

CORRESPONDENCE

The Joint Editors

14 December 1962

*The Journal of the Institute of
Actuaries Students' Society,*

Dear Sirs,

Since the publication of my letter in *J.S.S.* **16**, 6, my attention has been drawn to two papers in the *Transactions of the Society of Actuaries*. Both are by Mr Henry S. Huntington, the first (**10**, 329) being entitled 'Derivation of Premium Rates for Renewable Term Insurance', and the second (**12**, 526) 'Gross Premium Rates for Renewable Term Insurance'.

The fact that Mr Huntington's calculations are basically of assessment premiums is somewhat hidden by the fact that he brings in at the same time an option to convert to whole life assurance. A comparison between his formulae and my own is rendered a little difficult because he uses continuous functions and five-year renewal intervals, whereas I used discontinuous functions and one year renewal intervals, whilst any rough check on arithmetical results is vitiated by our using different tables of mortality.

In order to compare Mr Huntington's work with my own I have applied discontinuous functions and yearly renewal intervals on $A_{24/29} 3\%$ to his idea, that is to say instead of coming back from a point of expiration at age 81, I have come back from an option to convert to whole life assurance at that age. The following figures emerge:

x	$z_x \%$	$h_x \%$
70	5.222	5.222
71	5.723	5.723
72	6.255	6.255
73	6.816	6.815
74	7.407	7.409
75	8.036	8.035
76	8.724	8.697
77	9.542	9.398
78	9.479	10.102
79	8.703	10.780
80	6.330	11.369

The figures in the column under the heading $z_x\%$ are an expansion of the end section of the table I gave on page 481 of *J.S.S.* 16, 6, while the figures in the column under $h_x\%$ allow also for the option to convert to whole life assurance at age 81 in accordance with Mr Huntington's idea.

It will be seen that below age 76 there is a difference of not more than 2 in the last decimal place, while below age 73 (as far back as one likes to go) the columns are identical to three decimal places.

The fact that the scales coincide to three decimal places if the last few years before the option age are ignored, links up with the fact that there is a similar coincidence with a variation in termination ages in the case of pure assessment premiums, and a little more light might be thrown on this if an explicit formula for z could be produced and take the place of the chain method used by Mr Huntington and myself. The problem is not difficult if taken in steps and I give below the first two steps which are sufficient to indicate the pattern.

Step one (for mortality tables with one year of selection):

$$z_x = \frac{C_{[x]}}{D_{[x]}} + \frac{D_{x+1}}{D_{[x]}} \left[\frac{C_{x+1}}{D_{x+1}} - \frac{C_{[x+1]}}{D_{[x+1]}} \right] - \frac{D_{x+1}}{D_{[x]}} \left[\frac{C_{x+1}}{C_{x+1}} - \frac{C_{[x+1]}}{D_{[x+1]}} \right] \\ \times \left[\frac{C_{x+2}}{D_{x+2}} - \frac{C_{[x+2]}}{D_{[x+2]}} \right] + \dots - \dots$$

Step two (for mortality tables with two years of selection):

$$z_x = \frac{C_{[x]}}{D_{[x]}} + \frac{D_{[x]+1}}{D_{[x]}} \left[\frac{C_{[x]+1}}{D_{[x]+1}} - \frac{C_{[x+1]}}{D_{[x+1]}} \right] \\ - \frac{D_{[x]+1}}{D_{[x]}} \left[\frac{C_{[x]+1}}{D_{[x]+1}} - \frac{C_{[x+1]}}{D_{[x+1]}} \right] \left[\frac{C_{x+2}}{D_{x+2}} - \frac{C_{[x+2]}}{D_{[x+2]}} \right] + \dots - \dots \\ + \frac{D_{[x]+1}}{D_{[x]}} \cdot \frac{D_{[x]+1+1}}{D_{[x+1]}} \left[\frac{C_{x+2}}{D_{x+2}} - \frac{C_{[x+1]+1}}{D_{[x+1]+1}} \right] - \dots$$

There can be seen explicitly in these formulae what is implicit in the chain method, i.e. that the value of z_x depends upon the degree of selection exhibited by the table at every age after age x . It is apparent also how small is the effect on the value of z_x of subsequent terms.

To give a numerical example using $A_{49/52} 3\%$ and employing the formula for mortality tables with two years of selection,

$$\begin{aligned} z_{40} &= \cdot 00100 + \cdot 96988(\cdot 00143 - \cdot 00110) \\ &\quad - \cdot 96988(\cdot 00143 - \cdot 00110)(\cdot 00224 - \cdot 00121) \\ &\quad + \cdot 96988 \times \cdot 96977(\cdot 00224 - \cdot 00159) \\ &= \cdot 00100 + \cdot 00032 - \cdot 00000 + \cdot 00061 \\ &= \cdot 00193. \end{aligned}$$

The chain value is $\cdot 00194$.

Incidentally, although z_x exceeds $q_{[x]}$ it does not necessarily exceed q_x at the older ages.

Yours faithfully,

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P.S. It has been pointed out to me that M_{x+n} (p. 480, *J.S.S.* 16, 6) is incorrect. It should be $M_{[x+t]+n-t}$.

The Editor,

18 July 1962

*The Journal of the Institute of
Actuaries Students' Society*

Dear Sir,

I have recently had occasion to borrow from the Library of the Institute a volume of interest tables by Bošnjak, the instructions for which are in four languages. Whilst I fully realize that the *Journal* does not normally accept hilarious articles, I thought this quotation from the English version of the 'Publisher's Preface', written in 1931, too good to let slip:

'... a crisis of international confidence is threatening to strangle international intercourse'.

'A heavy responsibility is put on the shoulders of undertakers to apply the economic values at their disposal to really productive application.'

Yours faithfully,

H. A. R. BARNETT

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The Joint Editors,
*The Journal of the Institute of
Actuaries Students' Society*

30 May 1962

Dear Sirs,

In reply to Mr H. A. R. Barnett's letter in your May issue I put forward my 'Germ' for consideration, and no doubt it will cause inflammation in some circles.

As a new reader of the *J.S.S.* I only had to glance at the *Journal* to see its fault. It is not a journal for students as the title would imply, but a journal for the qualified esoteric of 'at least three years standing'.

Like Mr Barnett, I too searched through a previous *Journal* for some interesting points in the correspondence section, which as a student I might be able to comprehend and find interest in, but found nothing despite the assurance early on in the *Journal* that these items do exist.

Students are not bothering to comment in the *Journal*, not due to timidity, as was suggested by Mr Barnett, but because there is little in the *Journal* for them to reply authoritatively on.

I would therefore beg to suggest that Fellows wishing to propagate any advanced subjects send them to the *J.I.A.* for inclusion in that journal, and have them criticized or applauded at the right level, rather than burden the student with them. Either change the contents of the *J.S.S.* or the name.

When the editors find more matter for the less advanced students, interest and correspondence from them would undoubtedly increase. Perhaps this may be the beginning.

Yours faithfully,

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