CHAPTER XIII

GRAVITATIONAL LENSES

https://doi.org/10.1017/S0074180900159844 Published online by Cambridge University Press

GRAVITATIONAL LENSES AS TOOLS IN OBSERVATIONAL COSMOLOGY

Claude R. Canizares
Department of Physics and Center for Space Research
Massachusetts Institute of Technology

1. INTRODUCTION

The study of gravitational lenses is intimately tied to observational cosmology. When we observe a gravitationally lensed quasar, we are viewing a single object along two or more neighboring paths (null geodesics) of cosmological dimensions (Figure 1). What we see depends on bulk properties of the universe, such as H_0 and q_0, on the large scale structure and inhomogeneities along the paths, and on the small scale structure in and around the primary deflector. Furthermore, the deflection of light depends on the gravitational field along the line of sight, so it is sensitive to all forms of matter: luminous or dark, baryonic or exotic. Thus the images of gravitationally lensed quasars contain an imprint of the universe that is virtually inaccessible by any other means. The hope of decoding this imprint has stimulated observers and theorists to expend many thousands of hours of telescope time, computer time and cogitation on the elucidation of gravitational lens properties.

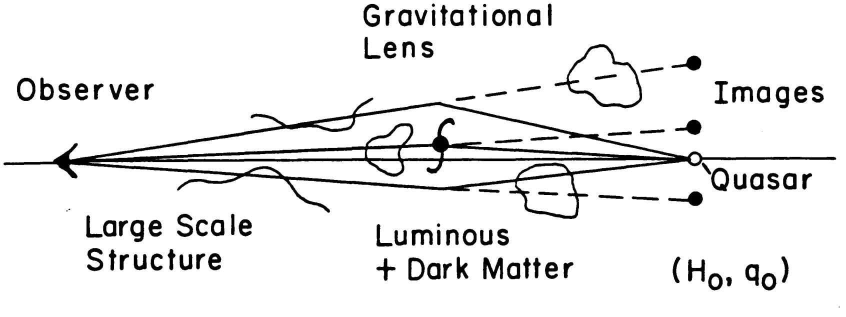

Figure 1. Schematic of a gravitational lens.

As a reviewer of the vast literature that this subject has generated, I must be selective. I have chosen to focus on topics directly relevant to this meeting, though this forces me to ignore much interesting work. (See Peacock 1983, Turner 1987, Gott 1987, and Burke 1986 for other reviews).

A. Hewitt et al. (eds.), Observational Cosmology, 729–746.

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Before embarking on a discussion of the data, let me recall some of the basics. Gravitational lensing involves the bending of light from a distant quasar by matter along the line of sight. For the moment I will make the usual assumption that most of the deflection takes place in a localized region, within a galaxy or cluster, and that the universe is otherwise reasonably homogeneous (but see below). Then a mass concentration will bend the light sufficiently to make multiple images of a distant object if its surface mass density Σ exceeds $\Sigma_c \approx 0.7$ g cm^{-2}, for reasonable assumptions about the distances to the lens and quasar (Narayan, Blandford and Nityananda 1984, Turner, Ostriker and Gott (1984), Subramanian and Cowling 1986). If the lens has a density distribution characterized by a core radius r_{core}, this translates to a limit on its line-of-sight velocity dispersion of $\sigma > 1000\ (r_{core}/100\ \mathrm{kpc})^{0.5}$ km s^{-1}. Individual galaxies generally satisfy this criterion, whereas most clusters do not. The separations between images are roughly comparable to the angular scale of the deflector, ~1-2" for a galaxy. Image separations > 1-2" can be caused by a combination of galaxy plus cluster -- in effect the galaxy makes the images and the cluster spreads them (Turner, Ostriker and Gott 1984, Narayan, Blandford and Nityananda 1984, Dyer 1984, Kovner 1986). Solid objects like stars, of course, have surface densities >> Σ_c and so they also can make multiple images. These so-called microlenses give image separations $\sim 10^{-6}(M_L/M_\odot)^{1/2}$ arc sec, where M_L is the mass of the lens (e.g. Press and Gunn 1973).

2. FERMAT'S PRINCIPLE

In general, the images formed by anything other than a highly symmetric lens can be very complex. A major and elegant theoretical advance has been the application of Fermat's principle to this problem, which greatly facilitates the computation and visualization of image formation (Schneider 1985, Blandford and Narayan 1986, Narayan 1986). I remind you that Fermat's principle in classical optics states that light will travel from a source to an observer along paths for which the propagation time is an extremum:

$$\delta \int d\tau = 0.$$

In the relativistic extension, the propagation time for a given path has two terms, the geometric travel time for the deflected ray and the gravitational time delay. For a given distant source, one can compute the light travel time for a possible ray that reaches the observer from each position (α,δ) on the sky (Figure 2). This gives a two dimensional "Fermat time surface" $\tau(\alpha,\delta)$. The locations of the extrema of this surface then correspond to the locations of the images of the source on the sky. In a transparent homogeneous universe with no lens, there is one extremum: the path with minimum propagation time that defines the line of sight to the source (Figure 3a). Adding mass along the line of sight increases the propagation time for the central rays. For example, a cylindrically symmetric transparent mass with

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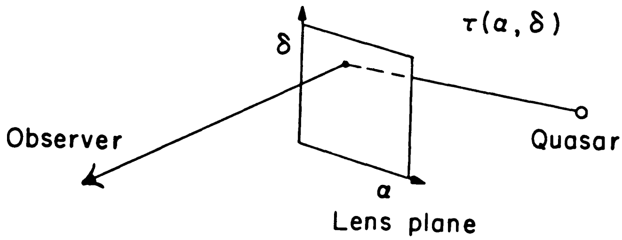

Figure 2. A possible light path from a quasar to an observer. In the "thin lens" approximation, bending can only take place in the plane containing the excess mass associated with the gravitational lens (e.g., a glaaxy of cluster). The total propagation time τ is expressed as a function of position on the sky ($\alpha \cdot \delta$) as seen by the observer. Images occur where τ is an extremum (Fermat's principle).

$\Sigma > \Sigma_C$ centered on the line-of-sight introduces an inflection in the Fermat time surface. The central ray is delayed so it is no longer a minimum but rather a local maximum (Figure 3b). The surface has a trough around the line of sight, so this very special case gives a point image surrounded by a ring image. With less symmetry one would get a maximum, a minimum and a saddle point corresponding to three images on the sky (Figure 3c,d). More complex mass distributions will distort the time surface still further giving additional images.

One thing that emerges immediately is that any smooth, non-singular, transparent mass distribution must give an odd number of images (Dyer and Roeder 1980, Burke 1981, Blandford and Narayan 1986): beyond the original image, new images come in pairs corresponding to either a maximum or minimum in the time surface plus a saddle point (except for singular cases like the ring in Figure 3b). The more compact the mass distribution, the narrower its primary maximum and the fainter the corresponding image (the radius of curvature of the surface near the extremum gives the magnification of the image). For a single microlens the maximum becomes a cusp, leaving only two images (or more generally and even number). Images corresponding to maxima and minima have positive parity, whereas for saddle points they have negative parity -- the latter are mirror reflections of an extended source.

3. OBSERVATIONS

There are now more than ten candidates for gravitationally lensed quasars (see Table 1), several of which have recently emerged from a massive search program reported here by Hewitt (1986,1987), and one of which is newly reported at this meeting (Hammer 1987). The degree of public confidence that lensing is truly observed varies from object to object. Most practitioners would agree that the first four on the list are secure. The next two are less well established, 1146+111 is highly controversial, and the last three are new, unpublished candidates.

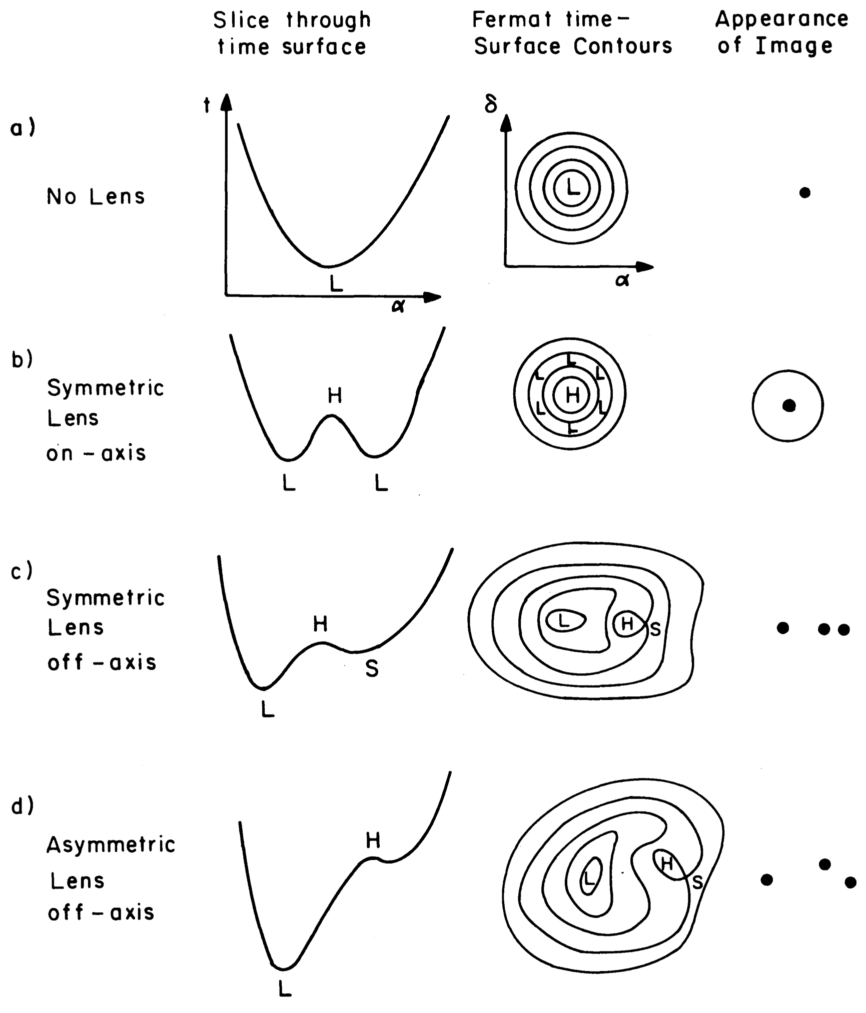

Figure 3. Illustration of the application of Fermat's principle for a distant point source seen through various possible distributions of transparent lens mass. The first column shows a slice through the Fermat time surface (τ vs. α at constant δ), the second column shows full two dimensional surface as contours of constant τ, the third columns shows the resulting images. Minima, maxima and saddlepoints are marked by L, H, and S, respectively. These figures follow Blandford and Narayan (1986) and Narayan (1986). a) No lens. b) Cylindrically symmetric lens centered on the (undeflected) line of sight to the source. c) Cylindrically symmetric lens off the line of sight. d) Asymmetric lens off the line of sight.

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Table 1

GRAVITATIONAL LENS [CANDIDATES]

Name	z_Q	images	$\Delta\Theta$	Lens?	Reference
0957+561	1.4	2	6"	Yes	1
1115+080	1.7	4	2	Maybe	2
2016+112	3.3	3	4	Yes?	3
2237+031	1.7	2	2	Yes	4
1635+267	2.0	2	4	No	5
2345+007	2.2	2	7	No	6
1146+111	1.0	2	157	No	7
3C324	1.2	2+	2	Yes	8
0023+171	1.0	2+	5	No	9
1042+178	0.9	4+	2	?	9

References: 1. Walsh, Carswell and Weymann (1979), Greenfield, Roberts and Burke (1985); 2. Weymann *et al.* (1980); Weymann and Folz (1983), Foy, Bonneau, and Blazit (1985), Henry and Heasley (1986); 3. Lawrence *et al.* (1984), Schneider *et al.* (1985, 1986); 4. Huchra *et al.* (1985), Tyson and Gorenstein (1985); 5. Djorgovski and Spinrad (1984); 6. Weedman *et al.* (1982), Foltz *et al.* (1984), Tyson *et al.* (1986); 7. Turner *et al.* 1986, Shaver and Cristiani 1986, Huchra 1986; 8. Hammer (1986); 9. Hewitt (1986, 1987);

I will not review each of them in detail, but will say a few words about some areas of current interest or controversy. In particular, I will focus on the degree to which the observations can be understood in the context of conventional theory. Some reasonable level of understanding is a clear prerequisite if lenses are to be used as tools of cosmology.

The first and by far the best studied and most secure lens is 0957+561 (Walsh, Carswell, and Weyman 1979, Greenfield, Roberts and Burke 1985 and references therein). The lensing mass is undoubtedly associated with a galaxy and cluster seen at $z = 0.36$. Because the imaged quasar is an extended radio source, it is possible to extract considerable information about the lens from the observations. Recently, Cohen *et al.* (1987; see also Gorenstein *et al.* 1984) have obtained exquisite VLBI maps of two radio jet images which show structure on milli arcsecond scales. The relative net magnification of the jets matches their relative brightness, as one would expect (neglecting extinction, gravitational lenses are achromatic and conserve surface brightness). Furthermore, the two jet images have identical shapes except for a parity inversion; this tells us immediately that one of the images corresponds to a saddle point in the Fermat time surface. The information from these maps should add constraints to models of the mass distribution in the lens (Greenfield, Roberts and Burke 1985, Cohen *et al.* 1987).

However, there are two major difficulties in the straightforward interpretation of the various lens candidates. These are not fundamental; they do not negate the lens hypothesis in general. But they do raise doubts about some of the candidates and complicate considerably the application of lensing as a tool of cosmology, even for 0957+561.

The Odd Image Problem As noted above, transparent lenses should generally produce an odd number of images, yet all but one of the lens candidates have an even number (Table 1). The exception is 2016+112, for which a faint ($>24^m$) third image was recently found lying practically on top of the image of a foreground galaxy (Schneider *et al.* 1986). Several explanations have been advanced for the general absence of the odd image:

(i) The odd image is unresolved or lies far from the other images (the latter is possible if clusters do most of the lensing; Narayan, Blandford and Nityananda 1984, Kovner 1986).

(ii) The lens galaxy has a compact core which, as noted above, causes one of the images to be very faint (Narasimha, Subramanian, and Chitre 1986). A core containing $\sim 10^{10}$ $M_\odot$ within 50 pc would reduce the image brightness by factors of 10^3-10^4.

(iii) The light of the odd image suffers extinction in the lens galaxy. This is not a viable explanation for 0957+561, because it cannot account for the absence of a radio image.

(iv) Stars in the lens galaxy act as microlenses which, under some circumstances, can preferentially dim the odd image (although they will also occasionally make it significantly brighter; Chang and Refsdal 1984, Subramanian, Chitre and Narasimha 1985, Paczynski 1986a). Again, this will not be effective for 0957+561, because the radio source is too large to be affected by a stellar microlens.

None of these explanations is entirely satisfying, although the degree of embarrassment for lens theory depends on how many of the candidates of Table 1 are actually lenses. Observations with the Hubble Space Telescope should be very useful for addressing this problem.

The Elusive Lens Problem This problem has two aspects. For several cases (e.g. 0957+561 and 2016+112) one can see foreground galaxies and clusters that are almost certainly responsible for the gravitational lensing. However, models which assign mass to these objects with constant mass-to-light ratios (M/L) do not account for the observed properties of the multiple images (Greenfield, Roberts, and Burke 1985, Schneider *et al.* 1985, 1986). Although this tells us something important about dark matter (see below), it hampers the use of straightforward lens models to constrain cosmological parameters (e.g. Falco, Gorenstein and Shapiro 1985)

For most of the candidates of Table 1, there is no evident lensing object. This is surprising. For an imaged quasar with $z_q > 1$, the most probable redshift of the lens is $z_l = 0.5 - 1$ (Canizares 1982, Turner, Ostriker and Gott 1984), so any reasonably massive galaxy that contributes (e.g. one with luminosity $\sim L^*$) will have $m_R \sim 20-23$ (here I have used $M_R^* = -21.8$, $H_0 = 100$ km s^{-1}, $\Omega_0 = 1$, and $K_R = 0.7$ or 2.0 at $z = 0.5$ or 1, respectively). Therefore, one would expect the lens to be clearly visible on deep CCD pictures. Deep searches appear to have been successful for 1115+080 (Foy, Bonneau, and Blazit 1985, Henry and Heasley 1986, Shanklan and Hege 1986, Christian, Crabtree and Waddell 1986) and for 2016+112 (Schneider *et al.* 1985, 1986). But for 1635+267 there is no galaxy brighter than 23.5 mag (Djorgovski and Spinrad 1984), while for 2345+007 the limit is J=25.5 for any galaxy located between the quasar images (Tyson *et al.* 1986).

One possibility is that some of the lenses are compact poor groups of galaxies. Such groups may be observed in 2016+112 (Schneider *et al.* 1986), 1115+080 (Henry and Heasley 1986), and perhaps 2345+007 (Tyson *et al.* 1986). As pointed out by Narayan, Blandford and Nityananda (1984, also Blandford and Jarosczynski 1981, Kovner 1986) such groups could match or even exceed rich clusters in their effectiveness as lenses because of their smaller core radii. X-ray images show that some poor clusters do have well developed gravitational potentials (Kriss, Cioffi and Canizares 1983), and that potentials are not always traced by galaxy distributions (Beers, Huchra and Geller 1983).

By far the most spectacular case of an elusive lens is 1146+111, which was discovered by Arp and Hazard (1980) and advanced by Paczynski (1986b) as a potential lens candidate. Turner *et al.* (1986) confirmed common redshifts (to ± 300 km s^{-1}) and showed that the spectra over λ4500 - 8000 were nearly identical. The very large separation of the two objects, 157", means that the deflector must have a mass of $\sim 10^{15}$ $M_\odot$. This makes it an extraordinary object. However, there is no obvious visible candidate for the deflector: Bahcall, Bahcall and Schneider (1986) show that a conventional very rich cluster would have been seen on deep CCD images. The wide separation and the very absence of an apparent lens led Paczynski (1986b) and Turner *et al.* (1986) to suggest that the lens might be a "cosmic string," a relic of the GUTS phase transition in the early universe. Vilenkin (1984) and Gott (1985) had previously predicted that such strings, if they exist, would best be detected in precisely this manner. Other suggestions for the lens include a 10^{15} $M_\odot$ black hole (Paczynski 1986b), an unusual cluster (Ostriker and Vishniac 1986), clusters of unstable dark matter (Dekel and Piran 1986), and burnt out galaxies (Silk 1986).

The gravitational lens candidacy of 1146+111 has been questioned on several grounds (see Turner's contribution to this meeting for a more complete discussion of this object). Shaver and Cristiani (1986) and Huchra (1986) have obtained spectra to the red and blue (respectively) of the Turner *et al.* spectrum in which, these authors stress, the two images are clearly not identical. Actually, although they are correct in noting differences, the overall spectral shapes are remarkably similar from the near UV to the near IR. Proponents find refuge in the fact that identical spectra are not required because the difference in light travel times for two such widely spaced images would exceed a thousand years, and as Huchra has stated, nobody knows how much quasars vary over several millennia. But any lens capable of causing such large separations ought to have other detectable features (such as double images of other nearby quasars or fluctuations in the cosmic microwave background). Some of these can already be ruled out (e.g. Paczynski 1986b, Gott 1986, Vilenkin 1986, Stark *et al.* 1986, Blandford, Narayan and Phinney 1987), and others are accessible to further ground based and space observations.

In addition to sparking excitement, the lens hypothesis for 1146+111 has provoked a kind of backlash against the reality of this and several other of the more tenuous lens candidates. Phinney and Blandford (1986) suggest that 1146+111 could well be a coincidence, an accidental pair of quasars with nearly identical redshifts. Bahcall, Bahcall and Schneider (1986) point out that if quasars are clustered (as may well be the case: see Shaver 1984), then chance coincidences could account for 1635+267 and 2345+007 as well as 1146+111. At some level all quasar spectra share common features, and unrelated quasars of comparable redshift might have very similar spectra by coincidence. Unfortunately, no one has ever quantified the probability of chance spectral similarity, so there is no way of including this in the statistical estimates.

The excitement over 1146+111 has increased the attention and the amount of telescope time being devoted to lenses, so it is likely that eventually the question of reliability will be settled. Although losing several lens candidates would be very disappointing, as a compensation it would alleviate somewhat both the odd image and elusive lens problems. And the lens searches should continue to provide new candidates.

Before leaving the observations, I want to mention the unusual case of 2237+031 (Huchra *et al.* 1985). Here the putative lens is very visible: it is a 15^m spiral galaxy at z = 0.04 with the spectrum of a z = 1.7 quasar superimposed on its nucleus. Huchra *et al.* immediately invoked lensing as the explanation, to the chagrin of Burbidge (1985), who prefers the interpretation that the quasar is local (i.e. its redshift is non-cosmological). Tyson and Gorenstein (1985) showed that the image is multiple, which confirms one test of the lens hypothesis. But the skeptics could reverse an argument often used against them by pointing out the lack of a control sample (how many nuclei of nearby galaxies have been so carefully scrutinized?), or they may suggest that all galaxies which eject quasars look like this. The Hubble Space Telescope should show if two (or even better, three) of the point sources have identical spectra, as the lens interpretation requires.

4. MEASURING COSOMOLOGICAL PARAMETERS

a) Hubble's Constant

Taking for granted the eventual improvement in observations, what are the prospects for using lenses as cosmological tools? By far the greatest attention has been given to a determination of H_0, following the work of Refsdal (1964, 1966), who pointed out that the difference in light travel time for two images of a gravitationally lensed quasar is inversely proportional to H_0. Happily, quasars are variable, so the difference in light travel time can be found simply by cross correlating the light curves of two images.

Unfortunately, as Einstein once remarked, nature is subtle, so there have been serious difficulties carrying out this important program. First, there are the observational realities: both 0957+561 (Schild and Cholfin 1986 [optical], Hewitt, Roberts and Burke 1984 and Hewitt 1986a [radio]) and 1115+080 (Vanderriest *et al.* 1986), the only lensed pairs that have been monitored, have displayed frustratingly bland light curves that are hard to correlate. Schild (private communication) has an additional 50-60 nights of data, which could strengthen confidence in the ~1.1 year time delay for 0957+561 reported earlier this year (Schild and Cholfin 1986). In any case, Einstein also noted that nature is not malicious, so we can hope eventually to have clean time delay measurements for several quasar image pairs.

The second class of difficulties may be less tractable, however. These have to do with our ability to extract H_0 from the timing measurements, whatever their accuracy. This has been a subject of sharp controversy in the recent literature, with opinions ranging from optimism (Borgeest and Refsdal 1984, Kayser 1986, Borgeest 1986) to varying degrees of pessimism (Falco, Gorenstein and Shapiro 1985, Alcock and Anderson 1985, 1986, Blandford and Narayan 1986).

That different images have different travel times is evident directly from the Fermat time surface (e.g. in Figure 3 the high points correspond to late times, etc.). What is also clear is that the difference depends on the details of the lens: an accurate measure of H_0 requires a reasonably accurate lens model. As noted above, direct observation of the lens galaxies and clusters does not yield straightforward models; assumptions of constant M/L, for example, (e.g. Boorgeest 1986) are not warranted.

On the other hand, it may be possible to measure enough observables (image brightness ratios, parities, or even the full magnification matrices) to provide interesting constraints on H_0 from time delay measurements. First applications of this approach to 0957+561 were made by Boorgeest and Refsdal (1984) and, in more detail, by Falco, Gorenstein and Shapiro (1985). Falco *et al.* find H_0 < 100 km s^{-1} for Δt = 1 year (Schild and Cholfin 1986). The value itself corresponds

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to a "minimum mass" model that reproduces the relative position and brightness of the A and B images. One has the freedom of adding a uniform mass to the model, which would leave the observables unchanged if H_0 were smaller -- hence the upper limit (see Figure 4). The new VLBI observations should provide more constraints on the minimum mass model (Gorenstein, private communication), but it is also important to limit the freedom in model parameters by measuring, for example, the velocity dispersions of the primary lensing galaxy and cluster.

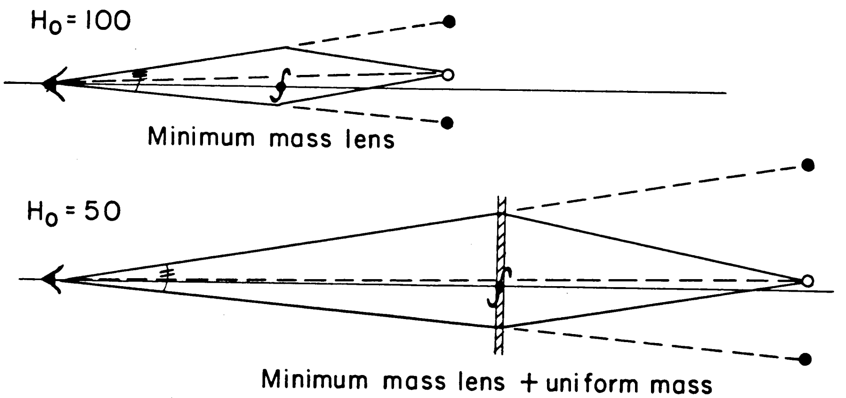

Figure 4. Illustration of the amiguity in determining H_0 from the observations of a lensed quasar. If a "minimum mass" lens model accounts for the observed relative image separations and time delays with H_0 = 100 km s^{-1} Mpc^{-1}, then one can always construct another model with additional uniformly distributed mass that also accounts for the observations but with a lower value of H_0 (e.g. 50 km s^{-1} Mpc^{-1}). The *true* position of the source for the two cases is different but is not observable. (After Falco, Gorenstein and Shapiro 1985.)

One remaining difficulty is the validity of the assumption of large scale homogeneity of the universe, which is incorporated into every model of a gravitational lens. Alcock and Anderson (1985, 1986) have questioned this assumption, noting that just as a lensed quasar may be a very rare alignment of a galaxy and/or cluster with a background quasar, so may it signal a rare line of sight with higher than average cosmological density. This reasoning is valid in general, but is probably weakest for 0957+561, for which the observed galaxy+cluster is already a plausibly sufficient lens. At any rate, the problem is most severe for higher redshift quasars; for 0957-561 at z = 1.4, even extreme variations by up to factors of ~2 in mean density in or near the line sight (causing either focusing or shear of the light beams of both images) result in < 30% uncertainties in the deduced value of H_0 (Alcock and Anderson 1985).

To summarize, gravitational lenses are no magic solution to the long standing problem of measuring H_0. So far they have provided at best a rather preliminary upper limit that encompasses the various values

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deduced from direct distance measurements. On the other hand, the promise of an independent constraint on this fundamental cosmological parameter is still very much alive, and it has taken only seven years to approach the accuracy of the results achieved after many decades of distance measurements. What is most important to me is that each lensed quasar could eventually yield a reasonably independent value of H_0. Consistency between several such measurements will provide a check on the validity of common assumptions, such as large scale isotropy. This will be the most, and perhaps the only, convincing argument for the validity of the method.

b) Cosmological Constant

If the cosmological constant Λ is non-zero, it could affect significantly the image separations for a given gravitational lens (Paczynski and Gorski 1981, Alcock and Anderson 1986, Gott 1987). Gott (1987) used the fact that 2016+112, with a redshift of $z_q = 3.3$, has relatively "normal" images to set a limit of $q_0 > -2.3$ or $\Lambda < 6.9\ H_0^2$ for $\Omega_0 = 1$.

c) Dark Matter in Galaxies and Clusters

As noted above, the models of 0957+561 and 2016+112 already tell us that the mass cannot follow the light in the lensing galaxies and clusters (Greenfield, Roberts, and Burke 1985, Schneider *et al.* 1985, 1986). As the lens models improve, they will undoubtedly contribute to our understanding of the quantity and distribution of dark matter (Gott 1987, Borgeest 1986).

d) Large Scale Structure

As mentioned above, large scale inhomogeneities in the matter distribution along the line of sight to a gravitationally lensed quasar could have a significant effect on the appearance of the images (Alcock and Anderson 1985, 1986). Large scale clumping of matter can introduce a stochastic variation in the effective distance (the angular diameter distance) for different lines of sight.

Anderson and Alcock (1986) showed that normal galaxy clustering has only a very minor effect on gravitational lens images. But there is increasing evidence that luminous matter is structured on scales > 100 Mpc (e.g. De Lapparent, Geller, and Huchra 1986, see also the contributions of Geller and Gott to this meeting). One estimate by Turner, Ostriker and Gott (1984) shows that gravitational lensing might be enhanced along some lines of sight through a universe in which most of the matter is distributed on shells. It would be interesting to see this explored in more detail. For example, I wonder under what conditions one could obtain multiple images from large scale inhomogeneities alone, in which no single galaxy would stand out as the lens.

As a long term goal, one can envision using the properties of an ensemble of lenses to set limits on the inhomogeneity of mass in the universe. Gravitational lensing may be the only way to decide whether or not the large voids are empty of mass as well as light.

e) Microlenses and the Nature of Dark Matter

Gravitational lensing may also be the only way of ascertaining if a significant fraction of the matter in the universe is in the form of compact objects.

If objects such as black holes or subluminous stars account for the dark matter in galaxy halos, then they should cause large fluctuations in the brightness of images whose light passes through the core of a galaxy (e.g. Chang and Refsdal 1979,1984, Young 1981, Gott 1981, Paczynski 1986a, Kayser, Refsdal and Stabell 1986; see also Grieger, Kayser and Refsdal 1986). The fluctuations have rather unusual signatures that should be distinguishable from other intrinsic variability. Some of the images of the objects in Table 1 should eventually show this effect, as should quasars like 0104.2+3153 (Stocke et. al 1984), which are viewed through a foreground galaxy at too large an angle to form multiple images.

Brightening of quasars by minilenses in galaxy halos will also cause enhancements of the apparent surface density of quasars near foreground galaxies (Canizares 1981, 1983, Vietri and Ostriker 1983, Zuiderwijk 1985, Schneider 1986a,b,c). Since my suggestion of this effect, several refinements have been made to the calculations, but the results are nearly the same (for a given assumed quasar luminosity function, the enhancements found by Vietri and Ostriker 1983 and Schneider 1986a,b,c differ by < 20% from those in Canizares 1981). The detectability of the effect depends on the true shape of the luminosity function. Samples of $>10^4$ galaxies, which corresponds to samples of many hundreds of quasars, are probably needed to see it (Vietri and Ostriker 1983, Canizares 1983, Schneider 1986b,c).

Brightening by microlenses may cause noticeable or even large effects on quasar luminosity functions (Turner 1980, Ostriker and Vietri 1986a, Vietri 1985, Schneider 1986d). The effect can be very strong if the intrinsic function is steep, as may well be the case (e.g. see the relevant contributions to this meeting). In fact, Ostriker and Vietri (1985) have made the intriguing suggestion that most BL Lac objects are microlensed quasars for which the small sized continuum emitting region is brightened while the larger line emitting region is not. Recently Wagoner (1986) and Schneider and Wagoner (1986) have suggested microlensing of a single evolving background supernova as a way of detecting compact dark matter.

The absence of certain lens effects is already sufficient to rule out cosmological densities in compact objects of various masses. The optical depth for microlensing is roughly equal to the density of

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compact objects in units of the critical density (Press and Gunn 1973). The effect of the lensing on the observed properties of quasars depends on the mass of the lenses: large masses will produce multiple images whereas smaller ones will cause variability and give differential brightening of continuum and emission lines, which will broaden any intrinsic distribution of equivalent widths (Canizares 1982). Table 2 lists limits on the contribution of compact objects to Ω_0 deduced from the apparent absence of such effects in quasar samples. The data of Hewitt (1986a,b) can also be used to limit the cosmological density of non-compact lenses (e.g. isothermal spheres), to be less than 0.1 - 1. Mpc^{-3} (depending on their central mass density). While these limits are still high, eventually this technique will tell us whether or not objects of galactic mass (from giant black holes to failed galaxies made of dark matter) are sufficiently numerous to close the universe.

Table 2

LIMITS ON COSMOLOGICAL DENSITY OF COMPACT OBJECTS

Ω_L	$M_L/M_\odot$	Reference
< 0.4	10^{11}-10^{13}	Hewitt (1986)
< 1.0	0.1-200	Canizares(1982,1983)
< 0.1	200 -10^5	"

5. CONCLUSIONS

I have chosen to stress some of the problem areas of gravitational lens research because that is where the activity will be for the next several years. But I want to conclude by listing the good points about lenses. First and foremost is their existence, which is very well established in several cases. Second, there are lots of things to measure: new candidates, image location,. brightness, shape, variability, lens properties, etc. This is sure to keep observers very busy for some time to come. Third, gravitational lenses are amenable to modeling. Theoretical tools like Fermat's principle can be used to probe the considerable parameter space of each lens. This is sure to keep theorists equally busy. Fourth, gravitational lenses probe cosmology. Studies will undoubtedly contribute to our understanding of H_0, Λ, Ω_0, large scale structure, etc. Fifth, unlike some other measures of cosmological parameters, a sample of lenses could yield independent measurements which will act as a consistency check on possible systematic errors. Sixth, gravitational lens phenomena are good probes of dark matter on various scales. Lastly, gravitational lenses remain exciting and surprising. Surely there are more discoveries in store for us.

ACKNOWLEDGEMENTS

I am deeply indebted to many colleagues for useful discussions, for detailed comments on the manuscript, and for providing information or preprints prior to publication. In particular I thank Charles Alcock, Roger Blandford, Marc Gorenstein, J. Richard Gott, Jacqueline Hewitt, John Huchra, Rudy Schild, and Edwin Turner.

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DISCUSSION

ARP: What would you expect the supposed galaxy underlying quasars to look like under magnification?

CANIZARES: That depends on the size of the lens. As a rule of thumb, if you project the lens scale back to the source, anything that falls inside will be magnified while anything outside will not. So a galaxy sized lens should be able to magnify the inner portions of a distant galaxy, whereas a cluster will magnify a whole galaxy.

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ARP: But that amplified and distorted image should extend beyond the 1 or 2 arc sec limit of optical resolution. This is not observed in the optical images of the lensed quasars.

CANIZARES: Recall that gravitational lenses really just magnify the source; they conserve surface brightness. I would think that even a magnified galaxy at z > 1 would be very hard to observe, but I don't know if anyone has modeled this for the specific cases of the known lenses.

NORMAN: Claude, isn't there still a serious problem with respect to the significant lack of small angular separation lenses on scales of ~1".

CANIZARES: I believe the consensus on this topic is that no serious problem exists if you include both galaxies and clusters acting together, as shown by Turner, Ostriker and Gott (1984) and Dyer (1984). In addition there are probably still selection effects that bias against detection of lenses with small separations. Jackie Hewitt (1986a) has been repeating the analysis for the VLA sample, where the selection effects are reasonably well understood. So far the statistics are too small to draw any conclusions. Also, it is precisely the candidates with the largest separations that are the most suspect, so some of them may go away.

LONGAIR: One important aspect of gravitational lensing for cosmological studies is the impact upon the observed intensities of quasars. If the luminosity function of quasars is steeper than $\phi(L) \propto L^{-3}$, gravitational lensing could significantly enhance the luminosities of distant quasars leading to the inference of stronger cosmological evolution for these objects. The luminosity function of the most luminous quasars may well be as steep as this. Is there evidence that these large redshift, highly luminous quasars might be enhanced by gravitational lenses?

CANIZARES: There is no "evidence" that I know of, although there have been some recent suggestions that lensing does indeed have a strong effect on the luminosity function where it is steep (Ostriker and Vietri 1986a, Vietri 1985, Schneider (1986d). It is the microlensing that seems to be most important. This led Ostriker and Vietri (1985) to suggest that BL Lacs were lensed quasars.

TURNER: In response to Dr. Longair's question, I would like to report that Burke, Roberts, Gott, and I carried out a VLA survey of ~30 of the highest optical luminosity, radio quasars a few years

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ago. No very good lens candidates with sizes $\gtrsim 0\overset{''}{.}3$ were found. This would seem to rule out lensing by galaxy mass lenses as significantly modifying the upper end of the optical luminosity function of radio selected quasars, at least.

YU XIN: The classical Fermat principle is based on the "Newtonian time". For the "lens effect" (based on general relativity) is the Fermat principle based on the "proper time"? In this case the (general relativistic) Fermat principle is really an assumption. Moreover, it appears that the gravitational field has to be known a priori before calculations can be made.

(2) For a non-Riemannian spacetime (with torsion) the quantities $\delta\int dt = 0$ and the autoparallel curve $k^{\alpha} \Delta_{\beta} k^{\beta}_{\alpha}$ are different. Would the Fermat principle still hold in this spacetime? Or would the Fermat principle rule out such a spacetime geometry.

CANIZARES: Yes, the General Relativistic extension of Fermat's principle does use the proper time along the light ray. Schneider (1985) discusses in more detail the conditions under which it appears to be valid and shows that it reproduces the results of the previous vector formalism. It is indeed necessary to assume a gravitational potential in order to calculate the lens properties; the advantage over previous methods is that the calculations are more efficient so one can iterate many times to explore the range of possible models. Also, certain general characteristics of lenses, such as the parity of the various images, become much clearer (see Blandford and Narayan 1986, Narayan 1986).

(2) I am not really competent to comment on this question. My own crude physical intuition would tell me that some form of Fermat's principle should hold in any universe in which light propagates along given paths. But I will leave that to the experts.