#### THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of The Mathematical Gazette, Mathematics in School, Primary Mathematics (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is 259 London Road, Leicester LE2 3BE, UK (telephone 0116 221 0013). The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to Teaching Committee should be addressed to TC Chair. The Association's Library is housed in the University Library, Leicester.

Views expressed in the *Mathematical Gazette* by authors or advertisers are not necessarily those of the Association.

### THE MATHEMATICAL GAZETTE

Editor:	Problem Corner:
Dr Gerry Leversha,	Mr Nick Lord,
15 Maunder Road,	Tonbridge School,
Hanwell,	Tonbridge,
London W7 3PN	Kent TN9 1JP
g.leversha@btinternet.com	njl@tonbridge-school.org
Production Editor:	Reviews Editor:
Mr Bill Richardson,	Mr Owen Toller,
Kintail,	4 Caldwell House,
Longmorn,	48 Trinity Church Road,
Elgin IV30 8RJ	London SW13 8EJ
wpr3145@gmail.com	owen.toller@btinternet.com

Potential advertisers e-mail Charlotte Dyason at: charlotted@media-shed.co.uk

Material for publication should be sent to the Editor. Books for review should be sent to the Reviews Editor.

#### Advice to authors of notes and articles.

Study the format of articles in the *Gazette*. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the *Gazette* was produced on an Acorn machine using TechWriter and Draw. 10.1017/mag.2021.151

### CONTENTS

Pairing theorems about parabolas through duality	Steven J. Kilner and David L. Farnsworth	385
Long medians and long angle bisectors	Sadi Abu-Saymeh, Yaqeen Al-Momani, Mowaffaq Haj, Mostafa Hayajneh	397 ia,
The probability certain random quadratics have real roots	Chris Boucher	410
Rectangles and spirals	J. N. Ridley	416
Johannes Kepler and his making of the Rudolphine Tables	Mark J. Cooker	425
Monotonicity of the midpoint and trapezium estimates for integrals	G.J.O.Jameson	433
Linearly periodic continued fractions	Kantaphon Kuhapatanakul and Lalitphat Sukruan	442
A pretty series revisited	Seán M. Stewart	450
Developing quadrature themes	John D. Mahony	458
Verifying non-isomorphism of groups	Des MacHale	467
Some more properties of the bisect-diagonal quadrilateral	Michael de Villiers	474
Aesop's moral on success	Edward J. Allen	481
Expected length and probability of winning a tennis game	Curtis Cooper and Robert E. Kennedy	490
A triangular exploration	Prithwijit De and Gerry Leversha	501

# Matter for Debate

Articles

Correct answer – dodgy method

Des MacHale 507

© The Mathematical Association 2021 Typeset by Bill Richardson

# **CAMBRIDGE** UNIVERSITY PRESS



Paper from responsible sources

Printed in the UK by Bell & Bain Ltd. ISSN 0025-5572

# **CONTENTS** (continued)

# Notes 105.38 to 105.49

A very simple proof of the two-squares theorem	Stan Dolan	511
The Eureka theorem of Gauss	Stan Dolan	512
A tale of two cubics	Prithwijit De	514
Topology haiku matrix	Josh Hiller	516
Generalised binomial theorem via Laplace transform technique	Kuldeep Kumar Kataria and Raj Kumar Mistri	516
$c^2 = a^2 + bd$ , a visual extension of the Pythagorean theorem	Francesco Laudano	520
A segment inside a square that is equal to the side of the square	Victor Oxman and Moshe Stupel	522
British flag theorem for isosceles trapezia	Quang Hung Tran	523
Proximity of the incentre to the excentres and inequality for the circumcevians of the incentre		529
On a synthetic proof for the conic sections orthoptics	Dario Pellegrinetti	533
Approximations for the Feller games	Toshio Nakata	539
Equality in mathematics	Des MacHale	544
Correspondence		549
Feedback		550
Problem Corner	Nick Lord	542
Student Problems	Beth Woollacott	559
Reviews		561
Acknowledgements		575