Statistical Properties of the IntraCluster Light from SDSS Image Stacking

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Abstract. The presence of a diffuse stellar component in galaxy clusters has been established by a number of observational works in recent years. In this contribution I summarize our results (Zibetti *et al.* 2005) obtained by stacking SDSS images of 683 clusters, selected with the maxBCG algorithm at $0.2 < z < 0.3$. Thanks to our large sample ($\gtrsim 30$ times larger than any other sample of individually observed clusters so far) and the advantages of image stacking applied to SDSS images, we are able to measure the *systematic* properties of the intracluster light (ICL) with very high accuracy.

We find that the average surface brightness of the ICL ranges between 26 and 32 mag arcsec^{-2}, and constantly declines from 70 kpc cluster-centric distance (i.e. distance from the BCG) to 700 kpc. Interestingly, the *fraction* of diffuse light over the total light (including galaxies), monotonically declines from ~ 50 to $\lesssim 5\%$ over the same range of distances, thus showing that the ICL is more easily produced close to the bottom of a cluster's potential well. On the other hand, clusters lacking a bright BCG, hardly build up a large amount of intracluster stellar component. The link between the growth of the BCG and the ICL is also suggested by the strong degree of alignment between these two components which is observed in clusters where the BCG displays a significant elongation. With the additional fact that the colors of the ICL are consistent with those of galaxies, all this appears to be evidence for intracluster stars being stripped from galaxies that suffer very strong tidal interactions in the center of clusters and eventually merge into the BCG.

Our measurements also show that intracluster stars are a minor component of a cluster's baryonic budget, representing only $\sim 10\%$ of the total optical emission within 500 kpc.

Finally, we discuss some open issues that emerge from a comparison of the present results with other observations and recent theoretical modeling.

Keywords. galaxies: clusters: general – galaxies: interactions – galaxies: elliptical and lenticular, cD – galaxies: halos – galaxies: evolution – techniques: photometric

1. Introduction

The existence of a diffuse stellar component in galaxy clusters is now a well established observational fact. After the early claim by Zwicky (1951) and pioneering work in the 1970's (Welch & Sastry 1971; Melnick *et al.* 1977), the advent of high-sensitivity panoramic CCD detectors allowed the unambiguous detection of intracluster light (ICL) in nearby clusters at very faint surface brightness levels, from ~ 26 down to $\sim 30\mathrm{mag}_R$ arcsec^{-2} (Bernstein *et al.* 1995; Gonzalez *et al.* 2000, 2005; Feldmeier *et al.* 2002, 2004). More recently, Krick & Bernstein (2007) have reported observations of the ICL in clusters up to $z \sim 0.3$.

Intracluster stellar populations have gained relevance in recent years as they can provide extremely useful constraints in a number of cosmological issues, among those: the history of galaxy interactions and mergers in dense environments, the baryonic budget

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of close systems (Gonzalez *et al.* 2007, this conference), the evolution of the stellar mass function of luminous red galaxies (Monaco *et al.* 2006).

Despite the growing number of observations fundamental questions about the nature and the origin of the ICL still remain open. The first issue concerns the very definition of the ICL. From a dynamical point of view one should consider as ICL only the light radiated from stars which are *not* bound to any individual galaxy, rather free floating in the cluster potential. Such a strict definition is not applicable to photometric measurements like those mentioned above, unless one believes parametric surface brightness decomposition based on analytical functions. To access the required dynamical information the observation of intracluster planetary nebulae (ICPNe) has been proposed and applied to nearby clusters (e.g. Arnaboldi *et al.* 1996; Feldmeier *et al.* 2004). These kinds of studies have demonstrated that the genuine intracluster stellar component and the extended halos of bright (elliptical) galaxies are often overlapping and can be hardly distinguished based on broad band photometry alone. Arnaboldi *et al.* (2004) convincingly show that the "binding" state of ICPNe that trace the diffuse stellar population critically depends on local density (cluster core or periphery), on the mass of neighboring galaxies and on the dynamical state of the cluster and its sub-components. All photometric studies of the ICL have to *define* criteria, like surface brightness thresholds and/or profile decompositions, to isolate the ICL from the rest of the optical cluster light, in a way that does not need to be consistent with dynamical definitions. Therefore attention should be paid at possible differences between "photometric" and "dynamical" ICL, for instance when comparing photometric studies with simulations. Also, different definitions in a number of recent studies can give rise to confusion in the quantitative results. The apparent general agreement between the results published in literature should be regarded with some care before deriving firm conclusions, as I will discuss later.

The way the ICL is defined and measured is also very relevant for studies that attempt at computing the total stellar budget in clusters (e.g. Gonzalez *et al.* 2007, this conference), as the light in galaxies and in the IC component must be measured consistently, for instance by applying the same SB thresholds to both components.

Although there is general consensus on the IC stars being formed in galaxies and subsequently being scattered into the intergalactic space, it is still not completely clear which are the main mechanisms of production. A number of theoretical studies, ranging from N-body+SPH cosmological simulations (e.g. Sommer-Larsen *et al.* 2005; Murante *et al.* 2007), to dissipationless N-body simulations (Rudick *et al.* 2006; Conroy *et al.* 2007), to pure analytic models (Purcell *et al.* 2007), give somehow contrasting explanations for the origin of the ICL, from the scatter of stars during the mergers of galaxies into the central cluster galaxy to tidal stripping and/or disruption of dwarf satellites. Observations show all these mechanisms to be at work to some extent. Gerhard *et al.* (2007) clearly show that a large number of intracluster stars are being "created" in the ongoing merger of the two brightest ellipticals in the core of the Coma cluster. On the other hand, the wealth of tidal structures ubiquitously observed in nearby clusters (as seen for instance in the spectacular image of Virgo published by Mihos *et al.* 2005) testifies that IC stars can be "created" all over the cluster environment. Given this large phenomenological variance, a systematic study of the properties of the ICL in a broad variety of clusters drawn from a large statistical sample is required in order to derive cosmologically meaningful constraints to the physics of cluster and galaxy formation. This was exactly the main goal of the study that we published in 2005 (Zibetti *et al.* 2005) and whose results are the focus of this contribution.

In the following I will first (Section 2) introduce the sample and the image stacking technique we have adopted in our analysis. In Sec. 3 I will summarize the main results.

Section 4 is devoted to a short discussion about the physical mechanisms of ICL production that are suggested by the present results. I will then discuss some critical issues that need to be clarified and addressed by future observations and modeling (Sec. 5) and finally give some conclusions (Sec. 6). A fully detailed description of data processing and analysis is beyond the scope of this contribution and can be found in Zibetti *et al.* (2005).

Throughout we adopt the standard cosmological parameters $\Omega_{tot} = 1.0$, $\Omega_{\Lambda} = 0.7$, $h = 0.7$.

2. Sample and photometric analysis

As already mentioned above, the diffuse ICL has very low typical surface brightness $\mu_R > 26$ up to 30–31 mag arcsec^{-2}, which is of the order of 10^{-3} or less times the surface brightness of a dark night sky. At these extremely faint levels not only sensitivity is an issue, but also and more crucial are systematic effects due to residual from flat field corrections, scattered light, internal reflections in the camera and fringing (in the red bands). These problems hamper the observations of ICL in individual clusters and make such observations very expensive in terms of telescope time and strategy (for adopted solutions see e.g. Gonzalez *et al.* 2005; Krick & Bernstein 2007). This has prevented to collect observations of the ICL in statistically large samples ($N \gtrsim 30$) of clusters so far.

The SDSS, with its large homogeneous drift-scan imaging coverage in 5 bands, offers the opportunity to tackle this observational problem with a completely alternative statistical approach. Instead of analyzing each cluster individually, it is possible to stack several hundreds images of clusters after masking out the light of all galaxies. In this way we can not only enhance the sensitivity to the levels required to study the ICL, but also the systematic "defects" mentioned above average out in the composite image.

For this experiment we select cluster candidates from ~ 1500 deg^2 of SDSS-DR1 as given in an unpublished preliminary version of the maxBCG cluster catalog (Koester *et al.* 2007a,b) kindly provided by Jim Annis. We selected clusters in the redshift range between 0.2 and 0.3 as a compromise between a sufficiently large image coverage of the cluster region over a single SDSS frame and minimal cosmological SB dimming. Before stacking, each image was centered on the brightest cluster galaxy (BCG), rescaled on a common physical size and photometric calibration, and checked against photometric defects (bright stars, evident blooming or scattered light). The final sample is composed of 683 clusters. Using the number of red sequence galaxies provided by the maxBCG catalog and the empirical relation between this number and R_{200} (Hansen *et al.* 2005), we can derive a rough estimate of the masses M_{200} using the formulae of spherical collapse theory. Our sample includes objects ranging from a few 10^{13} up to 5 10^{14} $h_{70}^{-1} M_{\odot}$, that is from (rich) groups to quite massive clusters†. Also interesting for following analysis is the fact that we span a large range in BCG luminosity, namely from ~ -22 to ~ -24 mag$_r$ (corrected to rest-frame $z = 0$).

We chose to define as ICL all the light emitted from regions having a surface brightness fainter than 25 mag arcsec^{-2} (r-band in the $z = 0.25$ observed frame). All galaxies, except the BCG, are masked to this extent (but see Zibetti *et al.* 2005, for details about masks and relative corrections). The BCG is left unmasked in the stacking process and is treated separately as its halo contribution to the ICL is debated, although in many respects this is just matter of semantic.

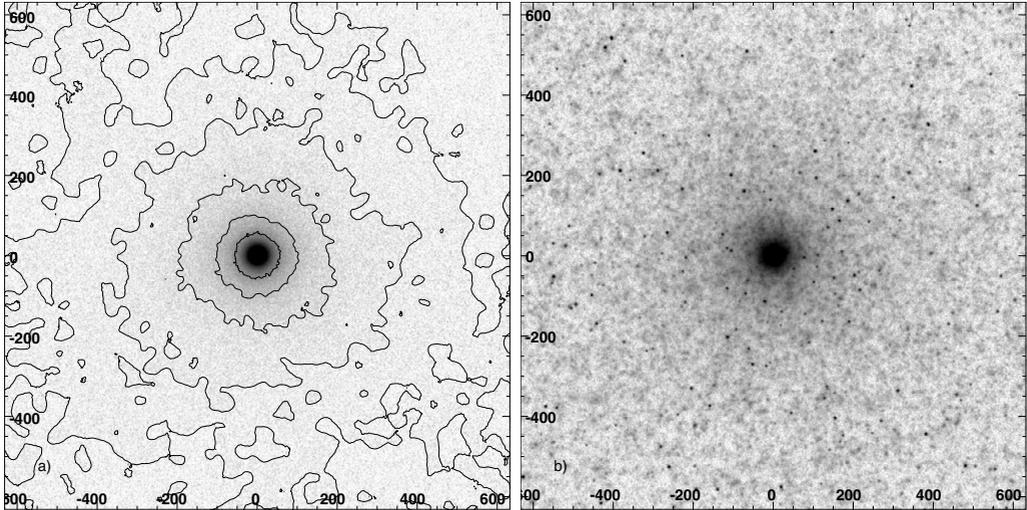† These estimates were not given in Zibetti *et al.* (2005).

Figure 1. The $r + i$ composite images resulting from the stacking of the main sample: the diffuse component plus BCG is in panel a), the total light in panel b). The same logarithmic grey scale is adopted in both images. Side-scale tickmarks display the distance in kpc from the center. Isophotal contours corresponding to $\mu_{(r+i),0.25}$ of 26, 27, 28, 29 and 30 mag arcsec^{-2} for the diffuse component are overplotted on panel a). Smoothing kernels of 3, 7, 11, 17, and 21 pixels respectively are used. Corresponding SB values in r-band are ~ 0.3 mag brighter.

3. Results

The pictorial result of the stacking for the whole sample of 683 clusters is presented in Fig. 1. Panel (a) shows the stack image where all galaxies except the BCG are masked out and represent the *average* distribution of light in the BCG and diffuse component. The faintest isophotes represent ~ 30.3 mag arcsec^{-2} (r-band observed frame) and extend to distances of $\gtrsim 600$ kpc. For comparison, panel (b) shows the stack image where only bright foreground sources have been masked, such that the total cluster light is represented. From stacked images like these we extracted azimuthally averaged surface brightness (SB) profile in each band and for different subsamples of clusters. An example of such SB profiles is reported in Fig. 2. Three main results can be extracted from these profiles.

• There is a diffuse luminous component (ICL+BCG) that extends to cluster-centric distance of ~ 700 kpc at the limiting SB of ~ 32 mag arcsec^{-2}.

• The SB of the diffuse component declines with distance more steeply than the total light of the cluster. Its contribution declines from roughly 30% of the total at 100 kpc to almost 0 at 700 kpc.

• The diffuse light is dominated by the DeVaucouleurs component of the BCG in the inner 70–80 kpc. Outside this radius the diffuse light profile significantly flattens and the light is dominated by galaxies other than the BCG (the fraction of ICL+BCG over total drops below 50%). If we assume that light traces mass, this implies that the flatter outer diffuse component lives in regions where the global cluster potential dominates over the potential of the self-bound central galaxy, and therefore can be dubbed as genuine ICL. It is worth noting at this point that, with respect to other studies of the ICL in individual clusters, the stacking analysis is extremely powerful in terms of depth and spatial extensions. In comparison to Gonzalez *et al.* (2005) we typically reach more than twice as large cluster-centric distances, while we go a couple of magnitude deeper than Krick & Bernstein (2007).
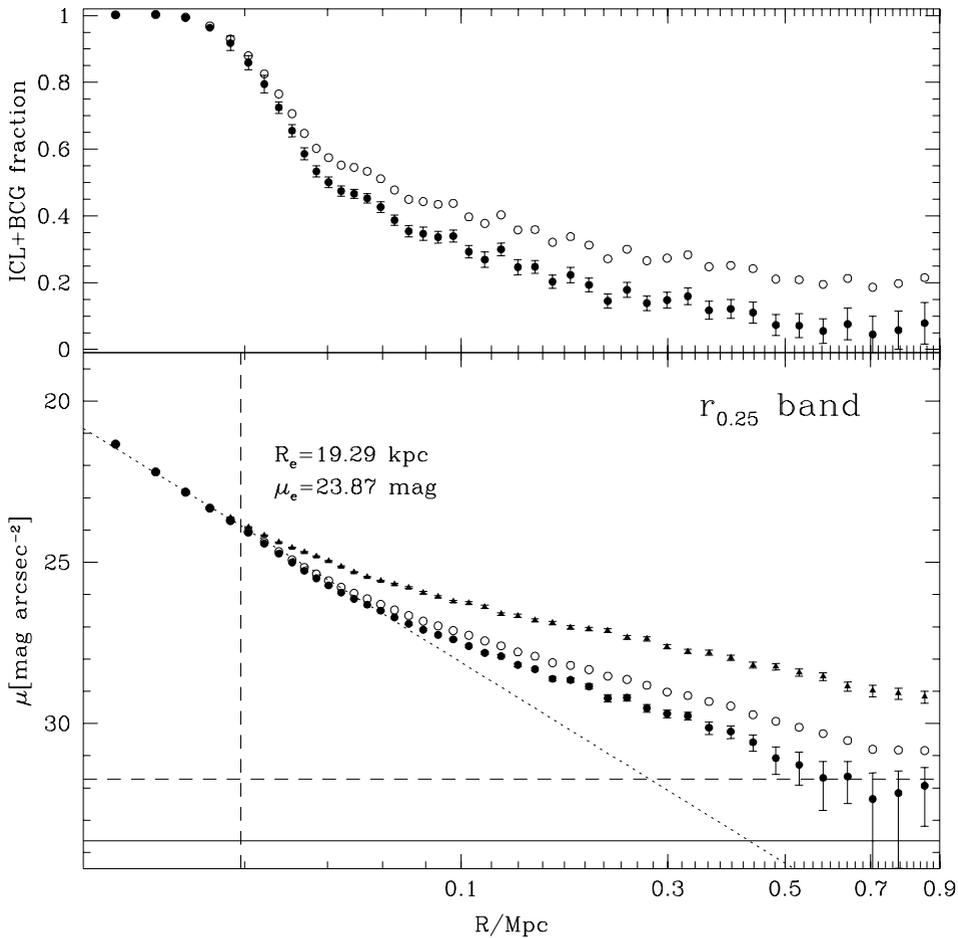
Figure 2. The surface brightness profiles and the local ratio of *ICL+BCG* and uncorrected *diffuse* light to the *total* cluster light in the r band for the complete sample of 683 clusters. The R axis is linear in $R^{1/4}$. Bottom panel: the SB is expressed in mag arcsec^{-2} in the $z = 0.25$ observer frame. Triangles with error bars represent the total cluster light, open circles the diffuse light (including the BCG) as directly measured from the stacked images. Filled circles with error-bars display the SB of the *ICL+BCG*, corrected for masking incompleteness. Horizontal dashed and solid lines display the SB corresponding to the 1-σ uncertainties on the background determination for the total light and for the *ICL+BCG* respectively. The dotted lines represent the best de Vaucouleurs fits to the inner regions: the effective radius of the best fitting model is indicated by the vertical dashed line and the corresponding parameters are reported nearby. Top panel: the local ratio of *ICL+BCG* (filled dots with error-bars) and uncorrected *diffuse* light (open circles) to *total* cluster light.

Using g, r and i photometry we were also able to measure the colors of the ICL and found that they are completely consistent with those derived for the total light of the cluster, thus implying that the ICL is emitted by stellar populations that do not differ significantly from those present in galaxies.

3.1. *Alignment of the ICL with the BCG*

An important test to understand the origin of the ICL is to check its alignment with other cluster components and with the BCG in particular. The stacking of randomly oriented images naturally produces a radially symmetric SB distribution. However, if one selects only clusters with significantly elongated BCG and aligns the images along its major axis, the symmetry is broken and it is possible to study the mutual alignment and elongation of the different components. We find that the stacked ICL is more elongated than the BCG, while the distribution of galaxies is less elongated. This implies that the ICL is well aligned with the BCG and is also more elongated on average. The distribution of galaxies, as opposed, is either less elongated or worse aligned with the BCG, or both.

3.2. *The ICL integral flux*

Measuring the total flux of the ICL requires to subtract the contribution of the BCG from the diffuse component. Unfortunately there is no physically motivated method that can be adopted for this task and, as I will discuss below, different authors adopt different strategies. We decided to measure the flux contributed by the BCG as the integral of the DeVaucouleurs inner component (as shown in Fig. 2). The second problem that arises in this measure is the aperture (if any) within which the fluxes are measured. Given the large uncertainties related to any extrapolation, we prefer to be conservative and limit our integration to within 500 kpc, where we have reliable data and uncertainties in the background subtraction are negligible. As it will be discussed below, a fixed metric aperture might not be the best choice when clusters of different masses are compared. However, given the large uncertainties of R_{200} for our clusters, a fixed metric aperture is the safest solution.

We find that $(10.9 \pm 5)\%$ of the optical luminosity of a cluster within 500 kpc has to be attributed to the ICL, another $(21.9 \pm 3)\%$ is contributed by the inner DeVaucouleurs component of the BCG. These fractions appear to have very little dependence on cluster richness or BCG luminosity.

3.3. *Dependence on cluster properties*

The broad range in richness and BCG luminosity enables us to study the dependence of the ICL brightness and flux upon those cluster properties. To this purpose we consider the clusters in the bottom and top tertiles of the richness distribution and similarly for the distribution in BCG luminosity. We end up with subsamples of "rich" clusters (estimated mass $M \gtrsim 5~10^{13} M_\odot$, as in Sec. 2), "poor" clusters ($M \lesssim 3.5~10^{13} M_\odot$), clusters with a "luminous" BCG ($M_{r,0} < -23.25$ mag) and clusters with a "faint" BCG ($M_{r,0} > -22.85$ mag). We then compare the SB profiles of the total and diffuse light for these four subsamples versus the average SB profiles of the complete sample, as shown in Fig. 3. The total light follows expected trends, with "rich" and "bright-BCG" clusters being brighter than the other two classes. On the other hand, the diffuse light is particularly suppressed in "faint-BCG" clusters only, while "poor" clusters do not show such a strong effect. This suggests that the presence of a bright central galaxy is a crucial ingredient for the formation of the ICL.

4. Interpretation

The results presented in the previous section lead to conclude that, although the ICL cannot be considered just as an extension of the BCG, there are strong links between
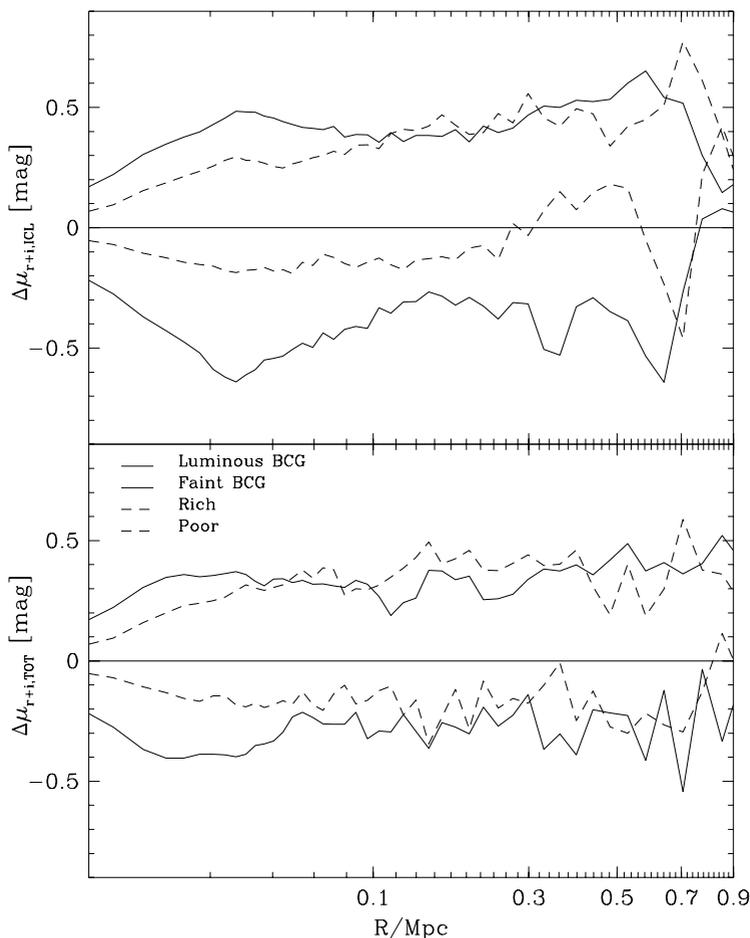
Figure 3. Comparisons between different subsamples: SB difference between the subsamples and the main sample. Thick solid lines are used for clusters hosting a luminous BCG, thin solid for faint BCG, dashed thick for rich clusters, and dashed thin for poor clusters. The bottom panel displays the SB differences as a function of the radius for the *total* light, the top panel those for the corrected *ICL+BCG* component alone.

these two components. The ICL tends to concentrate around the BCG; also, there must be some link between the growth of these two components since we found that the lack of a bright BCG implies a strong suppression in the ICL. It appears that most of the action in creating the ICL occurs close to the bottom of the cluster potential well, where galaxies experience very strong tidal fields and may eventually merge into the BCG. During these events a large number of stars becomes unbound as shown by numerical simulations (see for instance Murante *et al.* 2007). Interestingly, the high degree of alignment and elongation of the ICL with respect to elongated BCGs suggests that the accretion of galaxies along radial orbits (filaments) is particularly efficient in generating intracluster stars.

5. Open issues

The amount of ICL relative to the total cluster luminosity and the luminosity of the BCG is the quantity that is most often used to test against model predictions. Yet there are still many statistical uncertainties and bias in this measurement (and in the corresponding model predictions) that severely hamper a clean comparison. In particular, I discuss two very relevant aspects of this issue here below.

5.1. *ICL vs BCG vs Halos*

As already mentioned in the Introduction, pure photometric measurements of the ICL have to adopt operative definitions. In practice SB thresholds are introduced, below which the light is considered as ICL. However galaxies do not have sharp boundaries, but rather have smoothly declining SB profiles and are possibly surrounded by low-SB halos. The diffuse light which is measured in photometric studies is contaminated by these low-SB structures. Different choices of SB thresholds should be taken into account when comparing different studies, although the exact value of these thresholds do not appear to be critical (see Zibetti *et al.* 2005, their Sec. 5.4). On the other hand, the operational definition of ICL in terms of SB might lead to substantially biased results when comparing with simulations that define the ICL according to dynamical criteria. Recent simulations (e.g. Rudick *et al.* 2006; Conroy *et al.* 2007) mimic the definition of ICL in terms of SB and ease the comparison with the observations. It is unclear though how the dynamical and photometric ICL compare to each other and how much of the discrepancy between the predictions of different simulations arises from the very definition of ICL.

Separating the ICL from the BCG and its halo is another very controversial point, both in observations and simulation. The point is well illustrated by a comparison between Gonzalez *et al.* (2005) and Zibetti *et al.* (2005), who have the largest samples in literature and explicitly analyze the relationship between ICL and BCG. Gonzalez *et al.* (2005) decompose the SB distribution of BCG+ICL in two DeVaucouleurs components, of which the outer one is consider as genuine ICL and the inner one as proper BCG. Zibetti *et al.* (2005) instead fit only the inner DeVaucouleurs profile and consider it as BCG, while all the residuals are considered as ICL. Gonzalez *et al.* (2005) give $ICL/BCG > 5$ while Zibetti *et al.* (2005) find $ICL/BCG < 0.5$. Many factors may play a role in creating this discrepancy, for instance the different extent of the photometry and a different sample selection (the Gonzalez *et al.* sample, in fact, is biased toward clusters with a luminous dominant BCG). However, we calculate that by applying a 2-DeVaucouleurs decomposition to the Zibetti *et al.* (2005) data one would obtain $ICL/BCG \sim 2$, hence much closer to the Gonzalez *et al.* (2005) result. We conclude that the ratio of ICL/BCG is very poorly constrained by present observations and it would better not be used for comparison with models. As opposed, the ratio (ICL+BCG)/total is much more robust and there is apparently general consensus on a value around 30%.

5.2. *Trends with cluster mass and richness*

The literature about ICL of recent years presents a broad variety of claims about the dependence of the ICL fraction upon cluster mass or richness, including some contrasting claims of trends. As an illustration of the current state of the observations I report the trends of the ICL fraction in different works that analyze a broad range of cluster properties:

• Zibetti *et al.* (2005): no trend with richness nor with BCG luminosity (but brightness correlates with BCG luminosity).

- Krick & Bernstein (2007): no trend with mass, anti-correlation with presence of a cD.
- Gonzalez *et al.* (2007): negative trend with mass.

It is worth noting that the trends (or lack of trends) reported by Zibetti *et al.* (2005) could be intrinsically biased by the adoption of a fixed metric aperture of 500 kpc, which correspond to smaller fraction of R_{200} for more massive clusters. Given the steeper profile of the ICL with respect to galaxies, the ICL fraction of more massive clusters could be overestimated and a correction for this effect could reconcile these results with the negative trend found by Gonzalez *et al.* (2007).

From the short and incomplete compilation reported above it is evident that better determinations of the trends of the ICL with cluster mass and richness and with the luminosity of the BCG and the presence of a cD in the cluster are needed. These trends appear to be distinctive and crucial for theoretical models too. A better observational determination will certainly gain better insights in the dynamics of cluster formation.

6. Conclusions

I have reviewed the main results on the ICL from the stacking analysis performed by Zibetti *et al.* (2005). Thanks to the high quality of the SDSS drift-scan data, the large sample (683 clusters) and the effectiveness of the stacking technique against systematics, these results represent the most robust *statistical* assessment of the properties of the ICL so far and are an essential complement to detailed photometric and kinematic studies of the ICL in individual clusters.

I presented a tentative physical interpretation which is consistent with models where most of the ICL is created during violent tidal interactions in the deepest regions of a cluster's potential; these processes appear to be linked to the growth of the BCG.

We are witnessing a general convergence, within factor 2 or better for the basic observables, between different observations and models of the ICL. This is somehow astonishing, given the extreme difficulties of the measurements. However many open and controversial issues are still present. I indicate in particular the problems related to the operational definition of ICL luminosity and fractions, and to the assessment of trends with cluster global properties as priorities for the progress in this field.

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