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Conceptual design and analysis by sketching

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Abstract

The ability of a CAD system to perceive a three-dimensional model depicted in a single freehand sketch presents the practical possibility of bringing numerous established analysis tools into the early stages of design to institute conceptual analysis. In this article we hypothesize that the key to enabling systems to reason and communicate about conceptual design is the language of sketching. We explore this approach, outline the basic algorithms required, and provide several examples of an implemented system.

Keywords: Line Drawing; Sketch Interpretation; Conceptual Design; CAD

1. INTRODUCTION

Although there has been much debate about the nature of the engineering design process, its order is generally agreed upon: specification, conceptual design, embodiment, and detailed design. Moreover, it is further agreed that the conceptual design stage is by far the most critical in the design process. Yet despite this fact, computer-aided design systems are primarily geared towards the later, more detailed stage. In fact, even with today's abundance of powerful CAD systems, engineers typically begin their design exploration with a pencil and paper, turning to the computer only after their basic concepts have been established. In practice, the ubiquitous CAD systems in today's market are often no more than passive three-dimensional drawing boards. But can we develop intelligent CAD systems that will actually understand our designs at their early conceptual stage and offer creative feedback and true engineering insight? Current CAD system development has been focusing on the detailed design stage because at that stage information is well defined and easier to handle, whereas at the conceptual stage information is typically more vague and obscure. Yet a human engineer *is* able to reason about that same design while it is only a sketch and is even able to predict some of its basic properties. Thus, in this article, we hypothesize that in fact it is the language of communication—*sketching*—that should serve as a key ingredient in allowing computer-aided engineering tools to be applied at the very early design stage, where critical decisions are made. It is this precise ability that we seek to emulate in this research.

1.1. Structure of this article

In the first section of this article we will provide an overview and support our sketch-based approach, and briefly review existing sketch-interpretation algorithms. We will then outline the basic algorithms we used in two stages: 1) the sketch interpretation phase, used to “understand” the sketched conceptual design, and 2) an analysis tool used to reason about the conceptual design and output results in sketch format. We will then provide an example of an implemented system for the design and analysis of sheet metal parts.

2. THE IMPORTANCE OF SKETCHING

It is interesting to watch how an engineer, when given a design problem, instinctively reaches for a pencil and paper. The importance of drawing, both formal drafting and informal sketching, has been the subject of extensive research. In a series of experiments, Ullman et al. (1990) showed the necessity of drawing during all developmental stages of a mechanical design. Dan Herbert (1987) defined sketches as “informal, private drawings that architectural designers use as a medium for graphic thinking in the ex-

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ploratory stages of their work." Larkin and Simon (1987) concluded that sketches (diagrams) allow for grouping information in an easily accessible and retrievable format, as opposed to textual descriptions that are sequential in nature. Walderon and Walderon (1988) showed that mechanical designs are perceived on a variety of levels of sketch abstraction, ranging from simple geometrical entities to functional components. The use of sketching, therefore, avoids the necessity of transforming the designer's thoughts into a set of primitives and operations dictated by a particular software. In a survey of the adequacy of CAD tools for conceptual design, Puttre (1993) emphasized the primary benefit offered by the fluency, flexibility, and inaccuracy of sketches. An industrial designer relating to an existing CAD system is quoted saying "The interface is just not for us. I can do thirty sketches on paper by the time it takes me to do two on the computer." Jenkins and Martin (1993) also indicated that rough sketching is important in terms of flexibility and speed. They emphasized the fact that the reduced cognitive load obtained by avoiding the need for conversions suits short-term memory, which is typically fast but limited in capacity. Fang (1998) concluded from videotapes of designers at work that drawing and sketching have six primary uses: to archive the geometric and topologic form of a design; to communicate ideas among designers; to act as an analysis tool; to simulate the design; to serve as a completeness checker; and to act as an extension of the designer's short-term memory. An additional important aspect of sketches is that, because they are rough, one is less reluctant to discard them and try a different approach to the design. Indeed, in teaching design, we advocate sketching as means to promote "free" exploration of as many raw ideas as possible.

In summary, sketching appears to be important for the following reasons:

1. It is **fast**, suitable for the capacity of short term memory.
2. It is **implicit**, that is, describes form without a particular sequential structure.
3. It serves for **analysis**, completeness check, and simulation.
4. It is **inexact** and abstract, avoiding the need to provide unnecessary details.
5. It requires **minimal commitment**, is easy to discard and start anew.

2. SKETCH-BASED CAD SYSTEMS

Perhaps the earliest computerized sketching system (in fact, the earliest CAD system) is Sutherland's *Sketchpad* (1963). In that system, the user could draw using a light pen on a screen and manipulate graphic primitives such as arcs and lines. Since the development of *Sketchpad*, numerous graphic drawing packages have been developed, but only a few of them have tried to "understand" the picture being drawn, in the sense that they detect relationships not explicitly specified by the user, or connect individual components to form a "larger context," as humans may do when looking at a sketch. Moreover, not many of these systems support true *freehand* sketching, let alone freehand sketches of three-dimensional objects.

Kato et al. (1982) described a system for interactive processing of two-dimensional freehand-sketched diagrams. Jenkins and Martin (1993) described a system called *Easel* for online (interactive) freehand sketching of two-dimensional graphics comprised of lines, arcs, and Bezier curves. Their system is certainly aimed in the right direction, as it attempts to conform to some of the crucial aspects of sketching discussed in the previous sections by accepting direct freehand sketching and tolerating inaccuracies. The system avoids the use of menus so as not to impede the creative process, and therefore automatically distinguishes between stroke types and infers implicit constraints among them. Fatos and Ozguc (1990) described a system for two-dimensional architectural sketch recognition with lines, arcs, and corners. Hwang and Ullman (1990) described a system for capturing "back of the envelope" sketches. Eggli et al. (1995) proposed a solid modeler incorporating a sketching tool; their system is three dimensional but the sketching itself is always constrained to some plane, thereby avoiding the problematic inverse-projection reconstruction phase. A similar system for designing solid objects using interactive sketch interpretation was described by Pugh (1992).

3.1. Reconstruction of a three-dimensional object from a single view

When processing three-dimensional geometry, a system needs to extract spatial information from the inherently flat sketch. There are several reports of methods used to reconstruct a three-dimensional object from multiple views. However, systems for processing single-view sketches depicting three-dimensional scenes are less common. The primary difficulty is the need to perform an inverse projection from the plane of the sketch to three dimensions. This step is mathematically indeterminate, but humans seem to be able to accomplish this with little difficulty. Consequently, reports on systems attempting to interpret sketches as three-dimensional scenes typically focus on the reconstruction phase, whereas the type of input (freehand sketch or formal drawing) is of less interest.

3.1.1. Problem statement

Because the two-dimensional line drawing lacks depth information, the reconstruction process is ill posed mathematically. Consider, for example, the sketch shown in Figure 1 below. This sketch depicts a three-dimensional object; it is a projection of that object.

However, the sketch itself notes more than two dozen lines drawn on a flat media, that is, on this sheet of paper. It is typically very difficult for humans to conceive of a sketch as a collection of lines while disregarding their three-dimensional interpretation. For a computer, however, the

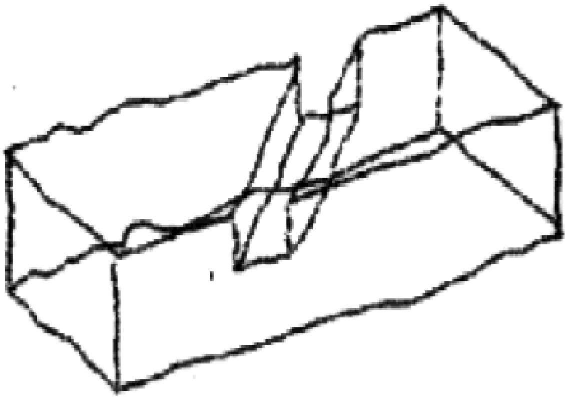

Fig. 1. A simple sketch of a block with a diagonal notch.

sketch is indeed nothing but a flat projection. Because the sketch is flat, it represents the projection of some three-dimensional objects. Indeed, there are an infinite number of objects which correspond to that particular projection. Some of them are shown in Figure 2.

Despite the many corresponding objects, most observers of this sketch will agree on a particular interpretation. That interpretation is shown in Figure 3. This consensus indicates that a sketch may contain additional information that makes us agree on the most plausible interpretation. It is exactly this capability that we wish to emulate.

Perhaps the earliest attempt to interpret line drawings was by Roberts (1963), who attempted to match a given line drawing to one of a set of primitives (predefined models), using a best-fit transformation. The matching is performed using a least squares fitting. Guzman (1968) developed a system called SEE to analyze line drawings of polyhedra without explicit models. His system uses simple heuristics about line junctions and relationships among nodes to establish an interpretation of a scene of multiple objects, with partial occlusions. Suffel and Blount (1989) described a system whose ultimate goal is to take an artist's original sketch made on paper, digitize it, and convert it directly into a three-dimensional computer model.

The literature contains several fundamentally different approaches to interpretation and reconstruction of objects and scenes from line drawings. These are briefly described below, along with key references. For a full survey see Lipson (1998).

Line labeling is a form of *interpretation* of a line drawing; it provides information about the drawing but does not yield an explicit three-dimensional representation. Each line in the drawing is assigned one of three meanings: convex, concave, or occluding edge. Junctions dictionaries and constraint graphs are used to find consistent interpretations (Clowes, 1971; Huffman, 1971).

The **gradient space** approach draws a relationship between the slope of lines in the drawing plane and the gradient of faces in the three-dimensional depicted scene. Assuming a particular type of projection, an exact mathematical relationship can be computed, and possible interpretations of the drawing can be constrained (Mackworth, 1973; Wei, 1987).

The **linear system** approach uses a set of linear equalities and inequalities defined in terms of the vertex coordinates and plane equations of object faces, determined by whether vertices are on, in front of, or behind the polygon faces. The solvability of this linear program is a sufficient condition for the reconstructability of the object (Sugihara, 1986; Grimstead & Martin, 1995). Linear programming optimization may yield a solution.

Interactive methods gradually build up the three-dimensional structure by attaching facets one after the other as sketched and specified by a user. The aim is to provide a practical method for constructing three-dimensional models in an interactive CAD/CAM environment (Fukui, 1988; Lamb & Bandopadhay, 1990).

The **primitive identification** approach reconstructs the scene by recognizing instances or partial instances of known primitive shapes, such as blocks, cylinders, and so forth. This approach contains a strict assumption that the depicted three-dimensional object is composed entirely of known primitives, but has the benefit of yielding the final three-dimensional structure in a convenient constructive solid geometry (CSG) form (Wang & Grinstein, 1989).

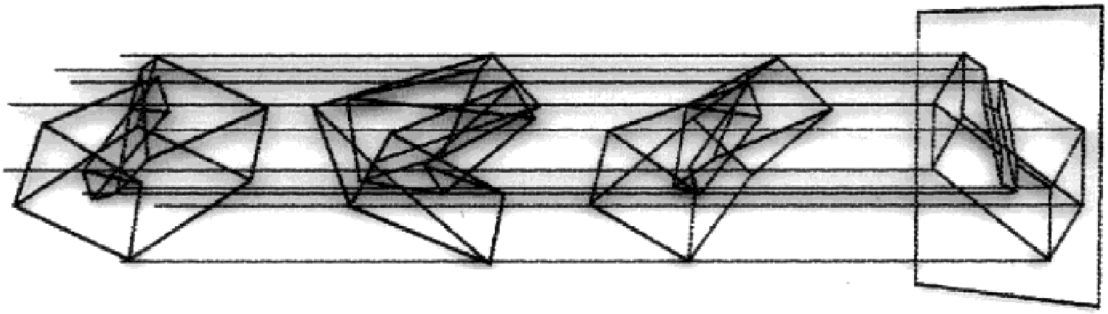

Fig. 2. There is an infinite number of three-dimensional objects corresponding to a single projection (sketch).

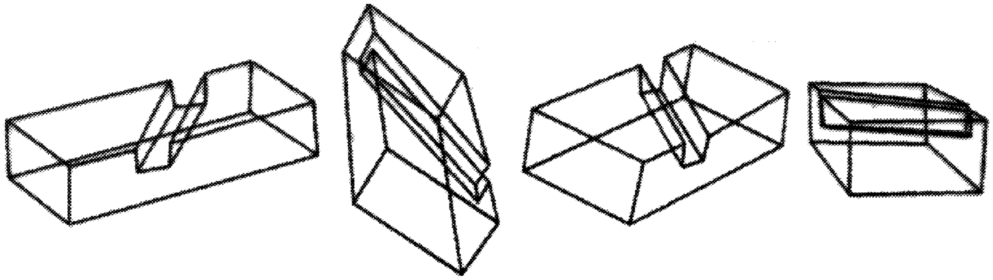

Fig. 3. A single sketch typically has a single plausible interpretation.

The **minimum standard deviation** approach focuses on a single and simple observation; that human interpretation of line drawings tends towards the most simple interpretation. In Marill's (1991) article, simplicity is defined as an interpretation in which angles created between lines at junctions are as uniform as possible across the reconstructed object, inflating the flat sketch into a regularized three-dimensional object (Marill, 1991; Leclerc & Fischler, 1992).

4. GEOMETRIC CORRELATIONS

The ability of human observers to directly perceive the three-dimensional object depicted by a sketch for complex drawings as for simple drawings and in spite of severe accuracies and drawing errors suggests that human perception does not employ techniques such as line labeling and mathematical gradient equations or logical deduction as described above. In this work we used the perceptual approach (Lipson, 1998), which tries to address the reconstruction problem from a more humanly plausible point of view, based on learning correlations.

The approach advocated here is based on the assumption that the human ability to understand sketches is primarily *experience*-based. Our accumulated visual and physical interaction experience helps us relate visual stimuli, such as arrangements of *lines*, to spatial structures we have experienced, such as physical corners. The capacity to accumulate such associative experience is well suited to observations in the primary visual cortex. Originally, Hubel and Wiesel (1977) noted that different areas of neurons represent different tilt angles of a line stimulus. Based on Hebbian learning and correlation among visual line stimuli, similar line orientation maps have then been shown to arise spontaneously on the basis of self-organization (Linsker, 1986). It is therefore plausible that at a higher level in the visual cortex, correlations among line orientations and other types of three-dimensional stimuli, such as stereoscopic or tactile, would yield some association between an arrangement of lines (say, in a drawing), and the corresponding three-dimensional structure (say, corners).

To quantify this notion of geometric correlations, it is necessary to seek correlations among various line arrangements in a drawing and the corresponding three-dimensional structure. In essence, the intuitive questions to ask are, for example, whether there are certain configurations of lines in drawings that are more likely to occur than others, and what are these configurations more likely to represent in three dimensions. Such questions can be broken down into more specific sets of questions at various orders of complexity. For example, at a first-order level, one can ask whether certain three-dimensional line orientations are more likely to be represented by preferred angles in the drawing plane. Intuitively, we know that the answer to this question is *yes*; in manually drawn objects, vertical lines in three dimensions are indeed more likely to be drawn vertically in the sketch plane. This is, of course, not a mathematical result, but one that arises from our experience of seeing things, perhaps because we usually experience the world upright. Consequently, this correlation is not true for computer-generated drawings of objects, but it is the case for manually drawn sketches of man-made objects. Moreover, because this is not a mathematical result, no direct geometrical analysis (Ulupinar & Nevatia, 1991; Ponce & Shimshoni, 1992) will yield it. At a second order of complexity, one can seek geometric correlations among *pairs* of line segments. For example, is there a correlation between the angle of a pair of lines u,v in three dimensions and the angle spanned between the projection of these two lines u',v' in a sketch? This correlation *can* be computed mathematically for arbitrary geometry, but for the geometry of man-made objects and manually drawn sketches, one has to measure this correlation empirically.

Figure 4 shows an empirical correlation plot for the case of second-order complexity. Each dot in the plot represents a measurement made on a pair of lines. The abscissa measures the dot product (corresponding to the angle) between the pair of lines in three dimensions, that is, $u \cdot v$, where u and v are the normalized vectors. The ordinate measures the dot product of the *projection* of these two lines when they are drawn in the sketch plane, denoted $u' \cdot v'$. Both axes range from -1 to $+1$. Looking at this plot, we see that denser high-correlation areas are immediately visible in the upper-right and lower-left corners. From a statistical point of view, this high correlation implies that when the dot product of two lines is near ± 1.0 in the sketch plane, it is *probably* also near ± 1.0 in three-dimensional space. More intu-

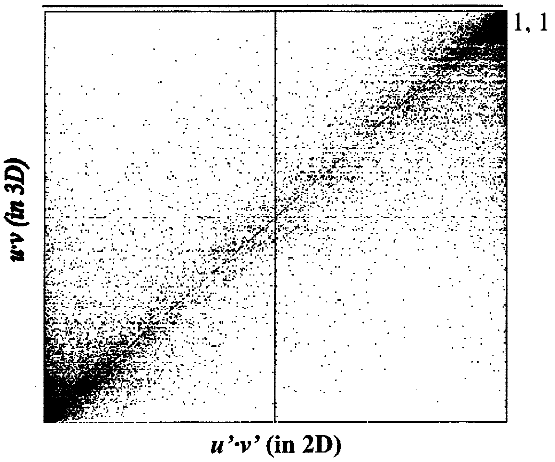

Fig. 4. Second-order angular-correlation plot.

itively, this correlation may be expressed verbally as "the more two lines are parallel in the sketch plane, the more they are likely to represent parallel lines in space." Of course, we know this specific correlation intuitively from our experience, but it is quantified in this plot. Figure 5 shows this type of plot applied to the first-order verticality correlation discussed earlier. The left plot shows the correlation for a general arbitrary object. No particular correlations are apparent. The right image shows the correlation for manually drawn sketches. Indeed, some areas of correlation exist.

Higher-order correlations can also be measured. For example, a third-order correlation determines whether there is a correlation between the three-dimensional volume spanned by a corner of three lines u, v, w, and the angles between the projected lines u', v', w' in the sketch plane. The answer to this question is expressed through a correlation plot that has three dimensions and cannot be easily shown on paper. However, a correlation plot for the three-dimensional volume *versus* the triangle area is shown in Figure 6. Note the thin and elongated areas of correlation near the upper and lower borders, both in the middle-left and middle-right areas. These areas correspond to rectilinear corners that prevail in man-made drawings.

Correlations are not restricted only to angular relationships, although angular correlations are in accordance with observed neurobiological details of the visual cortex. We may also seek correlations in length. Figure 7 shows the correlation between the length ratio of two lines in three dimensions *versus* their length ratio in the sketch plane. As can be expected in this case, there is a strong linear correlation.

As higher-order correlations are sought, more complex relationships can be discovered, such as those related to skewed symmetry and orthogonality among groups of lines. It is thus possible to consider systematically higher-order correlations among angles, lengths, and relative positions of lines, vertices, and, perhaps, curves.

The geometric correlation maps provide a quantitative meaning to the notion of "most plausible" interpretation. For the purpose of reconstruction, any arbitrary assignment of z coordinates to the vertices will yield a three-dimensional object whose projection is the sketch. The most plausible three-dimensional object, however, is the one that is most likely to correspond to the human understanding of the sketch. The geometric correlation hypothesis postulates that this interpretation is the one that best conforms to the accumulated correlations.

To obtain a three-dimensional reconstruction, it is therefore necessary to optimize the assignment of z coordinates so that the maximum geometric correlation is achieved. This can be formulated as an optimization problem. The process consists of two stages: 1) generation of a candidate solution, and 2) evaluation of the likelihood of the solution. These

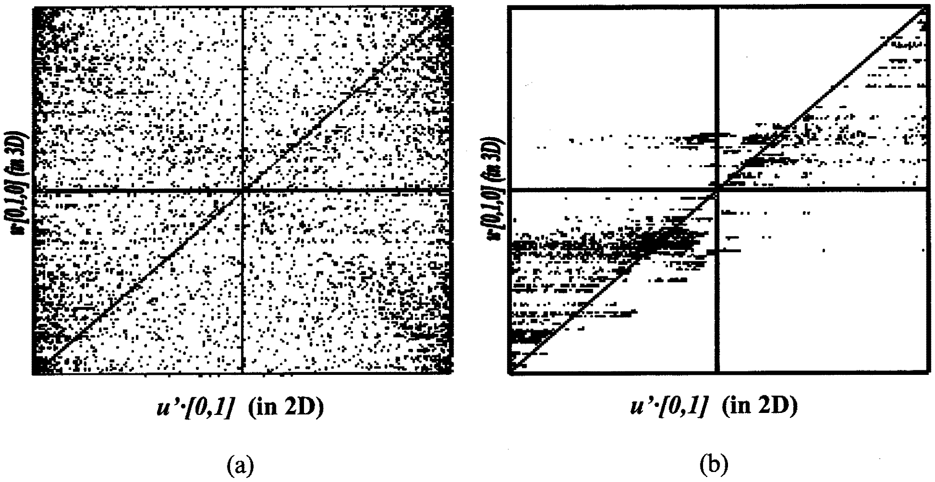

Fig. 5. Verticality correlation plot for (a) general objects, (b) man-drawn objects.

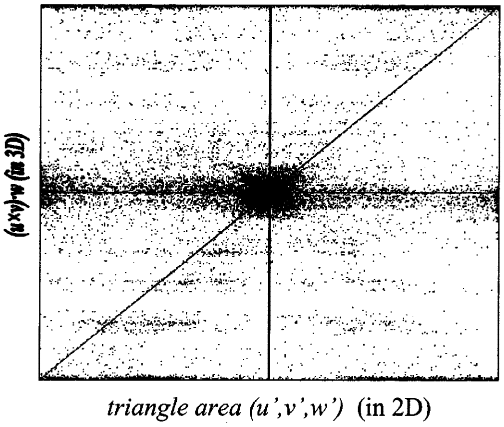

Fig. 6. Third-order angular correlation.

two stages may recur cyclically until convergence, using classical gradient or stochastic optimization methods.

In a set of experiments, we used training sessions consisting of 100,000 scenes with an order of one million correlations, in a total of approximately 30 s training time. The scenes consisted of right-angled wedges, as shown in Figure 8.

The correlations are accumulated and binned into a table whose axes correspond to each correlated parameter. Once the table has been normalized by the total number of stimuli, an approximation of the probability function is obtained. The likelihood of a pair of spatial lines being a solution to a sketched pair of lines can be determined by finding the probability of the corresponding bin. The reconstruction process itself produces candidate solutions and evaluates their plausibility by summing up the probabilities of all line pairs (or higher orders) in the candidate interpretation. Figure 9 shows how the original flat freehand sketch (a) was correctly reconstructed into a three-dimensional wedge depicted from several viewpoints in (b).

The correlation information can be stored in tables or neural networks or encoded directly into equations. In the following sections we used a more compact encoding in the form of explicit equations, as described in detail in Lipson and Shpitalni (1996). Figure 10 shows some models reconstructed using this method.

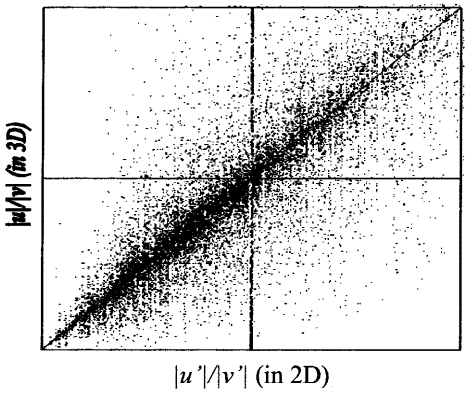

Fig. 7. Length ratios correlation plot.

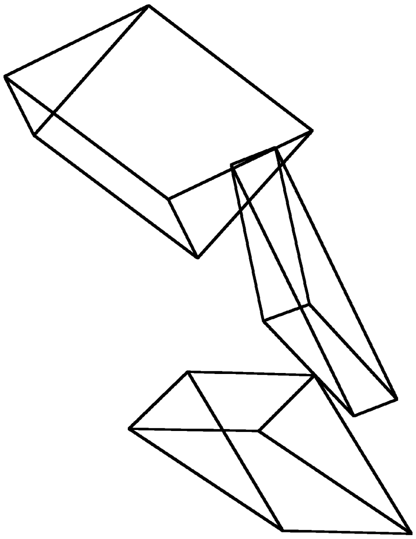

Fig. 8. A typical generated random scene.

5. CONCEPTUAL ANALYSIS

Returning to our initial motivation for sketch interpretation, we stress again that a surprising amount of information can be predicted by analyzing a rough model at a preliminary stage. For example, it would be desirable to obtain approximate predictions as to the viability of a product based on a rough sketch alone. Although the sketch interpretation process is generic, the analysis stage is domain specific. As an example, we will focus on the analysis of rough models of sheet metal parts obtained using the sketch interpretation techniques described in the previous section.

The system for analyzing the three-dimensional geometry of a product is based on concepts of classic expert systems for sheet metal products, such as DeVin et al. (1992) and Shpitalni (1993). The three-dimensional geometry obtained in the previous section is first decomposed into planar *facets* and *links* (between adjacent facets) to enable calculating some preliminary aspects of product cost and properties. Given a scale factor, a material, and a thickness, it is immediately possible to display a preliminary three-dimensional simulation of the product, as well as estimate the following overall properties:

- Number of bending operations;
- Total facet area (for painting);
- Total material volume and product weight;
- Overall packing volume.

The product is then analyzed for manufacturing. This stage determines the optimal unbent flat pattern associated with

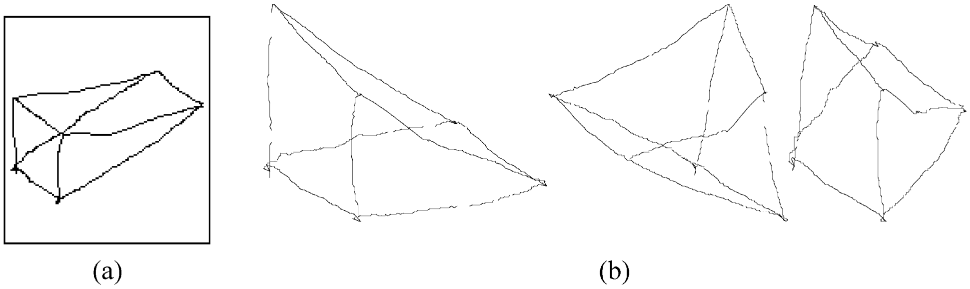

Fig. 9. Direct reconstruction: (a) original flat sketch, (b) three-dimensional reconstruction, viewed from three different viewpoints.

the product, under various criteria. The flat pattern (or sets of flat patterns) may predict additional information, such as:

- Number of components;
- Estimated flat pattern shape;
- Nesting efficiency;
- Raw material needed;
- Overall manufacturing cost.

In the following, we briefly describe the algorithm used to determine the flat pattern, and subsequently, amendments to this algorithm necessary to handle the inherent inaccuracy of the model.

5.1. Determination of flat pattern

The algorithm for determining the flat pattern is based on a heuristic search on the connectivity graph of the product. The topological connectivity of a sheet metal product can be represented by a graph, with nodes corresponding to facets and edges corresponding to connections between facets,

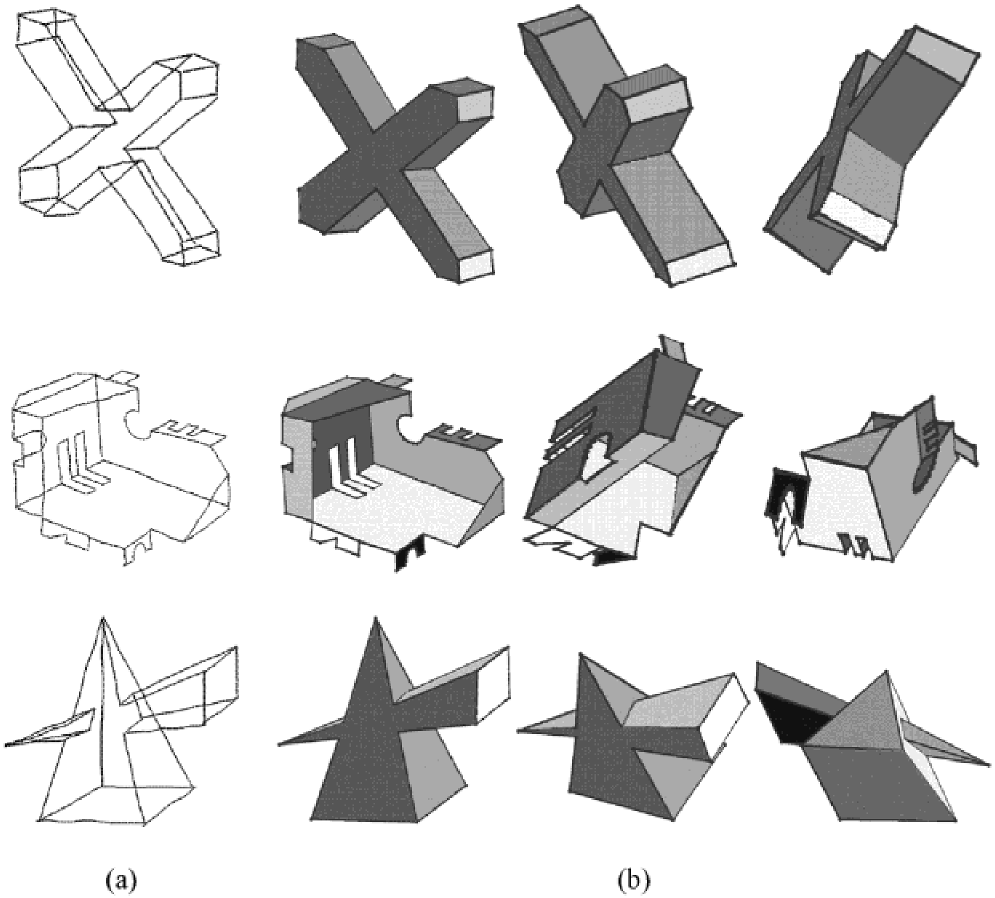

Fig. 10. (a) Product sketches, (b) the reconstructed three-dimensional models.

or *potential* bends. An example of a product and its corresponding graph is shown in Figure 11.

In principle, any flat pattern is a spanning tree of the graph. Depending on the optimality criteria, weights can be assigned to the links corresponding to their desirability as bends. Determination of the flat pattern, then, is reduced to finding a maximum weight spanning tree. However, in contrast with abstract spanning trees, the flat pattern must comply with certain constraints. The primary constraint is due to collisions: facets in the unbent flat pattern must not overlap, and sheets cannot cross at joints. This restriction introduces the possibility that a compliant spanning tree may not exist at all. It is therefore also necessary to consider the possibility of multiple components (corresponding to a "spanning forest"). When the product contains surface forks, two distinct "sides" cannot be defined uniquely, and therefore some facets may require flipping of flat patterns. This makes the problem domain *dynamic*.

The implementation of the solution algorithm is based on the A* approach, a well established AI technique for optimal heuristic search on a graph (Pearl & Korf, 1984). The bend assignment problem is formulated as a graph with nodes and edges. The nodes of the graph correspond to states in the problem domain, where each state is associated with a valid flat pattern set represented by a binary vector with a digit set to one for actives links. Initially (root node), all links are detached, and so the state vector is zero. This corresponds to a flat pattern set composed of the individual facets, where each is a detached unit that is always a valid solution. The graph edges correspond to transitions from state to state, achieved by joining two flat pattern components in a set while ensuring that no overlap is introduced, no product edge is used more than once, and no loop is closed. This ensures that all valid multicomponent layouts will be considered on the "path" to evaluating optimal single component flat patterns.

To define the search goal, a measure of state cost is necessary. In the A* algorithm, the cost associated with each state is composed of two terms: $g(n)$, corresponding to the actual cost required to reach the current state from the initial state, and $h(n)$, corresponding to an *optimistic* estimation of the remaining cost required to reach a goal state from the current state, where n is the current state. The cost functions are dependent on the overall goal of the unbending procedure. In a simple case, the cost $g(n)$ is a sum of the costs c_b of individual bend lines used so far, plus a cost c_d associated with the number of detached components. Typically, selecting long bend lines yields better layouts, so cost is assigned in inverse proportion to length. Because it is usually simpler to manufacture a product with fewer components, it is desirable that $c_d \gg c_b$, so that bends will be preferred over welds where possible. The remaining cost $h(n)$ can be estimated optimistically by assuming that at best, for k facets with m links already selected, the best $k - m - 1$ of the remaining links will be used. A bound on the number of components can be obtained from topological considerations.

The above formulation yields good solutions, including products requiring multiple components. However, the search is inefficient because it permits multiple states corresponding to permutations of the same flat pattern created by activating the same links in a different order. This situation is overcome by assigning arbitrary indices to the links and connecting links only in monotonous order. Moreover, a significant improvement in performance is obtained by assigning the indices in order of decreasing costs, so that the preferred links are, in general, tried first. This has the effect of pruning the search tree early in the search.

5.2. Results

The results of the analysis are conveyed to the user by displaying them back on the sketch. The results contain numeric data, drawings of possible flat patterns, and illustrations of selected bend lines, as well as error estimators for some of the results. Although these results can be expressed in an organized and neat report format, we have attempted to convey the results *in context* of their original

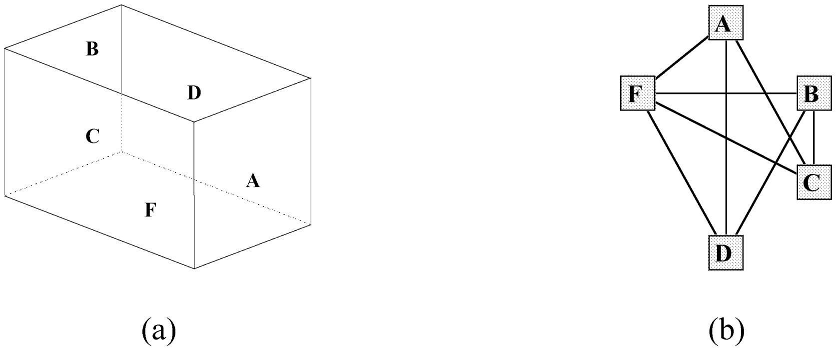

Fig. 11. (a) An open box, and (b) its graph representation.

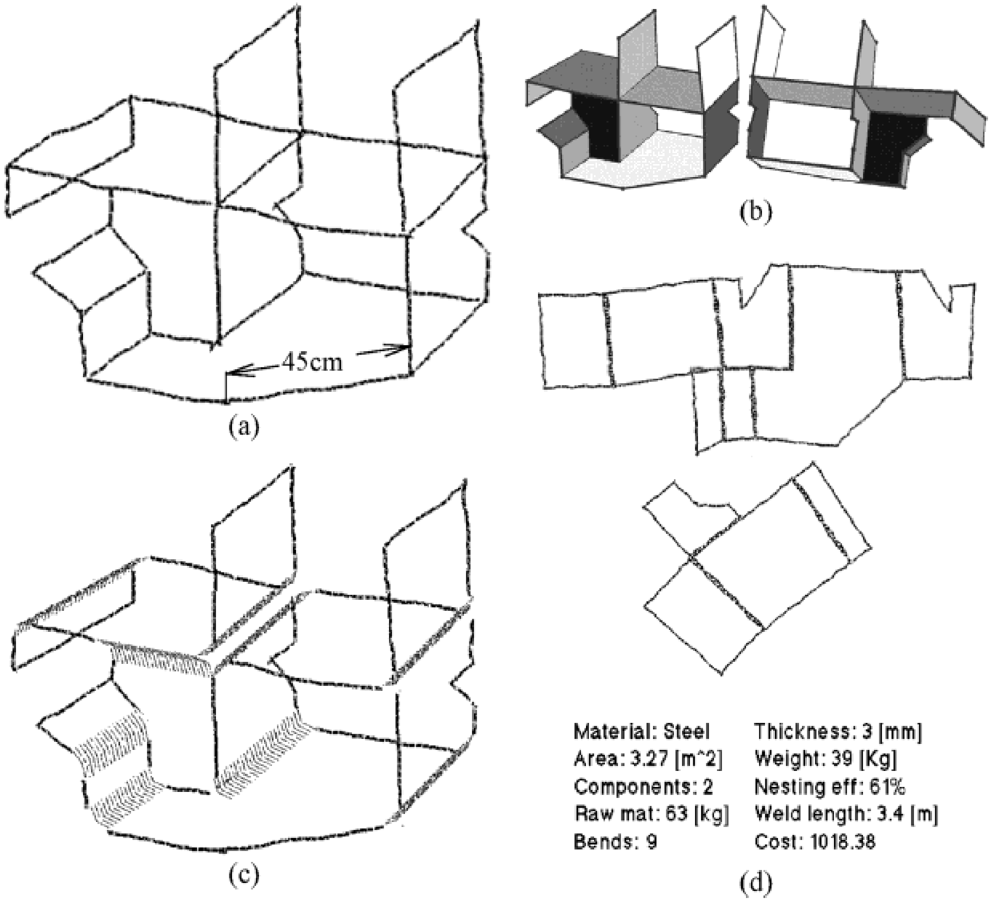

Fig. 12. (a) Original flat (two-dimensional) sketch, (b) reconstructed three-dimensional model, (c) optimal bend assignments overlaid on original sketch, and (d) optimal flat pattern and predicted product properties.

specification, so as to make them more easily accessible. This has been achieved by a) highlighting bend lines as rough marks overlaid on top of the original sketch using the reverse projection that was applied for reconstruction, and b) displaying the output flat patterns using rough synthesized sketch strokes, with amplitude corresponding to overlap error, to convey the notion that the results are not accurate and to indicate the expected uncertainty. Figure 12 illustrates an analysis sequence, and Figure 13 shows two additional examples.

6. CONCLUSIONS

The design of products can be a complex and elaborate process. However, many important aspects of a product's manufacturing characteristics are already determined at the early design stages. It is therefore important to allow the designer to try out and investigate many concepts of a product before starting the detailed design. This article has described a system for conceptual design of sheet metal products by sketching, based on principles of *early incorporation* of CAD, *imprecise* analysis, and *natural, in-context* interaction based on the language of *sketching*. With this approach, various preliminary aspects of a product such as manufacturability and viability can be estimated automatically based on only a rough, freehand sketch of a product, without requiring accurate details. This approach permits the designer to explore new concepts more freely and to make sounder decisions when selecting a particular concept for detailed design. We hypothesize that this form of abstract and rapid conceptual analysis is only possible when the communication format is itself fluent and natural.

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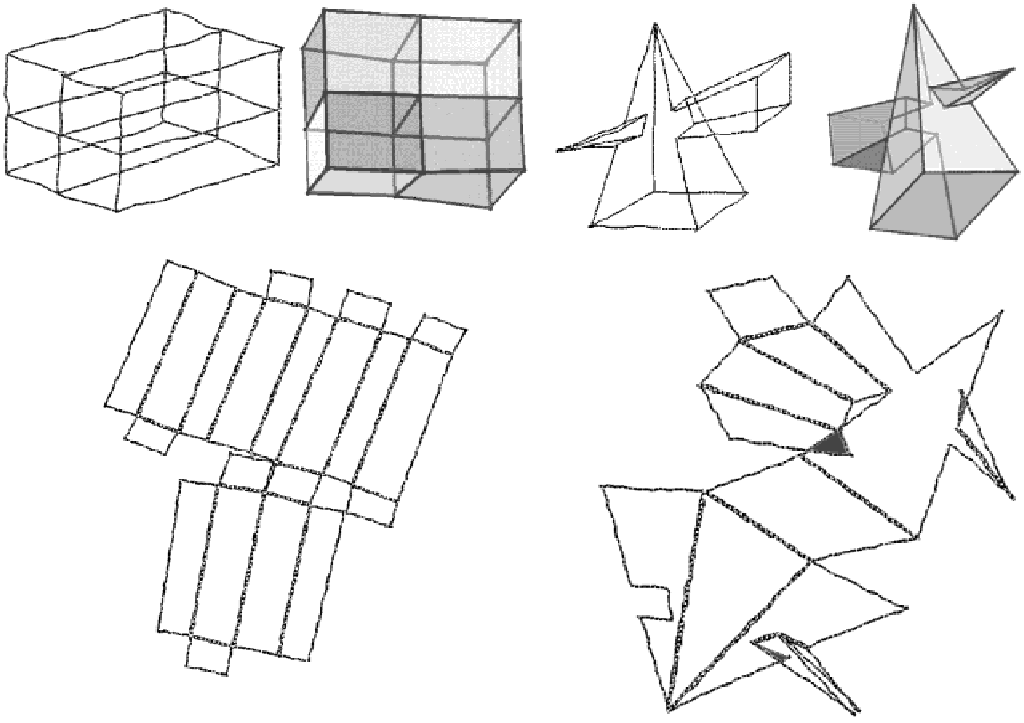

(a) (b)

Fig. 13. Two sample application: (a) nonmanifold box, (b) intersection pyramids. Both images show original two-dimensional sketch, automatically reconstructed three-dimensional model, and automatically generated flat pattern. Shaded areas in flat pattern indicate overlap determined to be incidental (due to sketch inaccuracy).

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