

The introductory of zeroth chapter is a heuristic description of generalized functions and direct operational methods. It also contains a brief resumé of the development of operational techniques in analysis. In chapters I-IV an easily understood development of generalized functions is presented and the resulting theory is applied to the solution of certain linear problems in analysis, primarily the solution of ordinary differential equations with constant coefficients. Chapters V and VI are more mathematical in nature and are devoted to studying structural properties of generalized functions. In these two chapters the author develops deep structural properties of the D'_+ functions, using as tools techniques of advanced calculus and arguments based on these techniques which he develops in the course of his exposition. One noteworthy example of the latter is the Lebesgue method of resonance in Chapter VI. Chapter VII introduces the Laplace transform of a certain subclass of D'_+ , which of course includes the usual functions to which the Laplace transform is applicable in the classical sense. It is then applied to the solution of linear ordinary differential equations, linear difference equations and linear partial differential equations. Chapter VIII is an introduction to periodic generalized functions and the theory of generalized Fourier series. The treatment in this chapter is more cursory than in chapters I-IV because, as the author points out, the exposition here is similar to that of Chapter II.

From a pedagogical point of view this is an exceptionally well written book. It could serve as the basis for an honors or graduate course for engineers and applied mathematicians or for self teaching provided the reader has a background in advanced calculus.

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Elements of Mathematics Logic, by P.S. Novikov. Translated by L. F. Boron, with Preface and Notes by R. L. Goodstein. Addison-Wesley Publ. Co., 1964. xi + 296 pages. \$7.95.

The author of this book is the same Novikov who showed the unsolvability of the word problem for groups and disproved Burnside's famous conjecture. The present book is a textbook in mathematical logic on the upper undergraduate level. The first four chapters are concerned with the predicate calculus, along the lines of the classic treatment of Hilbert and Ackermann, while the last two chapters are concerned with an axiomatic arithmetic and proof theory, including a complete proof of the consistency of a restricted system of number theory. One of the major themes throughout is the distinction between "actual infinity" (that is, the idea of an infinite set whose construction is completed and which can itself be operated on as a distinct entity) and "potential infinity" (that is, the idea of a process which can be carried out an arbitrary number of times to produce distinct objects,

but such that at no time are all of the objects produced by this process available for simultaneous consideration). The emphasis is on the Hilbert program, according to which metamathematical questions (such as the consistency and independence of axiom systems) are to be studied without using any method depending on the existence of an actual infinity.

Of the four chapters on the predicate calculus, the first two are devoted to the propositional calculus, while the last two are devoted to the predicate calculus proper. Within each of these pairs of chapters, the first is primarily devoted to heuristic and semantical questions, while the second gives a rigorous axiomatic development. In proving the completeness theorem for the predicate calculus, Novikov follows closely Hilbert and Ackermann (2nd ed.). In view of the well-known difficulties associated with rules of substitution, it is surprising that Novikov has followed Hilbert and Ackermann in defining the "true" (i. e. provable) formulas by means of axioms and rules of substitution instead of axiom schemata. Another curiosity is the name "Maltsev's Theorem" for a proposition essentially equivalent to the König Infinity Lemma.

The main novelty of the book lies in its last two chapters. Much of the material is contained in Kleene's Introduction to Metamathematics, but Novikov's treatment is more elementary and more leisurely. There is a good account of primitive recursive functions in axiomatic arithmetic, and a brief mention of general recursiveness. These notions are applied, in the last chapter, to give finitistic proofs of the consistency of restricted arithmetic (with all primitive recursive functions but without mathematical induction) and of the independence of mathematical induction from the other axioms. These results can be obtained more easily, perhaps, by the original Herbrand method. Nevertheless, they illustrate clearly the kind of reasoning characteristic of the Hilbert school while avoiding the complexities involved in a presentation of the full Incompleteness Theorem, and provide an excellent introduction for the study of that theorem.

The translation is adequate but not outstanding. There are a moderate number of misprints and translator's lapses. For example, on page 250, a certain operation is called "separation from the existential quantifier", while on page 280 the same operation is referred to as "isolation from the quantifier". R. L. Goodstein has contributed several helpful notes explaining slightly obscure passages in the text, as well as a one-page preface. In general, the reviewer feels that the book is useful as an introduction to mathematical logic, but that the definitive treatment of the material has not yet appeared.

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