This corresponds exactly to Cluley's case P_c (Appendix to his article). For the fault rates and times of interest in his application the approximations are amply adequate. But a formulation of the more general theory (which is of course well known) is not without interest and may very well turn out to be of practical value in other navigational problems.

The Sextant and Precision Celestial Navigation

from Captain C. H. Cotter

WHEREAS it is doubtless true that the two largest contributions to the errors of sights at sea are those referred to by Mr. Sadler in his Prefatory Note to Captain H. H. Shufeldt's paper on Precision Celestial Navigation Experiments[1], it cannot, I think, be gainsaid that the principal avoidable error in sights is due to faulty use of the sextant, especially in respect of collimation error.

It is important, when using a sextant for measuring the altitude of a star, to ensure that the direct ray from the point on the visible horizon vertically below the star, and the three parts of the doubly-reflected, or zig-zag, ray from the star, lie in a common vertical plane. If this is not the case when taking a sight, the plane of the instrument—that is the plane on which lies the sextant arc—will, in all likelihood, be held out of the vertical plane; and the measured angle will be larger than the required angle.

Three conditions are necessary for the three parts of the zig-zag ray from a star to lie in the same plane. First, the index mirror must be perpendicular to the plane of the instrument; second, the horizon glass must be perpendicular to the plane of the instrument; and third, the line of sight must be parallel to the plane of the instrument. If the first condition is not satisfied, the sextant possesses perpendicularity error: if the second condition is not satisfied, side error exists. These errors, if they are present, may readily be detected, and easily removed by making the first and second adjustments.

When adjusting a sextant; after making the first and second adjustments, the third adjustment—if necessary—is made. This involves slewing the horizon glass to bring it parallel to the index mirror, having first set the pointer on the index bar coincident with the zero mark on the sextant arc. The fourth adjustment, if necessary, should then be made to ensure that the axis of the telescope is parallel to the plane of the instrument. It is not until the four adjustments are made that the sextant is fit for use. Moreover, having made the adjustments once, it is not likely that they will have to be made again, provided that the sextant is handled in a way befitting of a scientific instrument of precision.

If the axis of the telescope is not parallel to the plane of the sextant, collimation error may result in all observed altitudes. It is commonly believed that collimation error is due solely to the axis of the sextant telescope not being parallel to the plane of the instrument. Collimation error may be due to one, or both, of two causes. One of these is faulty housing of the telescope; a cause which may readily be removed by making the fourth adjustment. The other possible cause of collimation error is faulty observing.

Accurate use of a sextant for nautical astronomical purposes demands that the image of the celestial object which is observed lies precisely at the centre of the field of view of the telescope. Should the field of view of the telescope be large, there is a big chance, unless great care is taken on the part of the observer, that this condition will not be satisfied.

The inverting or astronomical telescope, which, in the past, formed part of all sextant outfits, had a relatively high magnification but a small field of view. This telescope, having a small object glass, was useless for starwork: it was intended for Sun sights, for which purpose it was far superior to the star telescope which was provided with the outfit. The star telescope, being simpler to use than the inverting telescope, was generally employed for daylight- as well as for twilight-observations. Despite the fact that the astronomical telescope, with its high magnification, was best suited for Sun observations, it usually remained in the sextant case, never to lose the brilliant pristine freshness which it acquired at the hands of the makers. In some dozen or so years of practical experience at sea, I was shipmates with only one navigator who used the inverting telescope for Sun observations. Of course, a little practice is necessary before one can use an inverting telescope efficiently; and it appears that most navigators are not prepared to acquire this practice. I feel sure that those navigators who are adept at using the inverting telescope never use a low-powered erecting telescope for Sun observations, unless the movements of the ship make it impossible to use the inverting telescope.

It may be of interest to note that some navigators refer to the inverting telescope as a *star* telescope. This, presumably, is because it is an astronomical telescope. Because of this, the erecting telescope is thought of as being a *Sun* telescope, and thus the correct one to use for Sun observations. A well-known U.S. manual of navigation refers to the inverting telescope as a star telescope, and in the ensuing description on how to observe the altitude of a star, the star telescope is claimed to provide '*for increased magnification of stars*'. Further: '*in order that the amount of light shall be decreased as little as possible, erecting lenses are omitted and the star telescope is an inverting telescope*'! Our own Admiralty Manual —which, sad to say, is not widely used by merchant seamen—makes it quite clear that the bell-shaped telescope having a large object glass is the *star* telescope.

The inverting telescope used in a sextant usually has a reticle consisting of two pairs of cross-wires. This serves the twofold object of checking the alignment of the telescope axis; and providing the observer with the means of locating the exact centre of the field of view.

The single reference in Captain Shufeldt's paper to a telescope reticle gave me the impression that its principal use was to enable the observer to focus his eye at infinity. Surely a reticle is not needed for this purpose. Does not the eye focus itself to infinity naturally and without effort: muscular effort being required to focus otherwise? In the interesting paper in volume 6 of the *Journal*, on Astronomical Observations at Sea, the author, Commander Sharpey-Schafer, expresses the view that a telescope reticle may distract an observer taking a sight. I am sure than in my personal experience the reticle never distracted me. For a sextant telescope which has a large field of view—the type recommended in the second appendix to the paper by Captain Schufeldt—I contend that a reticle is a useful, if not a necessary, adjunct.

Fixing a ship at sea by conventional astronomical methods, with the highest degree of precision, demands—first and foremost—a skilful observer. Part of the skill at using a sextant is related to collimation error due to faulty observing. A telescope reticle would certainly help to eliminate a source of error which, in my opinion, has not been given the attention which its importance warrants.

If collimation error affects an observation the measured altitude exceeds the required altitude by an amount which varies as the square of the angle of inclination of the line of sight to the plane of the instrument, and as the tangent of half the measured altitude. It may readily be proved that:

$$\cos i = \sin \tfrac{1}{2} \theta / \sin \tfrac{1}{2} \phi$$

where:

i is the angle of inclination of the line of sight,

θ is the required altitude, and

ϕ is the measured altitude.

From this expression, the following formula for collimation error ($e = \phi - \theta$) is derived:

$$e'' = i^2 \sin 1''. \tan\tfrac{1}{2} \phi$$

For values of $i = 1°$, and $\phi = 30°, 60°, 90°$; $e = 0'.3, 0'.6$, and $1'.1$ respectively. Errors of these magnitudes are by no means improbable when using a wide-angle telescope, if the observer is unskilled.

A further point worth mentioning in connection with star-work is related to my belief that there is a tendency amongst navigators who practice stellar navigation to observe stars only from a favourite group which usually consists of the brighter stars such as Canopus, Sirius, Aldebaran, Capella, and so on. The less bright stars, outside the favourite group, are generally ignored. The brighter stars, of course, are most suitable for star-work when the sky is light; but when the light is failing during evening twilight, or during the early part of morning twilight, the best stellar fixes are obtained by using the less bright stars. With a horizon dim because of poor light, a first magnitude star appears in the sextant telescope like a veritable ball of fire; and it is not easy to get a good grazing arc with the horizon. There is a case, therefore, for providing the navigator with astronomical data of stars fainter than those which merit the appellation—*navigational stars*.

D. H. Sadler comments:

I am very glad that Captain Cotter has drawn attention to the large errors that can arise through the faulty adjustment of the sextant and through its careless use. In most theoretical discussions on the precision of astronomical sights, it has to be assumed that both instruments and observers have been 'corrected' for such errors; the analysis of actual sights often gives rise to the suspicion that they have not been entirely eliminated, thus making much more difficult the determination of those errors for which corrections cannot easily be made.

[1] Shufeldt, H. H. (1962).

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