BULL. AUSTRAL. MATH. SOC. MOS 16A76

VOL. 4 (1971), 143-144.

Near-rings and near-ring modules

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In the second chapter, the first of interest, we deal with a near-ring N under the blanket assumption of minimal condition on right ideals. This condition is shown, by examples, to be considerably weaker than the corresponding condition on right N-subgroups. Several results follow:

(a) the sum $Q(N)$ of all nil right ideals of N is a nil right ideal of N ; and

(b) if $\beta_1, \beta_2, \ldots$ is a sequence of elements of $Q(N)$, then there exists a positive integer k such that

$$\beta_k \beta_{k-1} \ldots \beta_2 \beta_1 = 0 .$$

One typical disadvantage of all so called radicals of a near-ring N is that if the radical is nil, very little can be said in general about the factor near-ring, and if the radical is such that the factor near-ring has special properties such as semi-simplicity, then that radical is in general non-nil. In the third chapter we obtain, under the assumption of minimal condition on ideals of N , a 'properly' ascending transfinite sequence

$$(*) \qquad \{0\},\ L_1(N),\ C_1(N),\ L_2(N),\ C_2(N),\ \ldots$$

of ideals of N with the property that each factor of the type $L_{\alpha+1}(N)/C_\alpha(N)$ is the unique maximal nil ideal of $N/C_\alpha(N)$ for all ordinals α , and $C_\alpha(N)/L_\alpha(N)$ is, in a sense, the unique maximal 'non-nil' ideal of $N/L_\alpha(N)$ for all non-limit ordinals α . It is shown that at least one property of semi-simplicity, that of having a

Received 28 September 1970. Thesis submitted to the Australian National University, July 1970. Degree approved, October 1970. Supervisors: Dr H. Lausch, Dr J. Wiegold.

distributive lattice of ideals, carries over to the factors of the type $C_\alpha(N)/L_\alpha(N)$ (α a non-limit ordinal). (The case where α is a limit ordinal is of no concern, since $C_\alpha(N) = L_\alpha(N)$ and for these ordinals, and only these ordinals, (*) fails to be properly ascending.) In that chapter it is also shown that if N has minimal condition on right N-subgroups, then the length of the sequence (*) is finite; the distributive lattices referred to above are finite; and the join irreducibles are readily displayed.

In Chapter 4 a tame N-module is defined as one in which every N-subgroup is a submodule (see [1]) and a tame near-ring is one with a faithful tame N-module. It is proved there that a tame near-ring with minimal condition on right ideals has maximal condition on the same.

Chapter 5 presents an N-module concept which is analogous to a Hirsch-Plotkin radical, but only in the case of unitary tame N-modules where N has maximal condition.

The most interesting result of Chapter 6 asserts that if the near-ring generated by the inner automorphisms of a group has minimal condition on right ideals, then it is finite.

Reference

[1] James C. Beidleman, "Quasi-regularity in near-rings", *Math. Z.* 89 (1965), 224-229.