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# RIGIDITY OF HYPERSURFACES IN A EUCLIDEAN SPHERE

QIAOLING WANG AND CHANGYU XIA

*Departamento de Matemática-IE, Universidade de Brasília, Campus Universitário, 70910-900 Brasília-DF, Brasil* (wang@mat.unb.br; xia@mat.unb.br)

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*Abstract* This paper studies topological and metric rigidity theorems for hypersurfaces in a Euclidean sphere. We first show that an $n(\geqslant 2)$-dimensional complete connected oriented closed hypersurface with non-vanishing Gauss–Kronecker curvature immersed in a Euclidean open hemisphere is diffeomorphic to a Euclidean $n$-sphere. We also show that an $n(\geqslant 2)$-dimensional complete connected orientable hypersurface immersed in a unit sphere $S^{n+1}$ whose Gauss image is contained in a closed geodesic ball of radius less than $\pi/2$ in $S^{n+1}$ is diffeomorphic to a sphere. Finally, we prove that an $n(\geqslant 2)$-dimensional connected closed orientable hypersurface in $S^{n+1}$ with constant scalar curvature greater than $n(n-1)$ and Gauss image contained in an open hemisphere is totally umbilic.

*Keywords:* rigidity; hypersurfaces; topology; sphere

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## 1. Introduction

A classical result in the theory of submanifolds states that an $n(\geqslant 2)$-dimensional connected closed oriented hypersurface in a Euclidean space $\mathbb{R}^{n+1}$ with non-vanishing Gauss–Kronecker curvature is diffeomorphic to an $n$-sphere [**6**]. Recall that the Gauss–Kronecker curvature of a hypersurface $M$ in a Euclidean space is defined to be the product of all the princpal curvatures of $M$. Similar differentiable sphere theorems for hypersurfaces in a Riemannian manifold have recently been obtained. For example, Sacksteder [**14**] showed that an immersed closed orientable hypersurface with non-negative sectional curvature in $\mathbb{R}^{n+1}$ is the boundary of a convex body and thus is diffeomorphic to a sphere. Since a closed hypersurface $M$ in $\mathbb{R}^{n+1}$ has at least one elliptic point, we know that if $M$ has non-vanishing Gauss–Kronecker curvature, then it has positive sectional curvature. Thus, Sacksteder's theorem generalized the above classical result. Furthermore, do Carmo and Warner [**7**] proved that an $n(\geqslant 2)$-dimensional connected closed oriented hypersurface with sectional curvature no less than 1 in $S^{n+1}$ is diffeomorphic to an $n$-sphere. Alexander [**3**] obtained a similar theorem for compact connected orientable hypersurfaces in a complete simply connected Riemannian manifold of non-positive sectional curvature. On the other hand, it is not true that an oriented closed hypersurface with non-vanishing Gauss–Kronecker curvature in $S^{n+1}$ is diffeomorphic to a sphere. Here, the Gauss–Kronecker curvature of a hypersurface $M$ in $S^{n+1}$ is also defined as the product of all the principal curvatures of $M$. For example, for any positive constants $r_1$, $r_2$ with $r_1 + r_2 = 1$ and any integer $k \in \{1, \dots, n-1\}$, the hypersurfaces

$$S^k(r_1) \times S^{n-k}(r_2) := \{(x, y) \mid x \in \mathbb{R}^{k+1},\ y \in \mathbb{R}^{n-k+1},\ |x|^2 = r_1,\ |y|^2 = r_2\}$$

of $S^{n+1}$ have non-vanishing Gauss–Kronecker curvature but they are clearly not diffeomorphic to a sphere. Thus it is natural to find conditions so that a closed hypersurface with non-vanishing Gauss–Kronecker curvature in $S^{n+1}$ is diffeomorphic to a sphere. In this paper, we obtain the following differentiable sphere theorem.

**Theorem 1.1.** *Let $M$ be an $n(\geqslant 2)$-dimensional connected oriented closed hypersurface immersed in $S^{n+1}$ with non-vanishing Gauss–Kronecker curvature. If $M$ is contained in an open hemisphere, then $M$ is diffeomorphic to a Euclidean $n$-sphere.*

We then prove the following theorem.

**Theorem 1.2.** *Let $M$ be an $n(\geqslant 2)$-dimensional complete connected orientable hypersurface immersed in $S^{n+1}$ and denote by $N$ a unit normal vector field globally defined on $M$. If the Gauss image of $M$ under $N$ is contained in a closed geodesic ball of radius less than $\pi/2$ in $S^{n+1}$, then $M$ is diffeomorphic to a Euclidean $n$-sphere.*

Hypersurfaces with constant scalar curvature in Euclidean space or spheres have been studied extensively recently. For surfaces in $\mathbb{R}^3$ the combined classical results of Hilbert and Hartman–Nirenberg classify complete surfaces with non-zero constant curvature as standard spheres and those with zero curvature as planes or cylinders [**8**]. Also, a well-known result of Cheng and Yau [**4**] classifies a complete hypersurface $M^n$ $(n > 2)$ with constant scalar curvature and non-negative sectional curvature in $E^{n+1}$ as a round hypersphere, a hyperplane or a generalized cylinder $S^k(c) \times E^{n-k}$, $1 \leqslant k \leqslant n-1$. Similarly, work by Li [**10**] showed that a compact hypersurface $M^n$ of constant scalar curvature $n(n-1)r$ with $r \geqslant 1$ in $S^{n+1}$, whose squared norm of the second fundamental form is bounded above by a certain constant which depends only on $n$ and $r$, is isometric to the totally umbilical sphere $S^n(r)$ of radius $r$ or the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$ for a certain value of the constant $c$. Recently, Alencar *et al*. [**1**] have given an interesting gap theorem for closed hypersurfaces with constant scalar curvature $n(n-1)$ in a unit sphere. Aslo, one can find some interesting results about minimal hypersurfaces in a sphere with constant scalar curvature in, for example, [**5**], [**12**], [**13**] and [**15**].

In the present paper, we obtain the following characterization of totally umbilic spheres in a unit sphere in terms of scalar curvature and Gauss map.

**Theorem 1.3.** *Let $M$ be an $n(\geqslant 2)$-dimensional connected closed orientable hypersurface of constant scalar curvature $n(n-1)r$ with $r > 1$ immersed in $S^{n+1}$. Let $N$ be a unit normal vector field of $M$ and assume that $N(M)$ is contained in an open hemisphere. Then $M$ is totally umbilic.*

## 2. Proofs of the theorems

Before proving our results, we first fix some notation. For an oriented hypersurface $M$ of $S^{n+1} \subset \mathbb{R}^{n+2}$, we shall denote by $N$ a unit vector field globally defined on $M$. Let $\langle\,,\rangle$ be the Riemannian metric on $\mathbb{R}^{n+2}$ as well as those induced on $S^{n+1}$ and $M$. Assume that $\tilde{\nabla}$, $\bar{\nabla}$ and $\nabla$ are the Riemannian connections of $\mathbb{R}^{n+2}$, $S^{n+1}$ and $M$, respectively. We have

$$\tilde{\nabla}_X Y = (Xy_1, \dots, Xy_{n+2}) \tag{2.1}$$

for $X, Y = (y_1, \dots, y_{n+2}) \in \mathcal{X}(\mathbb{R}^{n+2})$. Let $A$ stand for the shape operator of $M$ in $S^{n+1}$ associated with $N$. Then

$$\tilde{\nabla}_X Y = \bar{\nabla}_X Y - \langle X, Y\rangle x = \nabla_X Y + \langle AX, Y\rangle N - \langle X, Y\rangle x \tag{2.2}$$

and

$$A(X) = -\tilde{\nabla}_X N = -\bar{\nabla}_X N \tag{2.3}$$

for all tangent vector fields $X, Y \in \mathcal{X}(M)$.

**Proof of Theorem 1.1.** Let $a \in S^{n+1}$ and assume that $M$ is contained in the upper open hemisphere determined by $a$. That is,

$$M \subset \{y \in S^{n+1} \mid \langle y, a\rangle > 0\}. \tag{2.4}$$

Let

$$S^n_a = \{x \in S^{n+1} \mid \langle x, a\rangle = 0\}$$

be the equator determined by $a$ and define a map $\psi$ from $M$ to $S^n_a$ as

$$\psi(x) = \frac{N(x) - \langle N(x), a\rangle a}{\sqrt{1 - \langle N(x), a\rangle^2}}.$$

Since $\langle N(x), x\rangle = 0$, $\forall x \in M$, it follows from (2.4) that $N(x) \neq \pm a$, $\forall x \in M$, and so $\psi$ is a well-defined map. We shall show that $\psi$ is a diffeomorphism. In order to see this, let us calculate the tangent map $\mathrm{d}\psi$ of $\psi$. For any $x \in M$ and any $v \in T_xM$, it is easy to see from (2.1) and (2.3) that

$$\mathrm{d}\psi_x(v) = \frac{-Av + \langle Av, a\rangle a}{\sqrt{1 - \langle N(x), a\rangle^2}} - \frac{\langle N(x), a\rangle\langle Av, a\rangle}{(1 - \langle N(x), a\rangle^2)^{3/2}}(N(x) - \langle N(x), a\rangle a).$$

Thus, noticing that $\langle Av, N(x)\rangle = 0$, we get

$$\langle \mathrm{d}\psi_x(v), \mathrm{d}\psi_x(v)\rangle = \frac{|Av|^2(1 - \langle N(x), a\rangle^2) - \langle Av, a\rangle^2}{(1 - \langle N(x), a\rangle^2)^2}.$$

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By the Schwarz inequality,

$$\langle Av, a\rangle^2 = \langle Av, a^T\rangle^2 \leqslant |Av|^2 |a^T|^2,$$

where $a^T$ denotes the projection of $a$ in $T_xM$. Thus it follows from

$$1 - \langle N(x), a\rangle^2 - |a^T|^2 = \langle a, x\rangle^2$$

that

$$\langle \mathrm{d}\psi_x(v), \mathrm{d}\psi_x(v)\rangle \geqslant \frac{|Av|^2\langle a, x\rangle^2}{(1-\langle N(x), a\rangle^2)^2}. \tag{2.5}$$

Observe that $M$ has non-vanishing Gauss–Kronecker curvature, which implies that, if $v \neq 0$, then $Av \neq 0$. Since $\langle a, x\rangle \neq 0$, $\forall x \in M$, we conclude from (2.5) that, if $v \neq 0$, then $\mathrm{d}\psi_x(v) \neq 0$. Hence $\psi$ is a local diffeomorphism by the inverse function theorem. Since $M$ is compact, $\psi$ is a covering map (cf. [**6**, **11**]). But $M$ is connected and $S^n_a$ is simply connected, so we know that $\psi$ is a diffeomorphism (cf. [**6**, **11**]). This completes the proof of Theorem 1.1. □

**Proof of Theorem 1.2.** Fix $p \in S^{n+1}$, $r \in (0, \pi/2)$ and denote by $d$ the distance function on $S^{n+1}$. Assume that $N(M)$ is contained in the geodesic ball

$$B(p, r) = \{x \in S^{n+1} \mid d(x, p) \leqslant r\}.$$

Let

$$S^n_p = \{x \in S^{n+1} \mid \langle x, p\rangle = 0\}$$

be the equator determined by $p$ and define a map $\phi$ from $M$ to $S^n_p$ as

$$\phi(x) = \frac{x - \langle x, p\rangle p}{\sqrt{1-\langle x, p\rangle^2}}.$$

Our condition '$N(M) \subset B(p, r)$' implies that $x \neq \pm p$, $\forall x \in M$. Hence $\phi$ is a well-defined map. For any $x \in M$ and any $v \in T_xM$, a straightforward computation shows that

$$\mathrm{d}\phi_x(v) = \frac{v - \langle v, p\rangle p}{\sqrt{1-\langle x, p\rangle^2}} + \frac{\langle x, p\rangle\langle v, p\rangle}{(1-\langle x, p\rangle^2)^{3/2}}(x - \langle x, p\rangle p).$$

It then follows from $\langle v, x\rangle = 0$ that

$$\langle \mathrm{d}\phi_x(v), \mathrm{d}\phi_x(v)\rangle = \frac{1}{(1-\langle x, p\rangle^2)^2}((1-\langle x, p\rangle^2)|v|^2 - \langle v, p\rangle^2). \tag{2.6}$$

We have from the Schwarz inequality that

$$\langle v, p\rangle^2 = \langle v, p^T\rangle^2 \leqslant |p^T|^2 |v|^2,$$

where $p^T$ denotes the projection of $p$ in $T_xM$. On the other hand, we have

$$p = p^T + \langle p, x\rangle x + \langle p, N(x)\rangle N(x),$$

which gives

$$1 - \langle x, p\rangle^2 - |p^T|^2 = \langle p, N(x)\rangle^2.$$

Thus we obtain from (2.6) that

$$\begin{aligned}\langle \mathrm{d}\phi_x(v), \mathrm{d}\phi_x(v)\rangle &\geqslant \frac{1}{(1-\langle x,p\rangle^2)^2}\langle p, N(x)\rangle^2|v|^2\\ &\geqslant \langle p, N(x)\rangle^2|v|^2. \end{aligned} \tag{2.7}$$

But

$$\angle(N(x), p) = d(N(x), p) \leqslant r,$$

so we know that

$$\langle N(x), p\rangle^2 \geqslant \cos^2 r.$$

Therefore,

$$\langle \mathrm{d}\phi_x(v), \mathrm{d}\phi_x(v)\rangle \geqslant \cos^2 r|v|^2.$$

It then follows that

$$\phi^*(\langle\,,\rangle_{S_p^n}) \geqslant \cos^2 r\langle\,,\rangle. \tag{2.8}$$

Thus $\phi$ is a local diffeomorphism. Since $\langle\,,\rangle$ is a complete Riemannian metric on $M$, the same is true for the homothetic metric

$$\widetilde{\langle\,,\rangle} = \cos^2 r\langle\,,\rangle.$$

Equation (2.8) means that the map

$$\phi : (M, \widetilde{\langle\,,\rangle}) \to (S_p^n, \langle\,,\rangle_{S_p^n})$$

increases distance. If a map from a connected complete Riemannian manifold $M_1^n$ into another connected Riemannian manifold $M_2^n$ increases the distance, then it is a covering map and $M_2$ is complete [**9**, Lemma 8.1]. Thus $\phi$ is a covering map. But $S_p^n$ is simply connected and we conclude that $\phi$ is a global diffeomorphism. This completes the proof of Theorem 1.2. □

If $M$ is a compact hypersurface in $S^{n+1}$ such that $N(M)$ is contained in an open hemisphere, then $N(M)$ is contained in a geodesic ball of radius less than $\pi/2$. Thus the following corollary of Theorem 1.2 holds.

**Corollary 2.1.** *Let $M$ be an $n(\geqslant 2)$-dimensional complete connected closed orientable hypersurface immersed in $S^{n+1}$. If the Gauss image of $M$ is contained in an open hemisphere, then $M$ is diffeomorphic to an $n$-sphere.*

**Proof of Theorem 1.3.** Let $a \in S^{n+1}$ and assume that $N(M)$ is contained in the upper open hemisphere determined by $a$. That is,

$$N(M) \subset \{y \in S^{n+1} \mid \langle y, a\rangle > 0\}.$$

Let $\Delta$ be the Laplacian operator acting $C^\infty(M)$. Now consider the height function $\langle x, a\rangle$ and the function $\langle N, a\rangle$, which are defined on $M$. From (2.1) and (2.2) we know that their gradients are given by

$$\nabla\langle a, x\rangle = a^T \quad \text{and} \quad \nabla\langle a, N\rangle = -A(a^T), \tag{2.9}$$

where

$$a^T = a - \langle a, N\rangle N - \langle a, x\rangle x \tag{2.10}$$

is tangent to $M$. Taking the covariant derivative of (2.10) and using (2.2) and (2.3), we get

$$\nabla_X a^T = \langle a, N\rangle A(X) - \langle a, x\rangle X \tag{2.11}$$

for $X \in \mathcal{X}(M)$. Thus, the Hessian of $\langle a, x\rangle$ is given by

$$\nabla^2\langle a, x\rangle(X, Y) = \langle \nabla_X a^T, Y\rangle = \langle a, N\rangle\langle AX, Y\rangle - \langle a, x\rangle\langle X, Y\rangle,$$

for $X, Y \in \mathcal{X}(M)$, and its Laplacian is

$$\Delta\langle a, x\rangle = nH\langle a, N\rangle - n\langle a, x\rangle, \tag{2.12}$$

where $H = (1/n)\operatorname{tr}(A)$ is the mean curvature function of $M$. The Hessian of $\langle a, N\rangle$ is given by

$$\begin{aligned}\nabla^2\langle a, N\rangle(X, Y) &= -\langle \nabla_X(Aa^T), Y\rangle \\ &= -\langle(\nabla_X A)(a^T), Y\rangle - \langle A(\nabla_X a^T), Y\rangle \\ &= -\langle(\nabla_X A)(a^T), Y\rangle - \langle a, N\rangle\langle AX, AY\rangle + \langle a, x\rangle\langle AX, Y\rangle,\end{aligned} \tag{2.13}$$

for $X, Y \in \mathcal{X}(M)$. It follows from the Codazzi equation that

$$(\nabla_X A)(a^T) = (\nabla_{a^T} A)(X).$$

Thus the Laplacian of $\langle a, N\rangle$ is

$$\begin{aligned}\Delta\langle a, N\rangle &= -\operatorname{tr}(\nabla_{a^T} A) - \langle a, N\rangle|A|^2 + nH\langle a, x\rangle \\ &= -n\langle \nabla H, a\rangle - \langle a, N\rangle|A|^2 + nH\langle a, x\rangle.\end{aligned} \tag{2.14}$$

Consider the vector field $Z = A(\nabla\langle a, N\rangle)$. Take a local orthonormal frame $\{e_i\}_{i=1}^n$ on $M$. Then the divergence of $Z$ is given by

$$\begin{aligned}\operatorname{div} Z &= \sum_i \langle \nabla_{e_i}(A(\nabla\langle a, N\rangle)), e_i\rangle \\ &= \sum_i \langle A(\nabla_{e_i}\nabla\langle a, N\rangle) + (\nabla_{e_i}A)(\nabla\langle a, N\rangle), e_i\rangle \\ &= \sum_i (\langle Ae_i, \nabla_{e_i}\nabla\langle a, N\rangle\rangle + \langle(\nabla_{\nabla\langle a,N\rangle}A)(e_i), e_i\rangle) \\ &= \sum_i \nabla^2\langle a, N\rangle(e_i, Ae_i) + n\langle\nabla\langle a, N\rangle, \nabla H\rangle. \qquad (2.15)\end{aligned}$$

Integrating (2.15) on $M$ and using the divergence theorem and (2.14), we get

$$\begin{aligned}0 &= \int_M \biggl(\sum_i \nabla^2\langle a, N\rangle(e_i, Ae_i) + n\langle\nabla\langle a, N\rangle, \nabla H\rangle\biggr) \\ &= \int_M \biggl(\sum_i \nabla^2\langle a, N\rangle(e_i, Ae_i) - nH\Delta\langle a, N\rangle\biggr) \\ &= \int_M \biggl(\sum_i \nabla^2\langle a, N\rangle(e_i, Ae_i)\biggr) + \int_M (n^2 H a^T H + nH\langle a, N\rangle|A|^2 - n^2H^2\langle a, x\rangle). \quad (2.16)\end{aligned}$$

Now assume that $\{e_i\}_{i=1}^n$ is a basis of orthonormal principal directions corresponding to the principal curvatures $\lambda_1, \dots, \lambda_n$. Then $Ae_i = \lambda_i e_i$, $i = 1, \dots, n$. Thus, by using (2.13), we have

$$\begin{aligned}\sum_i \nabla^2\langle a, N\rangle(e_i, Ae_i) &= \sum_i \lambda_i \nabla^2\langle a, N\rangle(e_i, e_i) \\ &= -\sum_i (\lambda_i\langle(\nabla_{e_i}A)(a^T), e_i\rangle + \langle a, N\rangle\lambda_i^3) + \langle a, x\rangle|A|^2 \\ &= -\sum_i (\langle\nabla_{a^T}(Ae_i), Ae_i\rangle + \langle a, N\rangle\lambda_i^3) + \langle a, x\rangle|A|^2 \\ &= -\tfrac{1}{2}a^T|A|^2 - \sum_i \langle a, N\rangle\lambda_i^3 + \langle a, x\rangle|A|^2. \qquad (2.17)\end{aligned}$$

It follows from the Gauss equation that

$$n(n-1)(r-1) = n^2H^2 - |A|^2. \qquad (2.18)$$

Since $r$ is a constant, we have

$$a^T((nH)^2 - |A|^2) = 0. \qquad (2.19)$$

Substituting (2.17) and (2.19) into (2.16), we get

$$\int_M \biggl((|A|^2 - n^2H^2)\langle a, x\rangle + \biggl(nH|A|^2 - \sum_i \lambda_i^3\biggr)\langle a, N\rangle\biggr) = 0. \qquad (2.20)$$

Since $(nH)^2 - |A|^2$ is a constant, we know from (2.12) that

$$\int_M (|A|^2 - n^2H^2)\langle a, x\rangle = \int_M H(|A|^2 - n^2H^2)\langle a, N\rangle. \tag{2.21}$$

Consequently, we have

$$\int_M \biggl(H(|A|^2 - n^2H^2) + nH|A|^2 - \sum_i \lambda_i^3\biggr)\langle a, N\rangle = 0. \tag{2.22}$$

Set $S = |A|^2 - nH^2$; then

$$\sum_i \lambda_i^3 = nH^3 + 3HS + \sum_i (\lambda_i - H)^3. \tag{2.23}$$

It follows by combining (2.22) and (2.23) that

$$\int_M \biggl((n-2)HS - \sum_i (\lambda_i - H)^3\biggr)\langle N, a\rangle = 0. \tag{2.24}$$

Before we can finish the proof of Theorem 1.3, we will need the following lemma.

**Lemma 2.2 (see [10]).** *Let $a_i$, $i = 1, \dots, n$, be real numbers satisfying $\sum_i a_i = 0$ and $\sum_i a_i^2 = S$. Then*

$$-\frac{n-2}{\sqrt{n(n-1)}}S^{3/2} \leqslant \sum_i a_i^3 \leqslant \frac{n-2}{\sqrt{n(n-1)}}S^{3/2} \tag{2.25}$$

*and one of the equalities holds if and only if at least $(n-1)$ of the $a_i$ are equal.*

Since $r > 1$, we know from (2.18) that $H(x) \neq 0$, $\forall x \in M$. Thus either $H > 0$ on $M$ or $H < 0$ on $M$, since $M$ is connected. Consider the case $H > 0$ on $M$. In this case, by using $r > 1$ and (2.18), we obtain

$$H > \sqrt{\frac{S}{n(n-1)}}.$$

Since $\sum_i (\lambda_i - H) = 0$, $\sum_i (\lambda_i - H)^2 = S$, it follows from Lemma 2.2 that

$$(n-2)HS - \sum_i (\lambda_i - H)^3 \geqslant (n-2)S\biggl(H - \sqrt{\frac{S}{n(n-1)}}\biggr) \geqslant 0.$$

Therefore, since $\langle a, N\rangle > 0$ on $M$, (2.24) implies that $S \equiv 0$. That is, $M$ is totally umbilic. The case $H < 0$ on $M$ is similar and will be omitted. This completes the proof of Theorem 1.3. □

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## Note added in proof

The authors were informed recently that Theorem 1.3 in this paper has also been proved, by a different method, in [**2**].

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