## THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of The Mathematical Gazette, Mathematics in School, Primary Mathematics (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is $\mathbf{2 5 9}$ London Road, Leicester LE2 3BE, UK (telephone 0116221 0013). The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to Teaching Committee should be addressed to TC Chair. The Association's Library is housed in the University Library, Leicester.
Views expressed in the Mathematical Gazette by authors or advertisers are not necessarily those of the Association.

## THE MATHEMATICAL GAZETTE

| Editor: | Problem Corner: |
| :--- | :--- |
| Dr Gerry Leversha, | Mr Nick Lord, |
| 15 Maunder Road, | Tonbridge School, |
| Hanwell, | Tonbridge, |
| London W7 3PN | Kent TN9 1JP |
| g.leversha@btinternet.com | njl@tonbridge-school.org |
| Production Editor: | Reviews Editor: |
| Mr Bill Richardson, | Mr Owen Toller, |
| Kintail, | 4 Caldwell House, |
| Longmorn, | 48 Trinity Church Road, |
| Elgin IV30 8RJ | London SW13 8EJ |
| wpr3145@gmail.com | owen.toller@btinternet.com |

Potential advertisers e-mail Charlotte Dyason at: charlotted@media-shed.co.uk
Material for publication should be sent to the Editor.
Books for review should be sent to the Reviews Editor.
Advice to authors of notes and articles.
Study the format of articles in the Gazette. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the Gazette was produced on an Acorn machine using TechWriter and Draw.
10.1017/mag. 2021.151

## Articles

| Pairing theorems about parabolas through duality | Steven J. Kilner and <br> David L. Farnsworth |
| :---: | :---: |
| Long medians and long angle bisectors | Sadi Abu-Saymeh, Yaqeen 397 Al-Momani, Mowaffaq Hajja, Mostafa Hayajneh |
| The probability certain random quadratics have real roots | Chris Boucher 410 |
| Rectangles and spirals | J.N.Ridley 416 |
| Johannes Kepler and his making of the Rudolphine Tables | Mark J. Cooker 425 |
| Monotonicity of the midpoint and trapezium estimates for integrals | G.J.O.Jameson 433 |
| Linearly periodic continued fractions | Kantaphon Kuhapatanakul 442 and Lalitphat Sukruan |
| A pretty series revisited | Seán M. Stewart 450 |
| Developing quadrature themes | John D. Mahony 458 |
| Verifying non-isomorphism of groups | Des MacHale 467 |
| Some more properties of the bisect-diagonal quadrilateral | Michael de Villiers 474 |
| Aesop's moral on success | Edward J. Allen 481 |
| Expected length and probability of winning a tennis game | Curtis Cooper and Robert E. Kennedy |
| A triangular exploration | Prithwijit De and Gerry Leversha |
| Matter for Debate |  |
| Correct answer - dodgy method | Des MacHale 507 |
| (C) The Mathematical Association 2021 Typeset by Bill Richardson |  |
| CAMBRIDGE UNIVERSITY PRESS |  |
| Printed in the UK by Bell \& Bain Ltd.ISSN 0025-5572 |  |

## CONTENTS (continued)

## Notes 105.38 to 105.49

A very simple proof of the two-squares theorem Stan Dolan ..... 511
The Eureka theorem of Gauss Stan Dolan ..... 512
A tale of two cubics Prithwijit De ..... 514
Topology haiku matrix Josh Hiller ..... 516
Generalised binomial theorem via Kuldeep Kumar Kataria ..... 516
and Raj Kumar Mistri
$c^{2}=a^{2}+b d$, a visual extension of the Francesco Laudano ..... 520 Pythagorean theorem
A segment inside a square that is equal to Victor Oxman and ..... 522
Moshe Stupel
British flag theorem for isosceles trapezia Quang Hung Tran ..... 523
Proximity of the incentre to the excentres and Martin Lukarevski ..... 529 inequality for the circumcevians of the incentre
On a synthetic proof for the conic Dario Pellegrinetti ..... 533 sections orthoptics
Approximations for the Feller games Toshio Nakata ..... 539
Equality in mathematics Des MacHale ..... 544
Correspondence ..... 549
Feedback ..... 550
Problem Corner Nick Lord ..... 542
Student Problems Beth Woollacott ..... 559
Reviews ..... 561
Acknowledgements ..... 575

