# CORRESPONDENCE.

## ON THE EQUATION OF ARBITRARY RATES OF LIFE PREMIUM.

*To the Editor of the Assurance Magazine.*

SIR,—I do not think that either Messrs. Wylie or Scott make fair allowance for the facility which is afforded for the adjustment of arbitrary life premiums by the computation of ordinary commutation tables.

Without adverting to the circumstance of Mr. Wylie having formerly addressed you on the subject, I have made frequent use of such tables for the calculation of assurances, where the duration of the terms of payment or of benefit was irregular, and can bear testimony to the facility afforded by the methods to which I briefly draw your attention.

Given a table of "whole life" premiums, to form therefrom a commutation table. Since $\pi_x = v - \frac{N_x}{N_{x-1}}$, then $N_x = (v - \pi_x)N_{x-1}$; *i.e.*, with an arbitrary radix, for N, one year younger than the youngest age of the column to be formed, we at once get the logarithms of the succeeding values, by the successive addition of the logarithm of $v - \pi_x$ for each year of age. From this single column we can find all the *annuity* values by the usual formulæ, substituting for $D_x$, $N_{x-1} - N_x$.

Again: since $\pi_x = \frac{M_x}{N_{x-1}}$, then $M_x = \pi_x N_{x-1}$, by the computation of which we are furnished with the only remaining column necessary for the calculation of all assurance values.

Finally: should the computer be desirous of eliminating the mortality table producing the results thus found, it can be done with facility from the ordinary column D, $D_x = N_{x-1} - N_x$.

1. To exhibit the fraction measuring the probability of the life surviving one year, the form in which the table admits of the easiest comparison with others, we have (Gray's *Tables and Formulæ*, p. 119), $D_{x+1} = D_x p_x v$,

and $p_x = \frac{D_{x+1}}{D_x} \cdot \frac{1}{v}$, or $p_x = \frac{D_{x+1}}{D_x}(1+r)$, as may be found most convenient.

2. Should it be preferred to exhibit a table of decrements of the usual form, it is very easily deduced by the formula $D_x = l_x v^x \therefore l_x = D_x \cdot \frac{1}{v^x}$, or $D_x(1+r)^x$.

I remain, Sir,
Your most obedient Servant,
H. A. S.

*Aberdeen, 4th August,* 1856.

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## ON FORMULÆ FOR USING TABLES OF LOGARITHMS.

*To the Editor of the Assurance Magazine.*

Sir,—I understand that you are about to publish some tables of logarithms to twelve places of decimals, and I have thought that it might be interesting to you to have your attention called to the following formulæ for using the tables:—

Suppose $\log.(x \pm h) = \log. x \pm y$, then

$$\log. y = \log.\left(\frac{mh}{x}\right) \mp \frac{1}{2}\left(\frac{mh}{x}\right) \text{ nearly} \quad . \quad . \quad . \quad (1)$$

$$\log. h = \log.(xMy) \pm \frac{1}{2} y \text{ nearly} \quad . \quad . \quad . \quad (2)$$

The first is obtained by taking the logarithm of $y$ in the equation

$$\begin{aligned} \pm y &= \log.(x \pm h) - \log. x \\ &= \pm \frac{mh}{x} - \frac{m}{2}\frac{h^2}{x^2} \pm \frac{m}{3}\frac{h^3}{x^3} - \frac{m}{4}\frac{h^4}{x^4} \pm \ldots\ldots \\ &= \pm \frac{mh}{x}\left\{1 \mp \frac{h}{2x} + \frac{h^2}{3x^2} \mp \frac{h^3}{4x^3} + \ldots\ldots\right\} \end{aligned}$$

and the complete expression for $\log. y$ is

$$\log. y = \log.\left(\frac{mh}{x}\right) \mp \frac{1}{2}\left(\frac{mh}{x}\right)\left\{1 \mp \frac{5h}{12x} \pm \frac{h^2}{4x^2} \mp \frac{251h^3}{1440x^3} \pm \frac{19h^4}{144x^4} \mp \ldots\right\}$$

I found this formula for myself, and am not aware that it has yet been published in print.

The second formula is due to Legendre, who establishes it nearly as follows:—Suppose $\log. X = \log. x \pm y$, $X = x \pm h$, whence $X = x\varepsilon^{\pm My}$;

$$\therefore \pm h = X - x = x(\varepsilon^{\pm My} - 1) = \pm x\varepsilon^{\pm\frac{1}{2}My}(\varepsilon^{\frac{1}{2}My} - \varepsilon^{-\frac{1}{2}My}).$$

Expanding the term in parentheses, we have

$$h = (xMy)\varepsilon^{\pm\frac{1}{2}My}\left(1 + \frac{1}{6}\frac{M^2y^2}{4} + \frac{1}{120}\frac{M^4y^4}{16} + \ldots\ldots\right);$$

and, finally, taking the logarithm

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