angle of propagation of the electromagnetic wave changes as a result of differences in the layer's index of refraction. The beam-height model assumes a nominal gradient in Earth's troposphere, $\frac{dn}{dh} = -\frac{1}{4a}$, where a is the radius of Earth. The implies that the radar beam does not travel along a straight line but is bending toward the surface as it propagates.

For estimating the beam height, it is useful to adopt a modified Earth's radius such that the ray path is straight, as illustrated in Figure 2.8. Earth is slightly oblate, and its

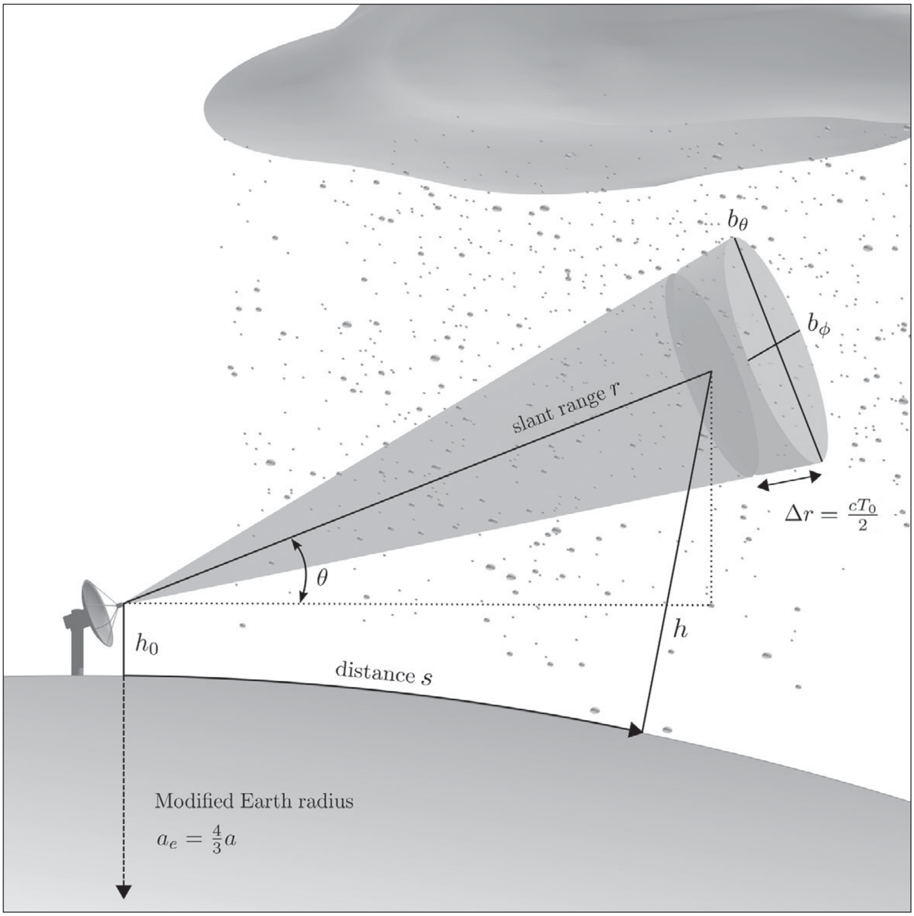

Figure 2.8 Schematic illustration of the propagation path and radar resolution volume.

radius varies from approximately 6357 km at the poles to 6378 km at the equator. For the purposes of estimating the beam's height, the variation of Earth's radius is small. The spherical Earth model with Earth's nominal radius of $a = 6371$ km is frequently assumed. Using the average gradient of the refractive index, $\frac{dn}{dh} = -\frac{1}{4a}$, leads to an effective radius of $a_e = \frac{4}{3}a$.

With simple trigonometric considerations, the equation for the height of the radar beam in standard atmosphere can be written as follows (see, e.g., Doviak et al. [9]):

$$h(r,\theta) = h_0 + \left(r^2 + a_e^2 + 2r\,a_e\,\sin\theta\right)^{\frac{1}{2}} - a_e, \tag{2.24}$$

where h_0 denotes the height of the radar antenna above the surface. The great-circle distance s (the distance along Earth's surface) is obtained with the law of sines, as follows:

$$s(h,\theta) = a_e \sin^{-1}\left(\frac{r\cos\theta}{a_e + h}\right). \tag{2.25}$$

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The ground relative distance and height above a spherical Earth's surface (i.e., neglecting topography) can then be approximated as

$$\begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = \begin{bmatrix} \cos\left(\pi/2-\phi\right) a_e \sin^{-1}\left(\frac{r\cos\theta}{a_e+h}\right) \\ \sin\left(\pi/2-\phi\right) a_e \sin^{-1}\left(\frac{r\cos\theta}{a_e+h}\right) \\ h = h_0 + \left(r^2+a_e^2+2r\, a_e \,\sin\theta\right)^{\frac{1}{2}} - a_e \end{bmatrix}. \tag{2.26}$$

Other useful relations are given by Doviak et al. [9] to calculate the slant range from the elevation angle:

$$r(s,\theta) = \frac{(a_e+h_0)\sin(s/a_e)}{\cos\theta}. \tag{2.27}$$

or the elevation angle from the beam height and great-circle distance:

$$\theta(h,s) = \tan^{-1}\left[\frac{H\cos(s/a_e)-a_e}{H\sin(s/a_e)}\right]; \quad H = a_e + (h-h_0). \tag{2.28}$$

As atmospheric conditions and the vertical gradients of the index of refraction vary from the nominal values assumed by the beam-height model, the actual beam height can change. Deviations in the propagating path from the model are typically referred to as *anomalous propagation*. Refraction of the atmosphere can vary the radar beam's path. A notable example in radar observations can be found in time lapses of scans at low-elevation angles around the time the sun sets: the presence of reflections from the ground and building can vary as the temperature gradient in the lower troposphere varies.

2.5 Radar Equation

The weather radar system transmits a signal and receives the echoes from scatterers in the troposphere. The radar equation is used to calculate the received power for a given radar system's and scatterer's parameters. The radar equation is an important tool to estimate the radar system's performance.

First, consider the objects that the radar will observe: the scatterers, such as raindrops, hail, or ice crystals. Generally speaking, the larger the size of the scatterer, the easier it is for the radar system to detect it. For smaller scatterers, a higher-sensitivity radar system is required to detect its weaker echo. With this simple concept of "smaller" and "larger" scatterers, when considered in the context of a radar, the size of the scatterer is formally referred as its RCS. Specifically, the RCS is directly related to the ratio of scattered power to the incident power, which is a measure of how much of the incident power (from the transmitter) is scattered back toward the receiving antenna. The RCS is given by

$$\sigma = \lim_{r\to\infty} 4\pi r^2 \frac{S_s}{S_i} \tag{2.29}$$

where r is range, and S_i and S_s are the incident and scattered power densities, respectively (in watts per square meter). In the case of a monostatic radar system, the scattered direction is opposite the incident direction (i.e., $\hat{s} = -\hat{\imath}$). (As an example, in Fig. 2.2, the direction of the incident wave's propagation, $\hat{k}_i$, would be $\hat{\imath}$.) In eq. (2.29), the limit as r approaches infinity is meant only to imply that the scatterer and the observing antennas are far apart (more accurately, the range is greater than the antenna's far-field range). The RCS is not dependent on the range.

The RCS, σ, is an area with units of meters squared and is discussed in greater detail in Chapter 4. The RCS corresponds to the apparent area of the target, assuming the scattered power is isotropic (scattered equally in all directions). The RCS's area and the target's physical area are not necessarily the same. The RCS can also depend on the radar's frequency. Typically, the larger the physical size of the object, the larger the RCS. For example, hail has a larger RCS compared with cloud water droplets. For more complex targets, the RCS can change significantly as the incidence and scattering angles vary.

2.5.1 Single-Target Radar Equation

The single-target radar equation (also called the *point-target radar equation*) is a simple concept used to calculate the echo power at the radar's receiving antenna for a target with a known RCS. For any geometry, the scatterer can be treated as a single point in space with a fixed RCS, σ. The radar equation is used to calculate the received power for a monostatic radar system, where the transmitter's and receiver's antenna are the same. Similarly, it can also be used for bistatic radar systems where the transmitter's and receiver's antennas are in different locations (and therefore at different ranges).

For a given transmit power (P_t), to determine the received power density (S_r), eq. (2.29) can be rearranged to calculate the scattered power density (at the scatterer). Assuming the transmitting antenna is an isotropic source (i.e., the transmit antenna's gain $G_t = 1$), the wave's power density incident on the scatterer at range r_t is

$$S_i = \frac{P_t}{4\pi r_t^2}, \tag{2.30}$$

where $1/(4\pi r_t^2)$ is the surface area of a sphere with radius r_t. The transmitting antenna focuses the transmitted power, which effectively increases the power density incident on a scatterer. If the transmitting antenna is not isotropic, the total transmitted power is confined to a smaller surface area, which increases the power density. The increased incident power density due to the transmit antenna's gain (G_t) is included as

$$S_i = \frac{P_t G_t}{4\pi r_t^2}. \tag{2.31}$$

The fraction of the incident power that is reflected (or scattered) is determined by the RCS. The scattered wave's power (at the scatterer) is

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$$P_s = S_i \sigma = \frac{P_t G_t}{4\pi r_t^2}\sigma, \tag{2.32}$$

where P_s has the unit of watts (recall that S_i has the unit of watts m^{-2}, and σ has the unit of m^2). The scatterer, in effect, becomes a second transmitting source whose power, P_s, propagates away from the scatterer. At the radar receiver, located at range r_r from the scatterer, the scattered wave's power density is

$$S_r = \frac{P_s}{4\pi r_r^2} = \frac{P_t G_t}{4\pi r_t^2}\sigma\frac{1}{4\pi r_r^2}. \tag{2.33}$$

Similarly, the receiver antenna acts to collect the scattered signal using its large surface area. The received power is increased as a result of the scattered power density being integrated over the antenna's effective area. To calculate the received power, multiply by the antenna's effective area (A_e; see eq. [2.42]) in square meters to give

$$P_r = S_r A_e = \frac{P_t G_t}{4\pi r_t^2}\sigma\frac{A_e}{4\pi r_r^2}. \tag{2.34}$$

The receiving antenna's gain is related to the antenna's effective area, A_e, following

$$G_r = \frac{4\pi A_e}{\lambda^2}. \tag{2.35}$$

Substituting G_r into eq. (2.34) gives the point-target radar equation:

$$P_r = \frac{P_t G_t}{4\pi r_t^2}\sigma\frac{\lambda^2 G_r}{4\pi}\frac{1}{4\pi r_r^2} \tag{2.36a}$$

$$= \frac{\lambda^2 P_t G_r G_t}{(4\pi)^3 r_r^2 r_t^2}\sigma. \tag{2.36b}$$

For a monostatic radar system, where the same antenna is used for both transmitting and receiving (i.e., $G = G_r = G_t$ and $r = r_r = r_t$), the point-target radar equation simplifies to the well-known form:

$$\boxed{P_r = \frac{\lambda^2 P_t G^2}{(4\pi)^3 r^4}\sigma.} \tag{2.36c}$$

It should also be noted that the antenna gain used for the point-target radar equation is the gain in the direction of the scatterer being observed. The antenna's gain varies as a function of angle, and therefore the actual antenna gain for the observed scatterer is not necessarily the antenna's boresight gain (the boresight gain is its peak gain). Note that, approximations for a parabolic antenna's boresight gain as a function of its diameter and wavelength, or its beamwidth, are given later by eqs. (2.43) and (2.44), respectively.

The signal's powers encountered in a radar system can span many orders of magnitude, from femtowatts (10^{-15}) for weak echoes to megawatts (10^6) for radar transmitter power levels. As such, the radar signal's power is frequently reported using a logarithmic scale with units of dBm or dBW. The power in dBm is the power ratio with respect to 1 milliwatt, in decibels. Similarly, dBW is the power ratio with respect to 1 watt. (Note that 1 dBW = 30 dBm.)

2.5.2 Weather Radar Equation

Precipitation is a collection of hydrometeors, which can each be treated as point targets from the radar's perspective. A typical approximation assumes that because of the low-volume fraction in space, there are no scattering interactions between hydrometeors (e.g., raindrops or snowflakes), and therefore the received power from each individual particle can simply be added together. This implies that the received power from a precipitation-filled radar volume is the summation of all particles' point-target responses within the radar volume. For the monostatic radar system, this is

$$P_r = \sum_V P_t(r) \frac{\lambda^2 G^2(\theta, \phi)}{(4\pi)^3 r^4} \sigma_n, \tag{2.37}$$

where σ_n indicates the RCS of the n^{th} particle, and V is the full spatial volume illuminated by the pulse's range resolution around range r (typically represented in spherical coordinates that correspond to the radar's azimuth, elevation, and range). The transmit power, P_t, is shown as a function of range to select the extent in the range of the radar volume (see Fig. 2.7). In practice, the extent over which the summation is carried out is the V_6 radar volume (see eq. [2.19]). It is also typical to characterize the scatterers within the volume as uniformly distributed (the particle sizes may vary according to a distribution, but the same distribution applies throughout the volume). With this, the total RCS of the radar's resolution volume (with units m^2) can be described by its reflectivity, $\bar{\eta}$, which is the mean volumetric RCS with units m^2 m^3. The total RCS and radar reflectivity are then related as $\sigma = \bar{\eta} V_6$. The concept of volumetric reflectivity is used to convert the discrete summation over a large number of precipitation particles to an integral. Using the volume-averaged mean reflectivity, the received power can be written as an integral:

$$P_r = \iiint_V P_t(r) \frac{\lambda^2 G^2(\theta, \phi)}{(4\pi)^3 r^4} \bar{\eta} \, dV. \tag{2.38a}$$

The antenna's pattern and the radar's range resolution act to spatially select the volume defined by eq. (2.19). The monostatic weather radar equation gives the received power:

$$P_r = P_t \frac{\lambda^2 G^2}{(4\pi)^3 r^4} \left(\frac{c T_{tx}}{2} \frac{\pi \theta_1 \phi_1}{8 \ln 2} r^2 \right) \bar{\eta}. \tag{2.38b}$$

The radar's observation volume is centered at range r. The transmit pulse's duration, T_{tx}, is used to define the pulse's range extent for integrating the reflectivity $\bar{\eta}$, and G is the antenna's boresight (peak) gain. Comparing the monostatic radar equations for the point-target and volume weather targets (eq. [2.36c] and [2.38b], respectively), the

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most significant difference is the received power's dependence on range. The point-target radar equation has $1/r^4$ dependence, whereas the weather radar equation has $1/r^2$ dependence.

The signal P_r is processed by the radar receiver's electronics. The receiver filter's output power is related to the observed volume's equivalent reflectivity factor by

$$P_o = G_{rx}\left(\frac{cT_{tx}}{2}\right)\left(\frac{\lambda^2 P_t G^2}{(4\pi)^3}\right)\left(\frac{\pi\theta_1\phi_1}{8\ln 2}\right)\left(\frac{\pi^5|K_w|^2}{\lambda^4 r^2}\right)\frac{Z_e}{10^{18}}. \tag{2.39a}$$

The 10^{18} factor converts from m^6 to mm^6 to give the appropriate unit for Z_e, which is mm^6m^{-3}. The receiver gain, G_{rx}, includes the gains (or losses) in the receiver and signal-processing paths. In practice, the majority of the terms are constant for a radar system, and as such, the equation for the equivalent reflectivity factor is simplified to

$$P_o = Z_e\frac{C}{r^2}. \tag{2.39b}$$

The equivalent reflectivity factor can be calculated from the received power and range as

$$\boxed{Z_e = \frac{P_o r^2}{C},} \tag{2.39c}$$

where the weather radar constant is

$$\boxed{C = \frac{G_{rx}P_t G^2\pi^3|K_w|^2 cT_{tx}\theta_1\phi_1}{10^{18}\lambda^2 1024\ln 2}.} \tag{2.39d}$$

Note that C assumes r is in meters, but some formulations may use range in kilometers. If the range in eq. (2.39c) is kilometers instead of meters, an appropriate scale factor (i.e., 1 km = 1000 m) must be included. If the range is presented in kilometers instead of meters, 10^{18} becomes 10^{24} in eq. (2.39d).

For dual polarization weather radar, the radar systems typically are designed to provide nearly identical performance at both polarizations. This would imply that the same radar constant, C, could be used for both the horizontal and vertical polarizations. In practice, there can be slight differences in performance between the two polarizations which can bias some of the radar measurements. While these differences are small, this results in different radar constants for each polarization state (i.e., C_h and C_v for the horizontal and vertical polarizations, respectively). As we'll discuss in Chapter 7, these differences can sometimes be included as calibration factors to apply to an otherwise-common radar constant.

2.6 Radar System Components

A block diagram of a dual polarization weather radar system is shown in Figure 2.9. This is an example of a weather radar that operates in the STSR mode, using one transmitter whose power is split between the horizontal and vertical

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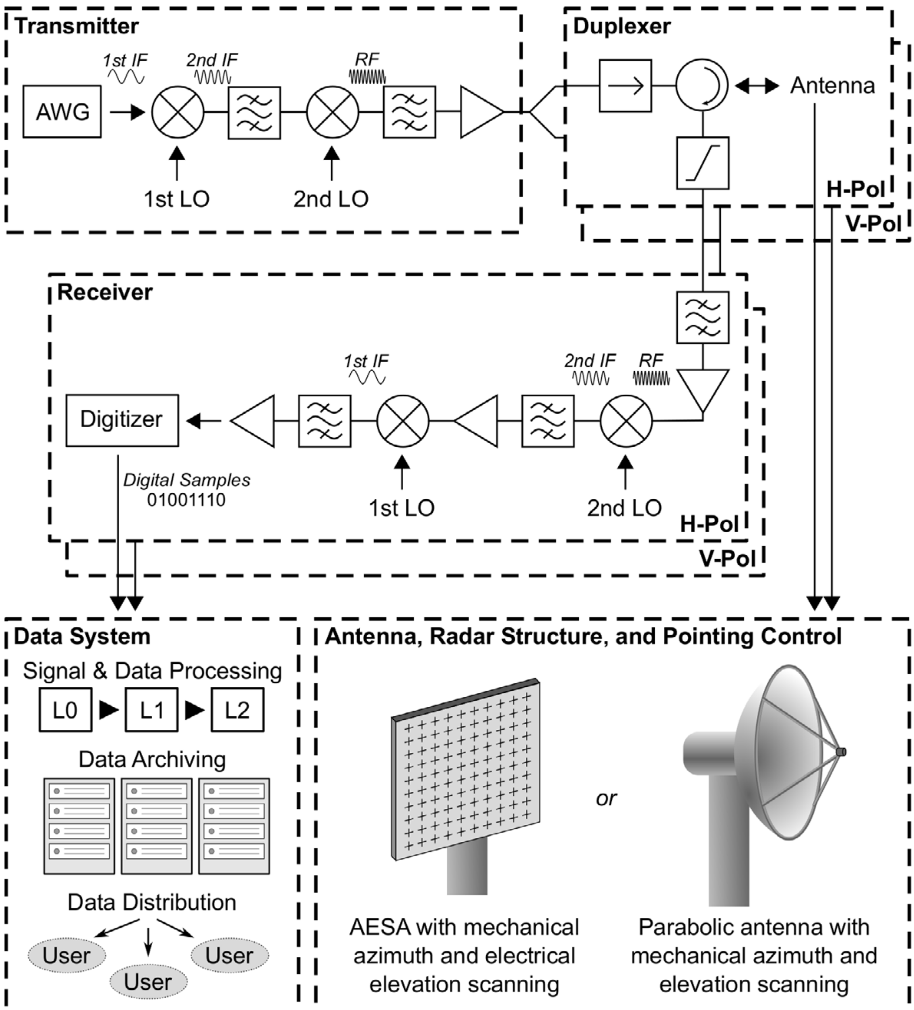

Figure 2.9 Block diagram of an STSR-mode dual polarization weather radar system (see the architecture in panel [b] of Fig. 2.5). Two antenna and scanning systems are shown: a mechanically scanning parabolic antenna and an active electronically scanned array (AESA), electronically scanning in the elevation and mechanically scanning in azimuth. Other antenna and scanning configurations could also be used for this block diagram. The relative frequencies of the radar's signal as it propagates through the radar are shown before and after the respective mixer stages. A two-stage up- and down-conversion is shown in this example.

polarizations (h-pol and v-pol, respectively). For the system shown, each polarization has a dedicated duplexer and receiver. The duplexer provides an interface between the radar electronics and the antenna. The antenna and pointing-control system could be mechanical scanning, electrical scanning, or a combination of both. The digitized data are communicated to the data system, where they are processed into data products

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(as reflectivity, velocity, rainfall rate, etc.). The processed data are archived for future use but are also distributed to radar operators, meteorologists, and other data systems.

The local oscillators (LOs) shown in Figure 2.9 are used to convert from the low-frequency signal of the transmitter's arbitrary waveform generator (AWG) to the radar's RF signal. An AWG is one of many techniques for generating the transmitter's signal. To ensure that only the frequency of interest is selected, it is sometimes necessary to use intermediate frequencies (IF), generated with multiple mixer stages and appropriate filters, to implement the frequency up- and down-conversion in the transmitter and receiver, respectively. With recent advances in high-speed analog-to-digital and digital-to-analog converters, directly sampling the RF signal, or only a single stage of analog frequency conversion, may be considered. This diagram and the explanation are meant to provide a high-level description of how the radar's RF signals are generated.

To estimate the Doppler velocity and the covariance between the dual polarization echo signals, the system's phase relationship between the transmitter and receiver must be known or measured so that it can be removed from the observations. When the phase of the received signal is known, or is accurately measured with respect to the transmitted signal, the radar system is said to be "coherent." The transmitter and receiver can be made coherent by phase locking all of the frequency sources to a common clock (e.g., a stable local oscillator [STALO]). When the transmitter and receiver are phase synchronized by design, this is known as a *coherent-on-transmit* design. Alternatively, instead of controlling the phase of the transmitter, the receiver can be used to sample the transmitted signal and estimate its phase for each pulse. This estimated phase can be used to correct the received signal on a pulse-by-pulse basis. This is known as *a coherent-on-receive design*. For magnetron-based Doppler radar systems, the transmitter's phase cannot be controlled, and therefore a coherent-on-receive design must be used. This has also become a standard technique for use with coherent transmitters to correct for second-order phase drifts in the radar system.

2.6.1 Electronically Scanned Arrays

The active electronically scanned array (AESA) antenna is typically designed as an integral system element rather than an interchangeable antenna component. The AESA radar is also commonly referred to as a *phased-array radar*. AESAs can provide a new paradigm for weather radar operations. With these new operational capabilities come additional considerations for system designs. Radar systems using an AESA antenna are highlighted as electronically scanned arrays or phased arrays to differentiate their enhanced capabilities compared with more traditional radar systems using reflector antennas.

Mechanically scanning systems are limited by the inertia of the antenna and the pointing system's solid structure. The radar's observation volume is set by the antenna pattern (which is fixed) and the direction in which the antenna is pointing. The pointing is mechanically controlled and therefore limited by the speed at which the antenna can change positions. For large S-band and C-band radar systems, the inertia of the antenna systems requires significant structures to withstand the large forces to move the antenna. Generally, mechanically scanning radars move continuously in the azimuth direction and step through multiple elevations (sometimes with research radars, they move continuously in elevation and step through the azimuth). This presents a trade-off between the time spent observing a specific radar volume and the total coverage that can be achieved within a given time period.

With electronically scanned systems, the observation volume is selected by electronically “steering” the antenna’s pattern. The antenna’s pattern is no longer fixed but is a controllable parameter in the radar’s operation. The antenna’s pattern and the pointing control are now combined together as one system. Because the pointing of the antenna beam is electronically controlled, the AESA does not have mechanical restrictions on how long it takes to position the antenna beam in the desired location, and the antenna’s beam can remain fixed on the same volume (as opposed to scanning across the volume, which results in a changing volume over time).

The AESA is composed of a number of smaller antenna elements whose signals are combined together to synthesize the desired antenna pattern. Numerous system designs are possible, and these continue to evolve with the availability of new technologies. Electronically scanning radar systems can be implemented in a number of ways with a combination of electrical and mechanical scanning or only electrical scanning. The AESA can be designed to scan in one direction only (e.g., in elevation) or both azimuth and elevation. More complex AESAs and receiver systems can simultaneously sample multiple beams at one time. With individually digitized elements (or subarrays of elements), the phased array can beamform on receive to sample multiple discrete radar volumes at once. Simpler forms of the AESA provide a single beam that more closely approximates the mechanical antenna. Figure 2.10 shows

Figure 2.10 An X-band radar system using a planar electronically scanned antenna. Image courtesy of the Raytheon Technologies.

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an example of an X-band AESA, which is implemented as a flat array of antenna elements, each of which can be electronically controlled to steer the antenna's pattern on transit and receive.

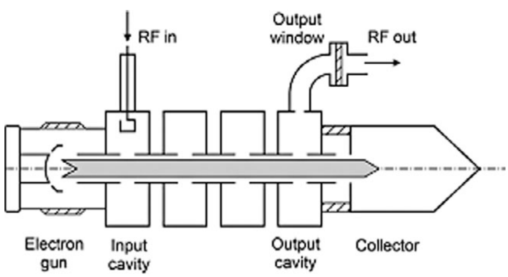
2.6.2 Transmitter

Radars are "active" instruments, meaning they provide their own signal to detect echoes. (In contrast, passive instruments only listen for other signal sources that may either be naturally occurring or are generated by other sources of opportunity.) As such, the transmitter is an essential component of the radar, providing the source signal to be used for "detection and ranging."

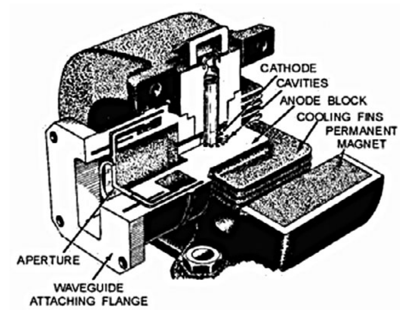
The key element of the transmitter subsystem is the source for the high-power RF signal. High-power amplifiers (e.g., klystrons, traveling-wave tubes, solid-state electronics) and high-power oscillators (magnetrons) are shown in Figure 2.11. The transmitter subsystem's coordination and timing are tightly controlled between the transmitter and receiver subsystems. For transmitters using power amplifiers, in addition to the high-power RF amplifier, the transmitter subsystem typically also includes the low-power RF signal-generation and frequency-conversion electronics. The power oscillator generates a high-power RF signal at its resonant frequency and does not use an input signal other than an on/off control. The most common transmitter types for weather radar systems are the magnetrons and solid-state power amplifiers (SSPAs). For some applications, the klystron and traveling-wave tube amplifiers (TWTAs), which use vacuum tube technology, provide the most effective solution.

Magnetrons (panel [b] of Fig. 2.11) were the first available techniques for generating microwave frequencies with sufficiently high power for use in radar systems. The most common magnetron design for radar transmitters is the resonant-cavity magnetron. The basic magnetron structure consists of a cathode located in the center of an anode ring. A permanent magnet is aligned so that the electrons interact with the magnetic field as they move through the cavity. As the electrons travel from the cathode to the anode, the magnetic field modifies their trajectory, forcing them to circle within the cavity. As they circle, a small percentage will eventually make it to the anode and generate a high-frequency electrical current. When correctly tuned, the frequency and stability of the electronic currents are suitable for generating RF or microwave signals. The operating frequency of a magnetron is largely a function of the mechanical dimensions of its cavity. A high-voltage modulator is used to energize the cathode, creating a voltage between the anode and cathode, which in turn generates the RF signal. The modulator typically has voltages in the tens of kilovolts and can generate peak RF powers in the megawatts.

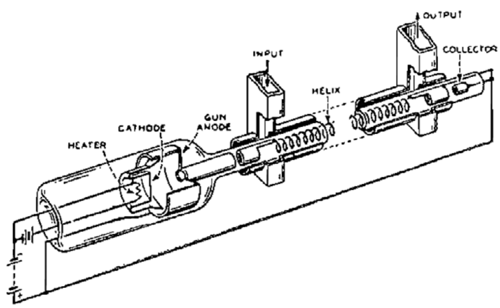
The magnetron and its associated modulator circuit are a cost-effective method of generating high-power RF for radar systems as a result of the benefits from the legacy of large-scale manufacturing. The magnetron's resonance frequency is determined by its cavity dimensions, and therefore the magnetron's frequency can drift over temperature. Modern magnetrons typically allow the operating frequency to be tuned during



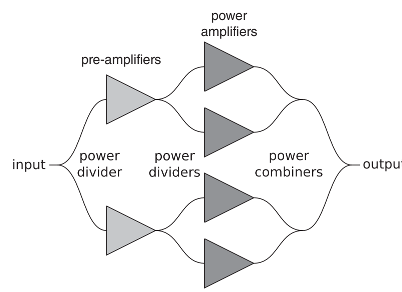
(a) Klystron [10]



(b) Magnetron [11]



(c) Traveling-wave tube amplifier (TWTA) [12]



(d) Solid-state power amplifier (SSPA)

Figure 2.11 Diagrams illustrating the concepts of operation for common technologies used as high-power transmitter sources or amplifiers in weather radar systems. The klystron (a) and traveling-wave tube (c) use vacuum tube amplifier technology. The solid-state power amplifiers (d) combine the power from multiple integrated circuits to achieve the desired transmitter power levels. These amplifiers reproduce the phase and frequency of an input reference signal while increasing the amplitude. The magnetron (b) is a power oscillator. The magnetron is both the transmitter's oscillator and a high-power source.

operation to conform to the radar's assigned frequency band. Similarly, because the magnetron is a power oscillator, the signal's starting phase is random. The frequency drift and random starting phase impose additional requirements on the receiver system to track the magnetron's RF frequency and phase via "automatic frequency control" circuits and coherent-on-receive signal-processing techniques.

The extended interaction klystron amplifier (EIKA; panel [a] of Fig. 2.11) is a power amplifier that uses vacuum tube technology. Unlike the magnetron, which oscillates at its resonant frequency, the klystron is a power amplifier that requires an RF signal to be provided as an input. The klystron tube amplifier takes a low-power input RF signal and amplifies it. Because the klystron is a power amplifier, the amplitude and phase of the output signal can be controlled via the low-power RF input signal. The klystron uses resonant cavities, which limits the range of frequencies for which it can operate to a relatively narrow frequency band. Klystron amplifiers are capable of generating high peak powers (>1 MW) but are typically limited to relatively low duty cycles ($<10\%$).

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Another type of tube-based power amplifier found in RF systems is the traveling-wave tube amplifier or TWTA (panel [c] of Fig. 2.11). TWTA designs are available for frequencies from 300 MHz to over 50 GHz. TWTA output powers can also vary from watts to tens of kilowatts or more. An advantage of the TWTAs is that they can operate over a wide bandwidth (the bandwidth can extend octaves for the helical configuration and may be 10–20 percent of the center frequency for cavity configurations). TWTAs are also commonly found in communication system transmitters where high-duty cycles are required.

The transmitter power sources mentioned so far – magnetrons, klystrons and traveling-wave tubes – all require high-voltage power supplies with voltages from multiple kilovolts to 100 kilovolts or more. High-voltage power supplies are complex devices and have potential hazards for the staff responsible for their maintenance. Although the actual power-amplifier elements themselves may be small, the power sources and support electronics can result in large, complex systems. As an alternative, solid-state RF power amplifiers for radar applications have recently become a viable alternative to these other high-voltage solutions.

Useful output power from solid-state systems at RF frequencies has largely been due to advances in semiconductors. Gallium arsenide (GaAs) and gallium nitride (GaN) have increased power density and efficiency to enable cost-effective RF power amplifiers at microwave frequencies. These solid-state power amplifiers rely on low-voltage power supplies commonly used by the other components of the radar system. An example of solid-state RF amplifiers power combined to increase the total RF power is shown in panel (d) of Figure 2.11.

The peak powers achieved with solid-state amplifiers are generally an order of magnitude lower that those achieved with the high-voltage counterparts previously discussed, but solid-state amplifiers can be operated at high duty cycles. From a review of the radar equation, the radar's sensitivity is related to the amount of energy that is transmitted ($E_{tx} = P_t T_{tx}$). To successfully utilize solid-state technologies as power amplifiers for weather radar, pulse-compression techniques have been adopted [13–15]. (Pulse compression is discussed in Section 6.5.) By using pulse-compression techniques and operating the solid-state transmitters at higher duty cycles, the solid-state power amplifiers (SSPAs) can transmit sufficient energy without sacrificing the radar's range resolution so that the radar systems achieve the required sensitivity. Although solid-state amplifiers may have a lower peak power than the other transmitters discussed, they can achieve similar transmitted energy with higher operating duty cycles and advanced signal processing.

SSPA transmitters provide characteristics that can be advantageous for numerous applications. They do not require high-voltage power systems, and their designs can be lightweight and compact. The solid-state integrated circuits that make up the power amplifier also enable modular configurations. These integrated circuits can be combined to increase the total available transmission power. For phased-array systems, where a physically distributed array of transmitting elements is desired, solid-state power amplifiers provide a cost-effective and compact solution. The combination of multiple lower-power devices to make a higher-power system inherently provides fault tolerance (albeit with degraded performance).

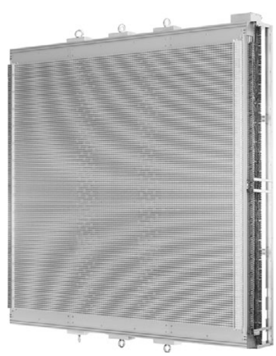
2.6.3 Antenna Systems

The antenna is the radar system's mechanism to transition the energy generated in the transmitter to free space (or from free space to the receiver). The antenna provides an electrical "match" from the nominal 50-ohm impedance of the radar system's RF components to the 377-ohm intrinsic impedance of the free space. Just as important for dual polarization weather radar, the antenna determines the radar's polarization characteristics. All antennas inherently have a polarization. In dual polarization radar systems, the polarization characteristics of the antenna are controlled and used toward a specific architecture, such as those presented in Section 2.2. In general, the antenna is the main device that controls the polarization state of the radar.

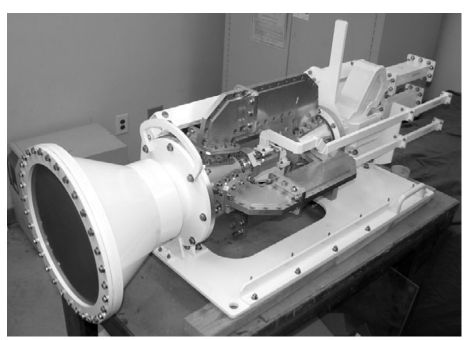
Antenna theory and technology is a well-developed field, with numerous textbooks and journals devoted to the latest research. The presentation here is a brief introduction in the context of weather radars. Figure 2.12 shows examples of a subset of antennas types (and types of antenna feed horns) that are used in dual polarization weather

(a) Center-fed parabolic
(NASA NPOL ©NASA)

(b) Dual offset Gregorian
(CSU-CHILL ©CSU)



(c) Slotted waveguide phased array
(Courtesy of Tomoo Ushio)



(d) Dual frequency horn and OMT
(Courtesy of Patrick Kennedy)

Figure 2.12 Common weather radar antenna types and a dual frequency, dual polarization feed horn with OMT.

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radar. For a reflector antenna, an electromagnetically reflective surface is used like a mirror to focus and direct the energy between the free space and a feed. The feed horn is the point where the transmitted and received fields of the radar are generated and collected, respectively. The reflector dish acts to collimate and focus the antenna beam, ultimately forming the antenna's pattern and determining its beamwidth. Reflector antennas are relatively simple and robust designs and provide excellent performance due to their legacy of scale manufacturing. For designs like the dual offset Gregorian antenna, multiple reflectors may be used.

The reflector antenna requires the feed horn (e.g., panel [d] of Fig. 2.12) to be placed in front of the antenna at a focal point; the forward distance F is defined by the parabolic surface curvature. For most parabolic antennas, the ratio for forward distances to diameter (also referred to as "F over D"), F/d_a, is 0.25–0.8 [16]. While D is commonly used to represent the antenna diameter, d_a is used here to differentiate it from the median drop diameter. The structures (or struts) to hold the feed in place and the feed itself can interact with the signal and distort the antenna's pattern (especially the cross-polarization pattern). Consider the center-fed parabolic antenna (panel [a] of Fig. 2.12) and the dual offset Gregorian antenna (panel [b] of Fig. 2.12). The center-fed antenna has its feed placed along a line orthogonal to the center of the reflector, where the peak of the antenna pattern is located, and the struts run across the entire antenna. The dual offset Gregorian antenna uses two reflectors and places the feed focal point outside of the direction of the main beam so that there is no interference within the antenna's primary observing area, which results in improved antenna sidelobe levels and reduces cross-polarization effects.

The orthomode transducer (OMT), shown in panel (d) of Figure 2.12, located behind the feed horn, combines or separates two orthogonal polarizations paths. The linear polarization OMT is a bidirectional waveguide device with three ports: a combined port, a horizontal signal port, and a vertical signal port. On transmit, the OMT combines the horizontal and vertical signals together to be output. On receive, the OMT splits the input signal into the horizontal and vertical components and then routes their signals appropriately.

For the dipole horn or slotted waveguide antennas, the signal is transmitted and received by the antenna element itself; the antenna directly transmits and receives the signal from the environment without requiring interactions with a dedicated reflector. Typically, multiple elements are combined as part of a larger system to generate a desired antenna pattern. An array of many simple antenna elements (e.g., dipoles or patches) can be designed to have equivalent antenna patterns as a reflector antenna.

The phase and amplitude of each element are precisely controlled so that the contributions of each element work together to form the final antenna pattern. The individual elements of the array are typically simple antenna designs, such as a patch, dipole, or waveguide. The amplitude and phase of each of the AESA's elements (on transmit, receive, or both) are then combined together to generate the array's antenna pattern. By adjusting the phase of each element, the direction of the AESA's antenna pattern can be steered. Amplitude weighting of the elements can be employed to control the antenna's side-lobe levels below the desired limits.

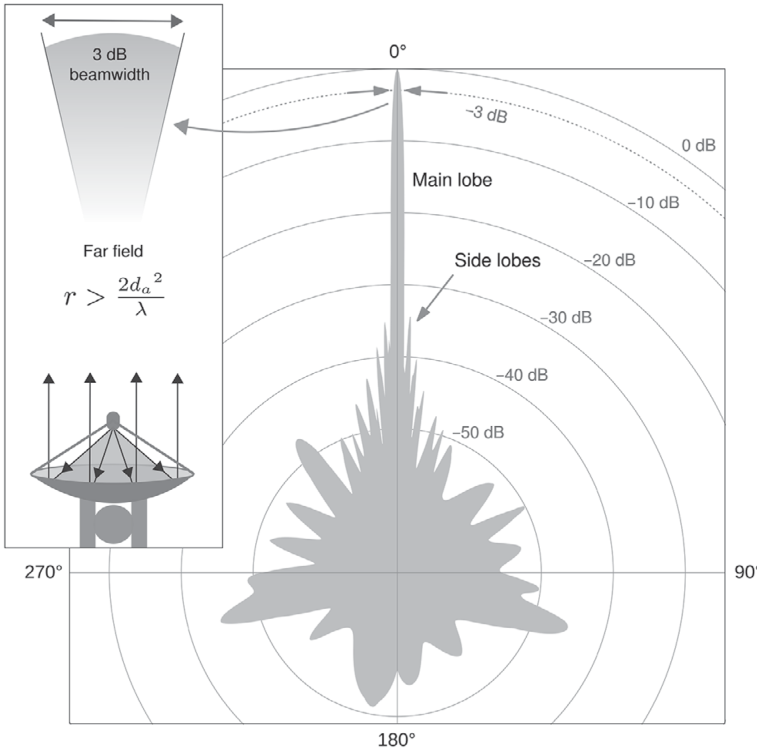

Figure 2.13 A typical two-dimensional normalized pattern of a weather radar antenna in polar coordinates (azimuth plane). The 3-dB beamwidth is $\sim 1^{\circ}$. The inset diagram on the left illustrates how the reflector collimates and focuses the wave from the feed horn. The resulting antenna pattern on the right is valid for the far field. Close to the antenna, the full area of the reflector contributes to focusing the wave.

The gain of the AESA is generally determined by the peak boresight gain of the individual element multiplied by the total number of elements used to form its beam. The gain for the transmit and receive may be different. As one might expect, AESAs are complex systems compared with a simple reflector antenna. The complexity can greatly increase the radar system's capabilities but also the cost. With multiple active elements in the AESA, calibration of the antenna may be required periodically, and compensation for component performance drifts (e.g., due to temperature variations) may be required.

For most weather radar systems, the transmitting and receiving antennas are the same (i.e., a monostatic configuration). Figure 2.13 is an example cut through an antenna's illumination pattern (here, the cut is across the antenna's azimuth plane). The pattern illustrates the antenna's one-way gain with respect to changes in the azimuth angles from the antenna's boresight direction. For monostatic weather radar systems, the one-way 3-dB beamwidth defines the azimuth and elevation extent of the V_6 radar volume.

The beamwidth of an antenna is generally inversely proportional to the diameter of the antenna. The half-power beamwidth (HPBW; in radians) of the antenna is approximated as

$$\boxed{\theta_{\text{HPBW}} = \frac{k_a \lambda}{d_a},} \tag{2.40}$$

where d_a is the antenna's diameter, and k_a is a factor that depends on the antenna's architecture and efficiency (a typical value for parabolic antennas is $k_a = 1.22$ [17]). The gain of a parabolic antenna is

$$G_0 = \frac{4\pi A_a e_a}{\lambda^2}, \tag{2.41}$$

where A_a is the antenna's physical area, and e_a is the antenna aperture's efficiency between 0 and 1 (a typical value of e_a is 0.6–0.7 for parabolic antennas). The aperture efficiency is a function of the geometry of the antenna, the illumination area, and the shape and roughness of the reflector's surface. For a circular reflector antenna with diameter d_a, the antenna's area is $A_a = \pi d_a^2/4$. The effective antenna area is then

$$A_e = e_a A_a = \frac{e_a \pi d_a^2}{4}. \tag{2.42}$$

The typical boresight antenna gain for a parabolic reflector ($e_a = 0.7$) is approximated as

$$G_0 \approx 0.7 \left(\frac{\pi d_a}{\lambda}\right)^2. \tag{2.43}$$

Combining these approximations for a parabolic reflector, the antenna's gain and beamwidth are related as

$$G_0 \approx 10.3 \left(\frac{1}{\theta_{\text{HPBW}}}\right)^2. \tag{2.44}$$

These approximations are valid for observations in the antenna's far field, that is, at ranges larger than the Fraunhofer distance r_{ff} [17]:

$$r_{ff} = \frac{2d_a^2}{\lambda}. \tag{2.45}$$

The far field also requires that $r_{ff} \gg \lambda$ and $r_{ff} \gg d_a$. For scatterers at ranges that are closer than the far-field distance, the antenna's pattern and gain can vary as a function of range, and near-field antenna patterns are required for interpreting the received signals.

Consider a 1.0° one-way, HPBW, center-fed parabolic antenna with $k_a = 1.22$ and $e_a = 0.7$. At the S band (2.9 GHz), the antenna is approximately 7.2 m in diameter. To achieve the same one-way beamwidth at the X band (9.4 GHz), the antenna is 2.23 m in diameter. If the antenna with $d_a = 7.2$ m is used at the X band, a one-way beamwidth of approximately 0.31° is achieved. For an S-band radar with a 7.2-m-diameter antenna, the far field typically starts at around $r_{ff} \approx 1000$ m, whereas the X-band 2.23-m antenna's far field starts at $r_{ff} \approx 311$ m.

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Dual polarization antennas are specifically designed with two objectives: (1) to provide nearly identical patterns at the two orthogonal polarizations and (2) to isolate orthogonal polarizations from one another. In particular for weather radar, minimizing the cross-polarization leakage and maintaining identical patterns at the two polarizations is important for analyzing weather observations. The differences in the observed signals between two polarizations are expected to be due to the precipitation and not the antenna patterns. If the copolar signal leaks into the cross-polar signal because of the antenna, this limits the radar's ability to accurately measure the cross-polarization signature of the scatterers (for the LDR) and can even bias measurements when comparing the dual polarization copolar signals, in particular, the differential reflectivity. For reflector antennas, an OMT enables the two polarizations. The OMT is a waveguide device that combines or separates two orthogonal polarizations. In antennas such as slotted waveguides, dipoles, or patch arrays, the linear polarizations' directions are defined by mechanically orienting the antenna elements 90° to each other. For radar systems using a reflector antenna, the system's polarization isolation depends on the performance of all elements, including the OMT, the feed horn, and the reflector and its mechanical supports. For the center-fed parabolic antenna, the mechanical struts that hold the feed horn in place are sources of cross-polarization that reduce the polarimetric isolation of the system [18].

The antenna pattern's amplitude and phase (F) are characterized as a function of elevation (θ) and azimuth (ϕ). For dual polarization, this gives us two copolar patterns, F_{hh} and F_{vv}, and two cross-polar patterns, F_{vh} and F_{hv}. F_{hv} is the amplitude and phase of the horizontally polarized radiated electric field when the vertical antenna port is energized, and vice versa for F_{vh}. Ideally for dual polarization antennas, the copolar and cross-polar antenna patterns are the same for the horizontal and vertical polarizations, giving $F_{co} = F_{hh} = F_{vv}$ and $F_{cx} = F_{vh} = F_{hv}$.

Recall that the antenna pattern is proportional to the electric field, and the antenna gain is proportional to power. The antenna's gain, as a function of look angle, is

$$G(\theta,\phi) = |F(\theta,\phi)|^2. \tag{2.46}$$

A parabolic antenna ideally has the boresight along look angle $\theta = 0$ and $\phi = 0$, with the peak copolar gains being equal for both polarizations, $G_{h0} = G_{v0}$. The gain of the antenna is included in F. It is sometimes useful to define a normalized antenna pattern (so that the peak gain is 1, or 0 dB, as in Fig. 2.13):

$$f(\theta,\phi) = \frac{F(\theta,\phi)}{\sqrt{G_0}}. \tag{2.47}$$

The copolar and cross-polar patterns are normalized by their respective polarization's copolar peak gains (e.g., $F_{co} = \sqrt{G_0} f_{co}$ and $F_{cx} = \sqrt{G_0} f_{cx}$).

The dual polarization performance of the antenna is typically quantified by its cross-polarization isolation. If the polarimetric antenna patterns are available for multiple planes (or "cuts") through the measurement space, the two-way (including the antenna's effects for both transmit and receive) integrated cross-polar ratio (ICPR_2) is calculated as follows [19]:

$$\text{ICPR}_2 = \frac{\iint |f_{hh} f_{vh} + f_{hv} f_{vv}|^2 \, d\Omega}{\iint |f_{hh}^2 + f_{hv}^2|^2 \, d\Omega}, \tag{2.48a}$$

where $d\Omega$ is the elemental solid angle ($d\Omega = \sin\theta d\theta d\phi$ in spherical coordinates) and the integration if over the sphere's surface. If we assume the copolar and cross-polar patterns are the same for the horizontal and vertical polarizations, the equation simplifies to

$$\text{ICPR}_2 = \frac{4 \iint |f_{co} f_{cx}|^2 \, d\Omega}{\iint |f_{co}^2 + f_{cx}^2|^2 \, d\Omega}. \tag{2.48b}$$

Although the antenna patterns are complex valued (the ICPR_2 is calculated using both the magnitude and phase), often only the magnitudes of the antenna patterns are available. Without the phase information, an upper bound of the ICPR_2 can be calculated as

$$\text{ICPR}_2^{(ub)} = \frac{4 \iint (|f_{co}|\,|f_{cx}|)^2 \, d\Omega}{\iint (|f_{co}|^2 - |f_{cx}|^2)^2 \, d\Omega}. \tag{2.48c}$$

For the CSU-CHILL X-band patterns shown in Figure 2.14, the ICPR_2 is calculated to be -42.0 dB. With only the antenna patterns' magnitude, the $\text{ICPR}_2^{(ub)}$ is calculated as -39.0 dB for the same antenna. The 3-dB difference is due to the effects of the antenna phase, which result in a slight decorrelation between the copolar and cross-polar patterns. A dual polarization antenna should have $\text{ICPR}_2 < -30$ dB to ensure negligible bias in the radar's dual polarization measurements for STSR mode (in particular, Z_{dr} and ρ_{hv}) [20]. The ICPR_2 determines the the LDR limit of the antenna for the ATSR mode (the antenna's LDR limit is discussed in Section 7.4.1). For interested readers, an in-depth examination of the polarimetric antenna pattern and its effect on the performance of dual polarization weather radar can be found in [18–21], and the references within.

The majority of radar systems operate with the antenna's surface covered by a radome. The radome protects the antenna and feed from rain, snow, and other debris and shields it from the forces of the wind. Consider the additional requirements placed on the antenna structure and the antenna's positioner systems if it must overcome the forces required to move a large antenna (which acts like a sail) in high winds. Ideally, radomes have a hydrophobic coating to limit the size of the droplets and the amount of water or ice adhering to the radome. Especially for high-frequency radars, the precipitation that adheres to the radome can add additional attenuation to the signal (see discussion in Section 7.4.3), which can also include polarimetric biases (e.g., the attenuation can be different between the horizontal and vertical polarizations as a result of the vertically oriented streams of water running down the radome). The radome's seams and brackets can also introduce biases in the dual polarization observations.

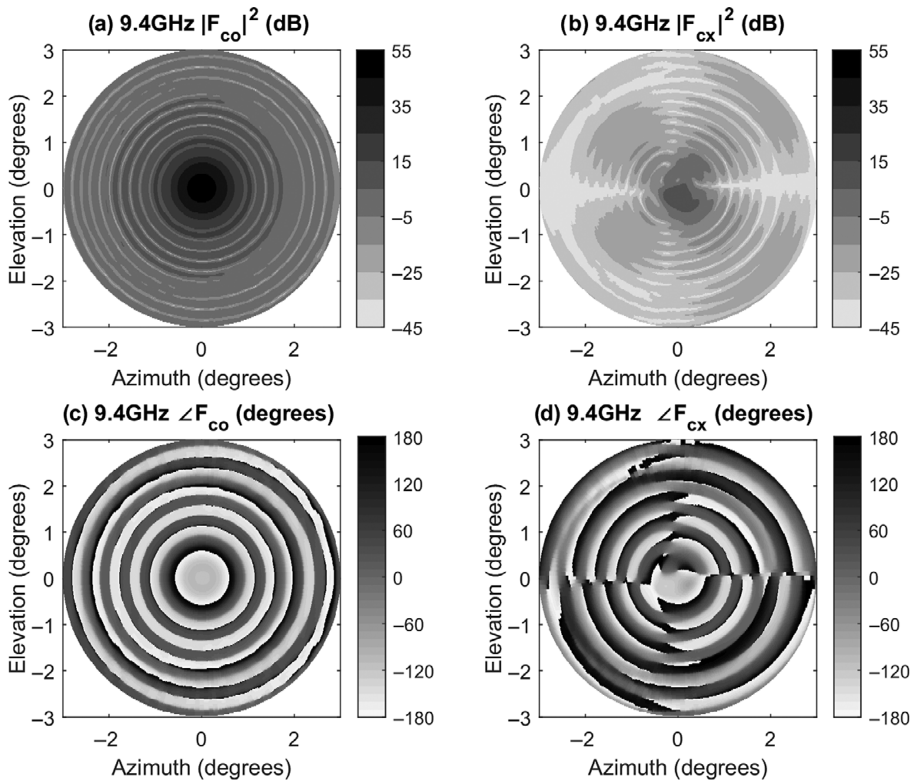

Figure 2.14 Antenna patterns from the CSU-CHILL's 8.5-m dual offset Gregorian measured at the X band (9.4 GHz). The patterns' magnitudes (in decibels) are $|F_{co}|^2$ (a) and $|F_{cx}|^2$ (b). Similarly, the antenna patterns' phases, $\angle F_{co}$ (c) and $\angle F_{cx}$ (d), are shown. Note that the cross-polar magnitude is lowest along the principal planes (i.e., the 0° cut, where elevation is zero and azimuth is varied, and the 90° cut, when azimuth is zero and elevation is varied). The cross-polar magnitudes exhibit increased antenna side lobes along both the 45° and 135° cuts. Also note that an odd symmetry about the boresight is observed for the cross-polar phase (d).

2.6.4 Duplexer

The duplexer allows the monostatic system to transmit and receive using a single antenna while providing isolation between the transmitter and receiver. Isolation between the transmitter and receiver is necessary to ensure that the transmitted power is routed to the antenna, the incoming echoes are routed to the receiver, and the amount of transmitter power that "leaks" into the receiver is sufficiently low to ensure that the receiver's electronics are not damaged. The transmitter's peak power levels can be on the order of 30 dBm (1 watt) to 90 dBm (1 megawatt), whereas the power of the received echoes can be as low as −110 dBm (10 femtowatts). The large differences between the peak transmitter's power and the received echoes' power can present a significant practical challenge.

Most of the monostatic pulsed radar systems operate using time-division multiplexing: this means the radar system is in either transmitting or receiving mode (but not both). Transmitting power and receiving echoes at the same time is not possible with time-division multiplexing. This is to ensure there is sufficient isolation between the two paths to prevent damage to the receiver electronics and ensure that maximum power is transmitted.

One of the most common elements of a radar duplexer is a ferrite circulator. The circulator is a nonreciprocal device with three RF ports, which ideally only allows RF energy to travel in one direction: from the transmitter to the antenna and also from the antenna to the receiver (the circulator also allows power from the receiver port to the transmitter port, but there is no RF power source in this direction). In practice, the circulator typically provides a low forward-insertion loss and high reverse isolation of 25 dB or more. When additional isolation is required, additional components are included in the duplexer to increase the isolation between the transmitter and receiver paths.

As part of the duplexer, a power limiter may be used to protect the sensitive electronics in the receiver. The power limiter is designed to prevent higher-than-expected power, which may enter the receiver path, from damaging the radar's receiver. The excess receive power may be due to large targets close to the radar or RF interference from other RF transmitters, such as wireless communication equipment or even other radars. Examples of power limiters include diode semiconductors and gas-discharge tubes. Diodes are used to limit the peak voltage in the receiver but have limited power-handling capability. For high-RF-power applications, ionized gas-discharge tubes are frequently used.

2.6.5 Receiver

The antenna collects the energy from the incoming signal (e.g., backscatter from precipitation) and routes each incoming polarization's signal to its respective antenna port. From the antenna port, the received signal passes through the duplexer and into the radar's receiver. The receiver includes all of the electronics to amplify, frequency convert, filter, and finally digitize the received signals. For radar architectures that simultaneously receive both polarizations, each polarization has its own receiver electronics, which are typically identical copies for each channel.

The receiver's components are selected to optimize the receiver gain so that the expected signal- and noise-power input from the antenna are within the range of powers that the digitizer can accommodate. The digitizer is an electronic device that periodically samples the analog voltage signal and converts it to a digital count that is proportional to the sampled voltage's magnitude. The digitizer is also referred to as an *analog-to-digital converter*. The dynamic range is the ratio of maximum to minimum power that the receiver can operate. The receiver's components, each with its individual gains and losses, are selected to maximize the receiver's dynamic range. As the receiver's input power becomes too high, components of the receiver can start to "compress," and the gain decreases. When the receiver's components are in compression, there is no longer a constant relationship between the change in input power and output power. Operating the radar receiver in the compression region is usually avoided for weather radars, and the gain should ideally remain constant over its dynamic range. Once the signal is digitized, digital signal processing can further increase the dynamic range of the radar system beyond the analog dynamic range of the receiver.

One of the purposes of the radar receiver elements is to convert between intermediate frequencies (IFs) and the radar's operating RF, which contains the signal's information. The radar's IFs are selected to accommodate available hardware resources, such as the digital-to-analog and analog-to-digital converters and signal-processing electronics. LO frequencies are constant frequencies that aid in frequency conversion only (they do not provide any new information about the transmitter's or receiver's signal). Although less common in modern radar designs, multiple frequency conversion stages may be used, which require multiple LOs (as shown in Fig. 2.9). The frequency conversion is typically accomplished using a mixer that effectively multiplies the two signals, one of which is the LO. The transmitter's frequency converter is referred to as an *upconverter* (because the frequency is increased), and the receiver's frequency converter is referred to as a *downconverter*. (Section 5.1.5 considers the mathematics and signal theory of frequency conversion.)

For coherent radar systems, a common reference clock is required between the radar's clocks (e.g., radar timing, sampling clock, LOs). The same LO and sampling clocks are commonly split to provide symmetry in the transmitter and receiver frequency converters. Doppler radars rely on the receiver and transmitter to be phase coherent, so the transmitted pulse's phase is determinstic with respect to the receiver's sampling time. The phase coherence (or phase lock) ensures that the phase difference between the system's transmitter and the receiver is constant, and therefore any measured change in phase is a property of the echo.

2.6.6 Radar Mechanical Structure and Pointing Control

Ground-based weather radar systems are typically designed to scan the hemisphere above the ground (some research radars make fixed, vertical-pointing observations). For operational scanning systems, the azimuth is typically scanned at a fixed speed, and the elevation is stepped in discrete intervals once per azimuth revolution. The specific elevation intervals are determined by the volume coverage pattern (VCP). Some radar systems require additional agility, with capabilities to provide full PPIs, sectors of a PPI, RHIs, fixed pointing, or combinations of all of these with various requirements for timing and scanning speeds.

For mechanically scanning systems, the radar system's pedestal anchors the radar to a solid platform (e.g., roof, tower, or ground), and the pedestal provides the mechanisms for full 360° motion in the azimuth direction. Elevation scanning from 0 to 90° or even 0 to 180°, which allows for horizon-to-horizon vertical slices, is possible. The maximum scanning elevation may be limited to something less than 90° if the radar application does not require this full range of motion.

For scanning radar systems, the mechanical assembly that does the scanning is balanced around the rotation axis (usually with the addition of counterweights) to minimize the stress on the motion-control system in static states. The motion control must still be designed to overcome the inertia of accelerating the mass, which can require substantial electrical power and mechanical strength in the structure.

For unrestricted azimuth operation, no fixed wires can be used, and a slip ring and/or waveguide rotary joint is needed to provide electrical or RF connections (the exact configuration depends on the location of the receiver or transmitter). These let the pedestal rotate continuously without concern for wrapping or binding wires or waveguides between the stationary platform base and the rotating antenna. Depending on the radar's design, one or both may be needed in various combinations to achieve the desired range of motion for the system. Slip rings in the pedestal are used to provide electrical paths for data communications and power. Modern solid-state and low-power radars frequently carry both the transmitter and the receiver on the rotating subsystems to reduce RF losses [15, 22].

The integration time, sometimes referred to as the *coherent processing interval* (CPI), indicates the duration over which N signal samples are collected for estimation of the volume's radar variables (Fig. 2.15). For a mechanically scanning system, the angular extent that is covered (ψ_{ant}) for a given scan rate ($\dot{\psi}_{\text{ant}}$) and pulse-repetition period (T_s) is

$$\psi_{\text{ant}} = \dot{\psi}_{\text{ant}} \cdot N \cdot T_s. \tag{2.49}$$

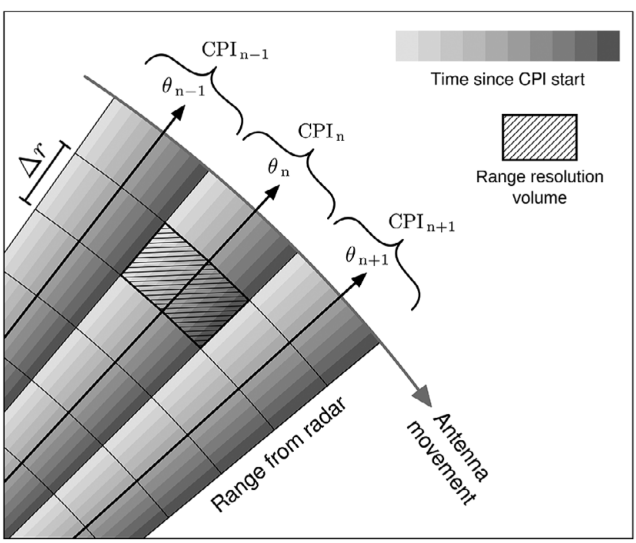

Figure 2.15 The received signal is integrated over multiple samples to estimate the radar variables. The integration time of the CPI is shown here for a scanning antenna. Each CPI includes a number of radar pulses that are averaged, and the estimated values within echo range resolution volume are typically assigned to the midpoint of the volume, both in range and angular positions (e.g., θ_n).

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Ideally, the radar antenna's scanning speed and integration time should be selected so that no more than half of the antenna's half-power beamwidth is traversed during the integration time (i.e., $\psi_{\text{ant}} < \theta_{\text{HPBW}}/2$). This is to limit decorrelation of the observation volume due to the antenna motion (which is discussed in Section 5.7). The antenna's speed can vary during the scan. Radar signal processors typically provide the option to process over fixed angular resolutions (e.g., every 0.5° or 1°, allowing N to vary) or for a fixed number of pulses (e.g., $N = 32$ or $N = 64$ and accepting the resulting variations in angular resolution). As an example, for a radar scanning at $\dot{\psi}_{\text{ant}} = 18^\circ\ \text{s}^{-1}$, $N = 64$ samples, and a pulse-repetition period of $T_s = 1$ ms, the antenna rotates $\psi_{\text{ant}} = 1.15^\circ$.

Electronically scanned systems can significantly deviate from their mechanically scanned counterparts, in both operational capacity and the design of the structure and pointing-control system. A fully electronically scanned system does not require any moving parts. The phased array can selectively sample volumes on a pulse-by-pulse basis. The phased-array system enables electronically steerable antennas, increasing the speed over which a volume can be scanned.

Mechanically controlled systems have a fixed antenna aperture (the beamwidth is fixed by the antenna design), and the pointing direction of the antenna is determined by the positioner system. Changing the pointing angle requires physically moving the antenna. The speed at which the antenna can be pointed to a new direction is largely a function of the antenna's mass and size and the capability of the positioner. For weather radars, the acceleration of the scanning system can be between $1^\circ/\text{s}^2$ and $60^\circ/\text{s}^2$, and the radar scanning rates can be between $18^\circ/\text{s}$ and $60^\circ/\text{s}$ (or 3–10 RPM).

For fully electronically steered antennas, the beam can be pointed to any direction within its field of view on a pulse-to-pulse basis (or nearly so), enabling an almost instantaneous scanning rate. With the use of beamforming techniques, multiple radar volumes can be observed at the same time. With intelligent scanning algorithms, the AESA radar can also track storms of interest (e.g., mesoscale convective systems) at high temporal rates while still providing surveillance of the entire domain at a lower (or standard) rate. A hybrid compromise, with electronic scan in the elevation and mechanical scan in the azimuth, is evolving as a more cost-effective compromise in weather radars. Although electronically scanned systems have operational advantages, mechanically scanned systems provide the most cost-effective solution for general-purpose weather surveillance.

2.6.7 Data System

The earliest techniques of recording weather radar observations were drawings or tracings by the operator or photographs of a cathode-ray tube's output taken at regular intervals. Modern computers revolutionized radar processing and data archiving, trading analog for digital. As digital data storage became more affordable and computer processing allowed, radar moments could be archived to magnetic tapes on a limited basis. As this trend of decreasing data-storage costs and increasing data-processing power continued, the fidelity of the radar observations similarly improved, and

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continuous data archiving became standard practice. For some radars (typically, research radars that are only periodically operated), the raw instrument data are archived and available for future reprocessing. This allows for new signal-processing algorithms to be applied to legacy data sets.

Radar data products are typically classified into "processing levels," where the higher the level, the more the data have been refined or processed. The higher-level data typically include calibration and unit conversions applied to the raw instrument data, as well as additional derived results from the lower-level data (e.g., rainfall rate), or placing data on a regular grid. The availability of these data to the general public varies by the desired data level and the radar's operator/owner. For example, the WSR-88D Level 2 and Level 3 data product archives are easily accessible, with data generally available within 1–3 days, depending on the data product. Chapters 6 – 10 cover the various aspects of signal and data processing used for weather radar.

Table 2.1 Example description of the levels of data processing. Each instrument may adopt variations of these descriptions.

Level 0	Raw, unprocessed instrument data. Calibration and geolocation information may be separate and at different sampling rates.
Level 1	The data from Level 0 are time referenced, with instrument and other ancillary information appended (e.g., radiometric calibration and georeferencing information). The ancillary data include the necessary information to process to higher levels, but no corrections or changes are made to the raw data at this point. The data are at the original resolution, without any data loss between Level 0 and Level 1. For some radar systems, Level 1 data are the lowest-level data available.
Level 2A	The geolocation and calibration information is applied to the Level 1 data at the original spatial resolution. Data are converted to the sensor's units (e.g., Doppler velocity, equivalent reflectivity factor). Data reduction may be performed (i.e., the Level 2A data cannot be reverted to Level 1 data because information, such as the uncalibrated raw data and calibration values, is missing). Data quality control is applied. Some processing systems may classify aspects of this processing as Level 1B. Note that Level 2 data are not always split into 2A and 2B processing.
Level 2B	Derived geophysical parameters in the instrument's spatial resolution and reference frame (e.g., range, azimuth, and elevation). Examples of derived geophysical parameters are attenuation corrected reflectivity, rainfall rate, and specific differential phase. Additional data quality control may be applied if necessary.
Level 3	Geophysical parameters from the instrument (e.g., Level 2 data) that have been spatially and/or temporally resampled (e.g., mapped to a Cartesian grid at specific time intervals). A common spatial and temporal sampling interval is typically used between multiple instruments of various types.
Level 4	Composite results from multiple sensors or model analysis from lower-level data. The data are typically at the same spatial and temporal grid resolution as Level 3 data. These Level 4 derived products are not directly from a single instrument (e.g., multi-Doppler velocity).

A general description of the data-processing levels is provided in Table 2.1. The specific details of the data-processing levels for a given instrument may vary. Additional letters may be used to define additional processing stages (1C, 2A, 2B, etc.). The data-processing levels build on prior processing levels. A single instrument can include process levels from 0 to 3. Multi-instrument data (or model results derived from the observations of a single instrument) are Level 4 data. Not all processing levels are used by all instruments. For some radar systems, the radar's signal and data-processing system may internally process the data so that the first data level archived is equivalent to the Level 1 data. The processed data are streamed to multiple users: radar operators and forecasters for real-time use, data archives for storage and future use, modern artificial intelligence (AI) applications for data discovery, and Level 4 data-processing systems for fusion with multiple sensors and models. The Level 4 data-processing systems can also provide nearly real-time data to applications as fused data products or near-term forecasts.

2.7 Radar System Sensitivity

There are a number of fundamental performance metrics of weather radar that are considered when selecting a system for a particular application. One of the most important considerations is the radar's sensitivity. The radar is typically limited in the dynamic range of the power (and therefore reflectivity) it can observe. These limitations are often due to system noise or transmitted power. In either case, a radar is designed to observed a range of reflectivities. The sensitivity of the radar is the lowest reflectivity the radar can observe and is usually given for a standard distance (e.g., 1 or 10 km).

The sensitivity also assumes different levels of signal-processing gains. As part of the design, the radar's minimum and maximum range and the range resolution must be defined. The selection of the range resolution has an impact on the reflectivities that can be observed. The selection of the maximum range must also be considered, along with the expected range of velocities the radar will need to unambiguously resolve.

These are some of the factors that must be considered when designing or operating a radar. Some of the choices are not limited by physics but by other constraints, such as available frequency bands or economics. This section discusses how the radar subsystems' performance metrics effect the radar's sensitivity.

2.7.1 Noise in Radar Systems

The radar receiver's signal includes noise in addition to the echo power. This noise power, P_n, determines the minimum detectable signal power of the receiver. This noise power comes from the environment, which is received by the antenna, but noise is also generated within the radar receiver's electronics. In the absence of any echo power (i.e., our signal), the power measured at the output of the receiver is noise. Interference from anthropogenic sources is neglected because it does not typically drive the estimates of radar system performance (other than making sure the radar can tolerate/survive it).

The noise power of a noise temperature (T_n) and a system bandwidth B is

$$P_n = k_B T_n B, \tag{2.50}$$

where $k_B = 1.380649 \cdot 10^{-23}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant. For determining receiver noise for the minimum detectable signal, B is the receiver filter's bandwidth. The total noise temperature is a function of the radar system and the environment. This noise temperature is largely dominated by the receiver's physical temperature but can be influenced by the background temperature in the direction the antenna is pointing (e.g., Earth's surface, the troposphere, or space).

Each component has its own noise contribution. Some can be negligible, some significant. The noise factor, F, defines the ratio of the input signal-to-noise ratio (SNR) to the output SNR as

$$\boxed{F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}},} \tag{2.51}$$

where $F \geq 1$. This means that the SNR of the output is the same as or lower than the SNR at the input of the device (without changing the bandwidth). The noise figure (NF) is typically used to describe a component's effect on the system's SNR and is simply

$$\mathrm{NF} = 10 \log_{10}(F). \tag{2.52}$$

For radar systems, the noise contribution of each component in the receiver's path can be cascaded together. The cascaded noise figure is used to determine the overall impact it has on the SNR between the antenna and the receiver's output (the output is typically sampled by the digitizer). The SNR determines the radar's ability to detect a given signal power, and therefore the radar system's performance is strongly tied to the receiver's noise figure.

The noise figure is defined with an input reference temperature of $T_0 = 290^\circ$ K. If the physical temperature of the device is varied, the input noise temperature and therefore noise factor are also varied. The additional noise temperature contribution of the system (T_{sys}) and the noise factor, F, are related as

$$F = \frac{T_{\mathrm{sys}}}{T_0} + 1 \tag{2.53a}$$

or

$$T_{\mathrm{sys}} = T_0(F - 1). \tag{2.53b}$$

The noise power can be calculated using the system and reference temperatures, or equivalently the noise factor, as

$$P_n = k_B(T_{\mathrm{sys}} + T_0)B = k_B T_0 F B \tag{2.54}$$

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Note that the radar signal's bandwidth, B, is typically the only parameter available to the radar operator once the radar is designed and built.

A radar's receiver consists of a number of devices where the signal is routed in series from one to the next from the input to the output. For a cascade of devices, starting with the first device ($n = 1$) and moving through all N devices, the cascaded noise factor is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{\prod_{n=1}^{N-1} G_n}, \qquad (2.55)$$

where G_n denotes the gain (in linear units) of each stage. Note that for some components, such as a filter or attenuator, their gain is less than 1 (or negative in decibels). For these components, a fraction of the input signal power does not make it to the output because it is either absorbed or reflected back toward the input by the device. Also note that for these passive components (no electrical power other than the signal goes into the device), the gain and noise factor are the inverse of one another. The cascaded gain is simply the product of the gains of all components in the series:

$$G = \prod_{n=1}^{N} G_n = G_1 G_2 G_3 \cdots G_N. \qquad (2.56)$$

The output signal power through the series of cascaded devices is the input signal power $P_s^{(\mathrm{in})}$ multiplied by the total gain of the entire set of components:

$$P_s^{(\mathrm{out})} = P_s^{(\mathrm{in})} \cdot G. \qquad (2.57)$$

With an input noise power of $P_n^{(\mathrm{in})}$, the noise power at the output of the series of cascaded devices is

$$P_n^{(\mathrm{out})} = P_n^{(\mathrm{in})} \cdot F \cdot G. \qquad (2.58a)$$

Like the output signal power, the output noise power includes the total gain of the series, but it also includes an increase in noise power due to the cascaded noise factor in the signal path (represented by F). The output SNR ($P_s^{(\mathrm{out})}/P_n^{(\mathrm{out})}$) is reduced compared with the input SNR by the noise factor (following its definition in eq. [2.51]).

A low-noise amplifier (LNA) is selected to provide amplification with a low-noise factor. With careful design, the first LNA dominates the receiver's overall noise figure. Any attenuation or loss before the first gain stage adversely affects the receiver's noise figure. This is why the first LNA is typically placed as close to the antenna feed as possible (on satellite dish receivers, you can find the electronics at the feed). For X-band radars, where losses are higher per length of waveguide, the electronics are again placed close to the antenna ports, usually on the back of the antenna if possible. For S-band radars, the waveguide losses are typically easier to tolerate (and the electronics are large), allowing (necessitating) longer distances between the antenna and receiver electronics.

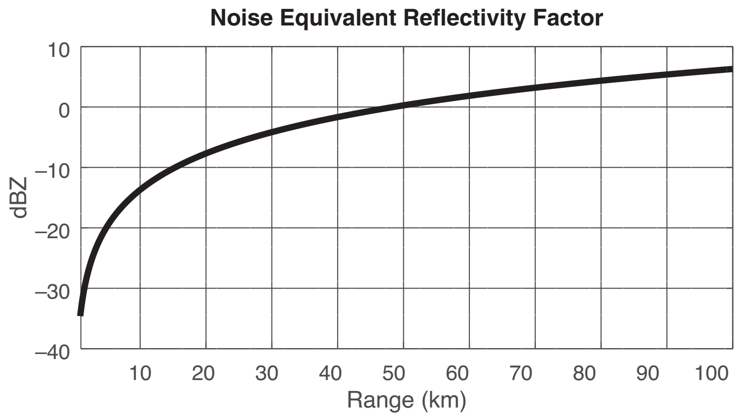

Figure 2.16 An example of the noise-equivalent reflectivity factor, Z_n, as a function of range for the CSU-CHILL S-band radar. Whereas the noise power P_n is constant for all ranges, Z_n varies in proportion to the range-squared (i.e., $Z_n \propto r^2$).

2.7.2 Noise-Equivalent Reflectivity Factor

When describing the radar system's performance, a metric that is used is the minimum detectable signal for a single pulse. This is frequently described as a minimum detectable reflectivity factor at a given range (1- or 10-km ranges are common). With the radar system's noise power P_n, a noise-equivalent reflectivity factor, Z_n, can be calculated from eq. (2.39c) as

$$Z_n = \frac{P_n r^2}{C}, \tag{2.59}$$

where weather radar constant C is calculated from eq. (2.39d). The noise-equivalent reflectivity factor is a measure of the single-pulse-detection performance of the radar and is a function of range, r. (This provides a performance metric to consistently evaluate radar performance for only one pulse. Improved detection can be achieved though signal processing techniques.) Figure 2.16 illustrates the noise-equivalent reflectivity factor for a constant noise power, $P_n = -105$ dBm. To estimate the noise-equivalent reflectivity factor, this example uses $G_{rx} = 1$, $P_t = 87$ dBm, $G = 43$ dB, $|K_w|^2 = 0.934$, $T_{tx} = 1\ \mu$s, $\theta_1 = \phi_1 = 1.1\ ^\circ$, and $f_0 = 2.725$ GHz. (Note that P_n assumes the signal's nominal bandwidth and includes the receiver's noise figure.)

For the design of weather radar systems, a typical objective is to maximize the sensitivity of the radar so that the radar can detect the weakest signal possible. The radar's sensitivity can be increased by reducing Z_n. Some of the parameters reduce the noise power, P_n, whereas other parameters act to increase the signal power, $P_s = P_o$. It is possible to evaluate how the parameters affect the system's SNR, which is directly related to the radar's sensitivity:

$$\text{SNR} = Z_e \frac{G_{rx} P_t G^2 \pi^3 |K_w|^2 c T_{tx} \theta_1 \phi_1}{10^{18} \lambda^2 1024 \ln 2} \frac{1}{r^2 k_B T_0 B F}. \tag{2.60}$$

The constants are neglected to focus on the radar system parameters that can be affected by the radar's design. From Section 2.6.3, the antenna's gain and beamwidth are related to one another following $G \propto 1/(\theta_1 \phi_1)$. After reduction of the previous equation, the parameters that can be controlled by the radar design or operation and how they affect the radar's sensitivity through consideration of the following relationship:

$$\text{SNR} \propto \frac{P_t G T_{tx}}{\lambda^2 r^2 B F}. \tag{2.61}$$

Some radar systems have constraints on the largest antenna size that can be accommodated. The antenna size and λ directly affect the antenna gain G. From eq. (2.61), it is clear that the SNR increases by increasing the amount of energy transmitted, either through a higher peak transmitter power P_t or through a longer duration of the transmitted pulse T_{tx}. (This is one motivation for using pulse-compression techniques to enhance radar sensitivity, as discussed in Chapter 6.) The SNR can also be increased by reducing the noise factor F, reducing the bandwidth of the radar signal B, reducing the operating range r, or reducing the wavelength λ. (Note that the scattering behavior is wavelength dependent and can be nonlinear. This is covered in Chapter 4.) Although not directly evident in this equation, the implementations and performance of the technologies are interdependent as well (e.g., the relationship between wavelength, antenna size, and antenna gain). If a network of radars can be used, the maximum range from any radar to a given volume can be reduced. (This and other performance characteristics of the radar network are covered in Chapter 10).

The minimum detectable reflectivity may be lower than the noise-equivalent reflectivity factor. The weather radar's minimum detection sensitivity includes additional signal-processing gains that enable the radar to estimate radar variables when SNR < 0 dB. A common technique to improve the minimum detectable reflectivity is to use noise correction (see Chapter 7). With noise correction and an integration time with N pulses, the radar's sensitivity, or its minimum detectable reflectivity, is approximated as

$$\boxed{Z_e^{\min} = \frac{Z_n}{\sqrt{N}}.} \tag{2.62}$$

When designing (or acquiring) a weather radar, the process typically starts with a cost cap and other practical implementation boundaries, such as limits on the available locations to deploying the radar and its social footprint. The availability of dual polarization capabilities for attenuation correction, solid-state transmitter technology, and the concept of networked radar systems have fundamentally altered the solution space for weather radars and have made higher-frequency bands practical (and attractive) options.

2.8 Selected Problems

1. What is the time delay of a radar echo that is received from a scatterer at a range of 250 km from the radar? What is the highest pulse repetition frequency that can be used for an unambiguous range of 250 km?
2. Calculate the altitude of the radar beam at 150-km slant range when the antenna is pointing at 1.0° elevation angle using eq. (2.24). Compare the result with the altitude obtained using a instead of a_e, that is, neglecting the vertical variation of the refractive index.
3. Assuming a simple pencil beam cone model, how wide will the antenna beam be at ranges of 40 km, 60 km, 100 km, 200 km and 300 km? For a range resolution of 150 meters, what is the radar observation volume in cubic kilometers?
4. A radar with a 1° beamwidth is scanning at a low elevation of 1° (to avoid local obstruction). How high will the beam center be at a distance of 40 km, 60 km, 100 km, 200 km and 300 km? To account for refractive index variation over the surface of the Earth, use a 4/3 Earth radius model.
5. For a bandwidth of 10 MHz and a noise temperature of 290 K, what is the noise power at the input of the antenna in dBm? If the bandwidth is changed to 1 MHz, what is the input noise power in dBm?
6. The CSU CHILL S-band radar operates at a frequency $f = 2.725$ GHz. It has a 1 MW transmitter and uses a 1 μs pulse width. Its dual-offset antenna has an antenna gain of approximately 43 dB and a half-power beamwidth of 1.0° . Assume $G_{rx} = 1$ (the power at the antenna port can be converted to reflectivity) and the dielectric factor of water is $|K_w|^2 = 0.93$:
   a) calculate the weather radar constant, C.
   b) Using CSU-CHILL's S-band weather radar constant and a receiver noise power of -110 dBm, what are the noise-equivalent reflectivity factor at 1 km, 10 km, and 100 km? (Note that dBm is the power ratio with respect to 1 milliwatt.)
7. Calculate the cascaded noise figure for three cascaded amplifiers, each with 20 dB gain and 3 dB noise figure. What is the system's noise temperature? Repeat the calculation, but with a 2 dB attenuator before each amplifier (for a 2 dB attenuator, its gain is -2 dB with a 2 dB noise figure). What is the contribution to the total noise figure of the first attenuator?
8. With a noise figure of 5.5 dB and gain of 24 dB, what is the noise power and signal power at the output of the receiver for an input noise power of -108 dBm and an input signal power of -70 dBm? What are the input and output SNRs for this receiver?

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9. If a the radar volume is uniformly filled with a density of one 2-mm raindrop per cubic meter, what is the radar reflectivity factor in dBZ? For a radar with a minimum sensitivity of -10 dBZ at 1 km, what is the maximum range that the radar would detect the precipitation echo? If the radar's minimum sensitivity was -30 dBZ at 2 km, what is the maximum detection range?
10. Metal spheres are commonly used as radar calibration targets. For spheres whose diameters are large with respect to the radar wavelength, and therefore in the optical scattering regime, the radar cross-section is the sphere's cross-sectional area: $\sigma_b = \pi r_s^2$. Assuming a wavelength of $\lambda = 3$ cm and the metal sphere has a dielectric factor $|K|^2 = 1$:
    a) What is the RCS of a 30 cm diameter metal sphere?
    b) For a radar observation with $V_6 = 25890$ m^3 (the approximate radar volume for $\theta_1 = \phi_1 = 1^\circ$ HPBW, $\Delta r = 150$ m, and r = 1000 m), what is the equivalent mean reflectivity $\bar{\eta}$ for the same RCS as the sphere?
    c) What is the metal sphere's reflectivity factor Z with $|K_w|^2 = 0.93$?
    d) If the radar range increases from 1 km to 10 km, what reflectivity factor would the radar measure?
    e) If the backscatter is calculated assuming the Rayleigh scattering regime, where particles must be much smaller than the wavelength, what is the metal sphere's RCS?
    f) If the Rayleigh approximation for the large calibration sphere's RCS is used instead of the appropriate optical regime approximation, what is the error in the reflectivity factor's calibration in decibels?