

COHEN, D. E., *Combinatorial group theory: a topological approach* (London Mathematical Society Student Texts 14, Cambridge University Press, 1989), pp. x + 310, cloth 0 521 34133 7, £30; paper 0 521 34936 2, £11.95.

There are two books with the title *Combinatorial group theory* that are very familiar to everyone working in the area. The 1966 book by Magnus, Karrass and Solitar is a classic, being the first book solely devoted to the subject. It remains a useful text today, although the more modern and encyclopaedic book of Lyndon and Schupp (1977) has become the standard reference. The book under review does not set out to compete with these texts. Instead, it concentrates on one aspect of the subject, namely the rich and fascinating interplay between group theory and topology. As the author explains in his introduction, he is trying to find a balance between on the one hand the purely algebraic approach of Magnus, Karrass and Solitar, and to a lesser extent Lyndon and Schupp, and on the other hand the more topological approach of Stillwell's *Classical topology and combinatorial group theory*, or Massey's *Algebraic topology*. He appears to have struck this balance very well.

The book is a revised, updated and extended version of the author's QMC lecture notes, which in turn were based on an MSc course. Having read both the previous book and attended the lecture course, I can attest that the material improves with maturity, like a good wine.

The book begins with a brief account of the standard constructions of combinatorial group theory, point set topology, and the theory of groupoids. This material can be found in more detail in other texts such as those mentioned above and Higgin's *Categories and groupoids*. There follows a chapter on the fundamental group and fundamental groupoid, which includes treatments of R. Brown's groupoid version of the van Kampen theorem, Brouwer's fixed point theorem in dimension 2, and the fundamental group of the Hawaiian ear-ring. The next chapter covers graphs and *complexes* (a complex being a 2-dimensional complex, defined combinatorially, in much the same way as a graph is a combinatorial version of a 1-dimensional complex). This chapter includes the proof due to Goldstein and Turner of Gersten's theorem: the fixed subgroup of an endomorphism of a finitely generated free group is finitely generated. The next two chapters develop (combinatorially) the theory of coverings of complexes, and apply it to give proofs of the Schreier and Kurosh subgroup theorems. There is then an extensive chapter on the Bass-Serre theory of groups acting on trees. Again, most of this is standard, and can be found in Serre's book *Trees*, but there are also some very recent results of Dicks and Dunwoody. There is no mention of recent generalizations of this theory to  $R$ -trees and  $A$ -trees, nor of the connection with length functions. The last main chapter concerns decision problems in groups. A final, very brief, chapter indicates some other recent advances in the subject that involve topological ideas and methods. The book ends with a useful set of chapter-by-chapter historical notes, an extensive bibliography, and an index.

I would find this an excellent book on which to base a graduate course. As well as being just about the right level, it is not beyond the pocket of most students (at least in the paperback edition). It takes the reader through Combinatorial Group Theory to the point of proving some fairly recent theorems, using a nice geometric approach which nevertheless does not demand too much topological sophistication.

JAMES HOWIE