NON-PERIODIC LATITUDE VARIATIONS AND THE SECULAR MOTION OF THE EARTH'S POLE

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1. The phenomenon known as the secular polar motion is, in essence, a relative displacement of the mean pole of the epoch of observation and the Conventional International Origin (CIO). The latter is attached to the zeniths of the five international latitude stations. If the zeniths were fixed relative to one another, or, in the terminology of A. Danjon, formed an invariable constellation, then the CIO could be considered as fixed with respect to such a constellation. But this is not the case: displacements of zeniths with respect to one another do occur. The question is how large they are.

In 1903 the CIO and the mean pole of the epoch of observation coincided; then they drifted apart and in 1970 were separated by approximately 0.″2. We could neglect relative displacements of the zeniths provided that they were small enough in comparison with the above value. Whether it is really so should be checked first of all.

Let A_i, A_j be two points on the Earth's surface and s_{ij} the arc of the great circle between the zeniths Z_i, Z_j of these points (Figure 1). Real changes of the arc s_{ij} could result from crustal displacements and changes in direction of gravity. Errors of the adopted proper motions of the observed stars and certain instrumental errors are capable of producing spurious variations. But the arc s_{ij} is completely independent of the polar motion.

Systematic latitude and time observations at the points A_i, A_j enable variation of s_{ij} to be found by the following equation

$$\Delta s_{ij} = -\cos a_i \cdot \Delta\varphi_i - \cos a_j \cdot \Delta\varphi_j - \kappa_{ij} \cdot \Delta(\lambda_i - \lambda_j) \tag{1}$$

$$\kappa_{ij} = \frac{\cos\varphi_i \cdot \cos\varphi_j \cdot \sin(\lambda_i - \lambda_j)}{\sin s_{ij}} \tag{2}$$

φ_i, φ_j and λ_i, λ_j are the latitudes and longitudes of the points A_i, A_j respectively; a_i, a_j are the azimuths of the arc s_{ij} at these points. If

$$|\cos a_i| \gg |\kappa_{ij}| \ll |\cos a_j|$$

the second term in (1) may be neglected and latitude variations alone used in computing Δs_{ij}.

For different combinations of observatories a total of 60 series of Δs_{ij} ranging from 5 to 37 yr has been obtained. It is sufficient to take from them several series as an example. Annual means of Δs_{ij} have been computed by the equation

$$\overline{\Delta s_{ij}} = -\cos a_i \cdot \overline{\Delta\varphi_i} - \cos a_j \cdot \overline{\Delta\varphi_j},$$

where $\overline{\Delta\varphi_i}$, $\overline{\Delta\varphi_j}$ are the annual means of latitude variations at the points A_i, A_j. One

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can see from Figure 2 that the arc between the zenith of Ukiah and those of Poltava, Pulkovo and Kitab shortened from 1963 to 1968 by 0$''$.12 on the average. This fact could not be ascribed to the errors of the proper motions of the stars, since all the curves are similar though the stars observed at the three above observatories are different. We may suppose that the changes in s_{ij} are due to the errors of the proper motions of the stars observed at Ukiah. Then, however, similar changes should have been observed at Geitersburg. But Figure 3 shows that the trend of Geitersburg is opposite to that of Ukiah, as if Geitersburg has been moving away from Ottawa, Blagoveschensk and Irkutsk and coming nearer to Richmond and Washington. Though these displacements can not be considered as real, it is important that they are of the same order as the relative shift of the CIO and the mean pole of the epoch. It means that in studying the secular polar motion the relative displacements of the zeniths should not be neglected.

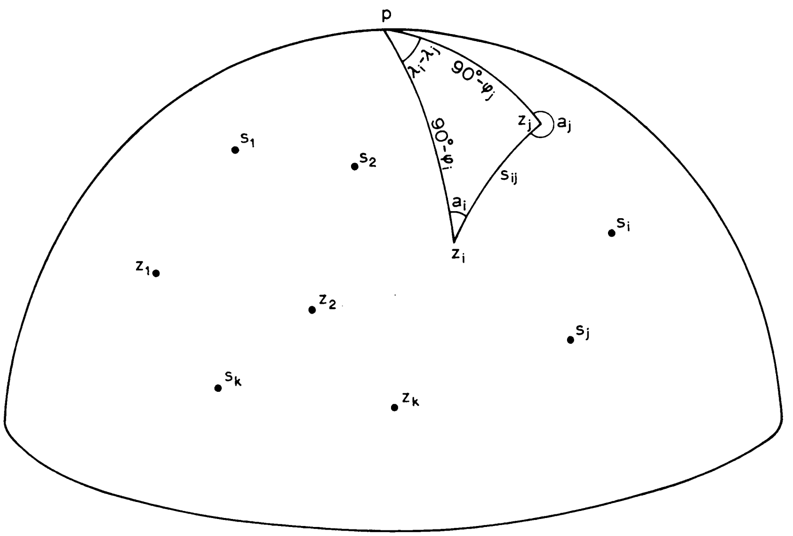

Fig. 1.

2. Let us take an arbitrary point 0 and draw from it unit vectors $\overline{0Z_1}, \overline{0Z_2}, \ldots, \overline{0Z_n}$ parallel to the vertical lines at the points $A_1, A_2, \ldots, A_n$. Now we have to decide how to attach the rotating axes $\xi\eta\zeta$ to the pencil of these vectors, if they do not maintain their direction relative to one another.

All further constructions will be made upon the surface of the auxiliary sphere of unit radius. The 'stars' $S_1, S_2, \ldots, S_N$, the zeniths of the observatories $Z_1, Z_2, \ldots, Z_n$ and the poles are all situated on this surface. Let XYZ be a set of rigid axes in some way or other attached to the stars, and let α_i, δ_i be the spherical coordinates of the zenith Z_i in the system XYZ. They can be determined from observation, for instance, with photographic zenith tubes. It is assumed that a catalogue of star positions is already available.

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It is impossible to attach the rotating axes $\xi\eta\zeta$ to the zeniths rigidly for the zeniths themselves do not form a rigid constellation. But the following condition can be imposed on the displacements $\bar{\varrho}_i$ of the zeniths Z_i in the system $\xi\eta\zeta$

$$\sum_{i=1}^{n} \varrho_i^2 = \min . \tag{3}$$

The motion of the axes $\xi\eta\zeta$ is governed by the condition (3) only. The choice

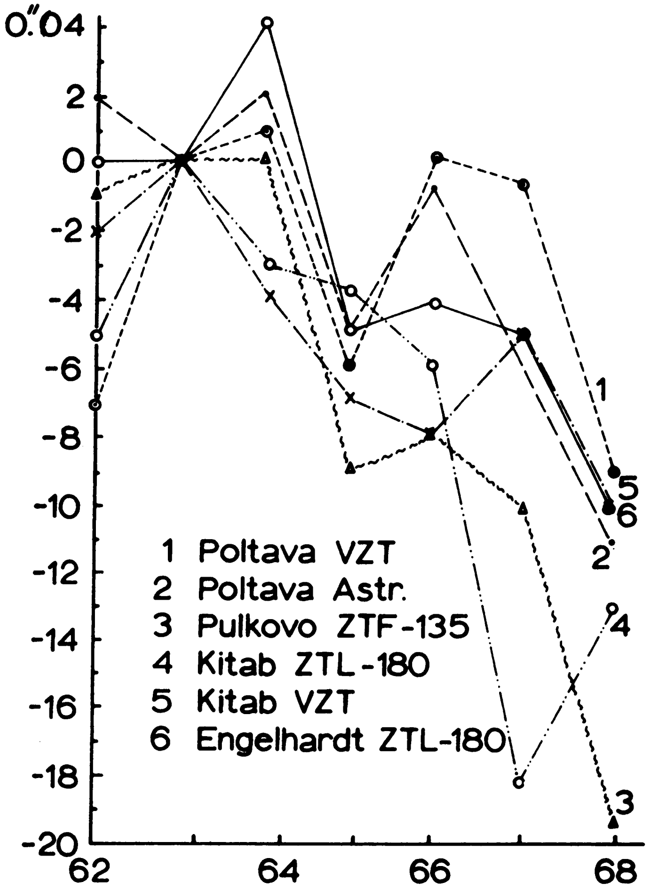

Fig. 2.

remains of determining their initial position which is completely arbitrary. We may, for example, choose some initial values of the Eulerian angles Ψ_0, θ_0, Φ_0 to define the position of these axes at the moment t_0. Then the relation linking the two sets of coordinates will be

$$s(t_0) \begin{Bmatrix} \cos\delta_i \cdot \cos\alpha_i \\ \cos\delta_i \cdot \sin\alpha_i \\ \sin\delta_i \end{Bmatrix} - \begin{Bmatrix} \cos b_i \cdot \cos l_i \\ \cos b_i \cdot \sin l_i \\ \sin b_i \end{Bmatrix}, \tag{4}$$

where $S(t_0)$ is the rotation matrix and l_i, b_i are the spherical coordinates of the zenith Z_i at the moment t_0.

In calculation of the elements of the matrix $S(t_0)$ it is convenient to represent it in the following form

$$s(t_0) = r(\Phi_0)\cdot p(\theta_0)\cdot r(\Psi_0), \tag{5}$$

where

$$p(\beta) = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{Bmatrix}, \quad r(\beta) = \begin{Bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{Bmatrix}. \tag{6}$$

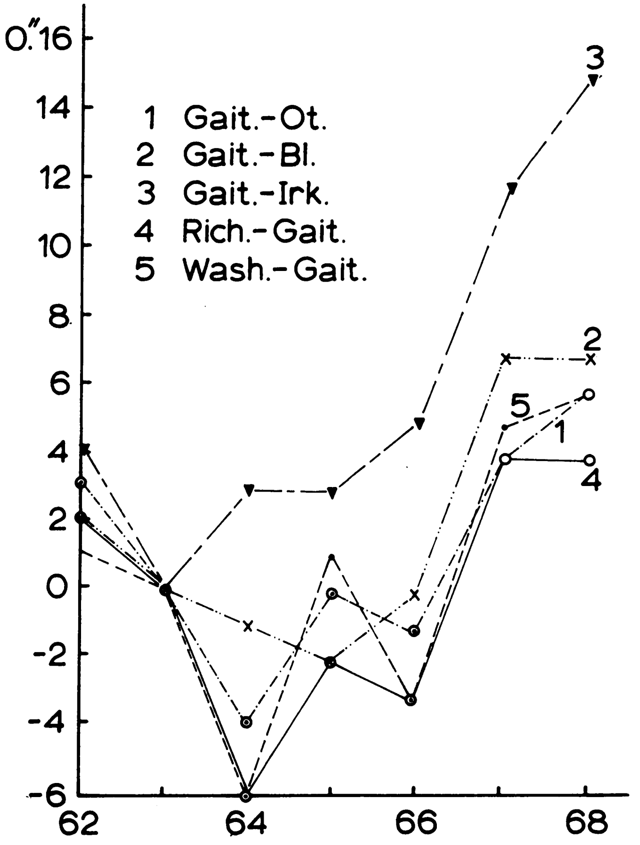

Fig. 3.

Since the matrix $S(t_0)$ is given and the coordinates α_i, δ_i are obtained from observation, the coordinates l_i, b_i, can be found by means of (4). But we should keep in mind that they are not the coordinates on the Earth's surface of the point A_i itself, but the coordinates of its zenith on the auxiliary sphere.

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For any other moment t we may write

$$s(t) = r(\Phi)\cdot p(\theta)\cdot r(\Psi).$$

and

$$s(t)\begin{Bmatrix} \cos\delta_i\cdot\cos\alpha_i \\ \cos\delta_i\cdot\sin\alpha_i \\ \sin\delta_i \end{Bmatrix} = \begin{Bmatrix} \cos(b_i + \Delta b_i)\cdot\cos(l_i + \Delta l_i) \\ \cos(b_i + \Delta b_i)\cdot\sin(l_i + \Delta l_i) \\ \sin(b_i + \Delta b_i) \end{Bmatrix}. \tag{7}$$

It is easy to express $\bar{\varrho}_i$ in terms of Δb_i, Δl_i and then to find values of the angles Ψ, θ, Φ which satisfy the condition (3). But it is more practical, taking certain approximate values of the Eulerian angles, to search for small corrections to them. We denote by Ψ', θ', Φ' these approximate values computed using the theory of rotation of the rigid Earth, the angles Ψ^1, θ^1 fixing the orientation of its moment of momentum. Predicted values of these angles are published in the *Astronomical Ephemeris*. We shall call Ψ', θ', Φ' the ephemeris Eulerian angles. They define the position in space of the auxiliary axes $\xi'\eta'\zeta'$ which may be also called the ephemeris axes. Let λ_i, φ_i be the coordinates of the zenith Z_i in this system and $\lambda_i - l_i = \Delta\lambda_i$, $\varphi_i - b_i = \Delta\varphi_i$. To pass to the system $\xi\eta\zeta$ let us revolve the axes $\xi'\eta'\zeta'$ through the small angles u, v, w shown on Figure 4. The matrix of transformation is $E+\sigma$, where by E we denote the unit matrix and

$$\sigma = \begin{Bmatrix} 0 & w & -v \\ -w & 0 & u \\ v & -u & 0 \end{Bmatrix}. \tag{8}$$

Then the displacement of the zenith Z_i relative to the system $\xi\eta\zeta$ is expressed as

$$\begin{Bmatrix} -\sin b_i\cdot\cos l_i \\ -\sin b_i\cdot\sin l_i \\ \cos b_i \end{Bmatrix}\Delta\varphi_i + \begin{Bmatrix} -\cos b_i\cdot\sin l_i \\ \cos b_i\cdot\cos l_i \\ 0 \end{Bmatrix}\Delta\lambda_i + \\ + \begin{Bmatrix} 0 & w & -v \\ -w & 0 & u \\ v & -u & 0 \end{Bmatrix}\cdot\begin{Bmatrix} \cos b_i\cdot\cos l_i \\ \cos b_i\cdot\sin l_i \\ \sin b_i \end{Bmatrix} = \bar{\varrho}_i. \tag{9}$$

Now our task is to find the values of the angles u, v, w which satisfy the condition (3).

For this to be done we should square the right side of (9), then take the sum from $i=1$ to $i=n$ and equate to zero the partial derivatives of this sum by u, v, w. We obtain

$$\begin{aligned} A_{11}u + A_{12}v + A_{13}w &= \sum_i \sin l_i\Delta\varphi_i - \sum_i \cos b_i \sin b_i \cos l_i\Delta\lambda_i, \\ A_{21}u + A_{22}v + A_{23}w &= -\sum_i \cos l_i\Delta\varphi_i - \sum_i \cos b_i \sin b_i \sin l_i\Delta\lambda_i, \\ A_{31}u + A_{32}v + A_{33}w &= \sum_i \cos^2 b_i\Delta\lambda_i \end{aligned} \tag{10}$$

where

$$A_{11} = \sum_i (1 - \cos^2 b_i \cdot \cos^2 l_i), \quad A_{12} = A_{21} = -\sum_i \cos^2 b_i \cdot \cos l_i \cdot \sin l_i,$$
$$A_{13} = A_{31} = -\sum_i \cos b_i \cdot \sin b_i \cdot \cos l_i, \quad A_{22} = \sum_i (1 - \cos^2 b_i \cdot \sin^2 l_i),$$
$$A_{23} = A_{32} = -\sum_i \cos b_i \cdot \sin b_i \cdot \sin l_i, \quad A_{33} = \sum_i \cos^2 b_i. \tag{11}$$

Equations (10) represent the general solution of the problem stated above: how to

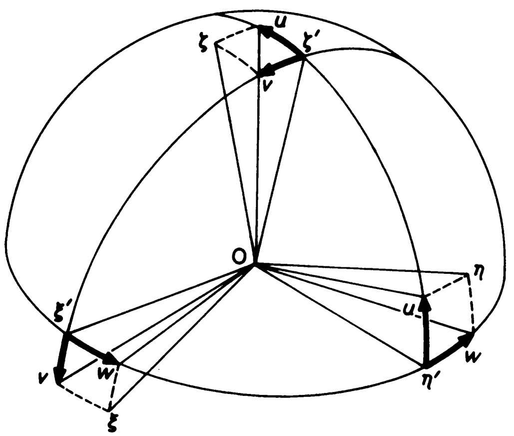

Fig. 4.

attach a system of coordinates to the pencil of vertical lines, if they are moving relative to one another. But usually two conditions are considered separately

$$\sum_i (\Delta b_i)^2 = \min. \quad \sum_i (\Delta l_i)^2 = \min. \tag{12}$$

From (12) we get

$$\begin{aligned} &\sum_i \sin l_i \cdot \Delta\varphi_i - v \sum_i \cos l_i \cdot \sin l_i - u \sum_i \sin^2 l_i = 0 \\ &\sum_i \cos l_i \cdot \Delta\varphi_i - v \sum_i \cos^2 l_i - u \sum_i \cos l_i \cdot \sin l_i = 0. \end{aligned} \tag{13}$$

3. Thus, on the surface of the auxiliary sphere we have the following points:

The ephemeris ζ' (which is usually taken for the pole of rotation, the celestial or instantaneous pole). The position of the point ζ' is defined by the adopted equations of precession and nutation and is independent of the combination of observatories used.

The point ζ which may be called a conditional reference pole. It moves so as to keep variations of its angular distances from the zeniths of a number of observatories as small as possible. Each combination of observatories will have its own pole ζ. That of the five international stations is called the Conventional International Origin.

If the coordinates x, y on the sphere are drawn from the CIO in the direction of Greenwich and 90° east of Greenwich respectively,

$$x = -v, \quad y = u.$$

It has been noticed that at the observatories which are close in longitude only the periodic latitude variations are similar while the nonperiodic could be quite different. For this reason A. J. Orlov recommended that the mean pole of the epoch of observation be adopted as the origin of polar coordinates. This is the point which is defined so that instead of (12)

$$\sum_i (\Delta b_i - \psi_i)^2 = \min,$$

where ψ_i is the variation of the mean latitude. To obtain ψ_i one can use the well known methods of filtration of observational data, for example, Orlov's formula which seems to be the simplest one.

Now instead of (13) we can write

$$x \sum_i \cos l_i \cdot \sin l_i - y \sum_i \sin^2 l_i = \sum_i \Delta\varphi_i \cdot \sin l_i,$$
$$x \sum_i \cos^2 l_i - y \sum_i \cos l_i \cdot \sin l_i = \sum_i \Delta\varphi_i \cos l_i$$

and substituting $\Delta\varphi_i - \psi_i$ for $\Delta\varphi_i$ we get the coordinates of the ephemeris pole ζ' relative to the mean pole of the epoch of observation. We denote them by x_p, y_p.

During the last 15 yr the number of observatories conducting latitude observations has considerably increased. This fact enables the comparison to be made of two systems of polar coordinates: the system/of the CIO and that of the mean pole of the epoch of observation in order to ascertain in which system the sum (12) is less.

We have computed for each observatory Δb_i using the following formulae according to the system of coordinates

$$\Delta b_i = \varphi_i - b_i - (x \cos l_i - y \sin l_i),$$
$$\Delta b_i = \varphi_i - b_i - (x_p \cos l_i - y_p \sin l_i),$$

where mean values of φ_i over the year 1962 were taken for the initial values of b. Then we formed the annual mean values of Δb_i. These were squared and averaged (the number of observatories was not the same over the whole period. It changes from 14 to 27). The results thus obtained are shown in Figure 5.

The difference between the two curves is due solely to the fact that the variations Δb_i were obtained with respect to different origins. Other effects such as, for example, the effect of the errors of proper motions are the same in both cases.

Curve 2 is seen to pass above curve 1. It means that the condition (12) is better satisfied for the system of the mean pole of the epoch of observation than for the system of the CIO.

It is in agreement with what would be expected on the theory of the rotation of the Earth. For any model of the Earth the pole of rotation revolves about the pole of inertia, so that the mean position of the former is always very near to the latter. So if we would feel it necessary to name a 'fixed' pole and we have to decide between the above two poles, preference should be given to the mean pole of the epoch of observation.

The XIII Meeting of the IAU recommended that coordinates of the instantaneous

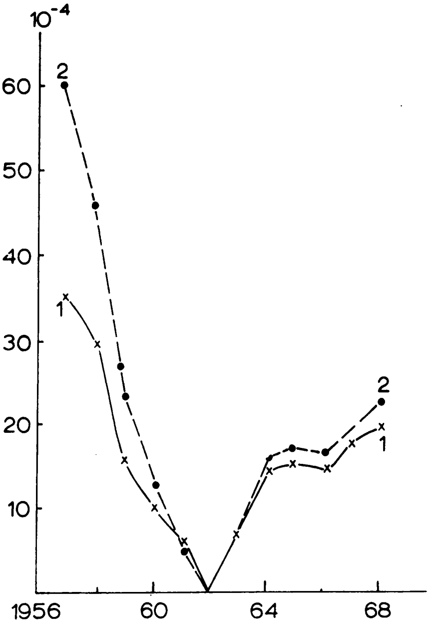

Fig. 5.

pole should be defined relative to the CIO. This is quite reasonable as far as observations of the IPMS alone are dealt with. But for astronomical and geodetic usage in general the mean pole of the epoch of observation is preferable.

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