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JOVIAN PERTURBATIONS PER ORBITAL REVOLUTION FOR SHORT-PERIOD COMETS: STATISTICAL PROPERTIES

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The gravitational perturbations by Jupiter per orbital revolution are calculated for six selected groups of 1000 hypothetical random comets of low inclinations, perihelia units 6 a.u. and aphelia in the range (4 a.u., 13 a.u.). The results are organized in the form of empirical distributions and their statistical properties are investigated in detail. They provide a rich material for the study of problems such as the orbital evolution of short-period comets and the origin of the Jupiter family.

The "periodic comets" move, broadly speaking, within the confines of the planetary system. They are sometimes classified into "comet families" associated with the giant planets. For each family the cometary aphelia are situated near the planet's distance from the sun. This classification with respect to aphelion distance can be made without regard to the real connection between cometary and planetary orbits. However, among the observed periodic comets one finds only one large group in definite connection with a planet, namely the Jupiter family.

Gradual "deactivation" of comet nuclei as well as planetary perturbations into orbits with larger perihelion distances restrict the lifetimes of observable comets. For the Jupiter family the estimates of mean lifetimes are usually $\lesssim 10^3$ years, and hence a relatively efficient mechanism of replenishment must be active, if the Jupiter family is a constantly observable phenomenon.

The possibility of replenishment of the family by capture from intermediate- or long-period orbits has been the subject of many investigations. To the present knowledge the parent orbits must have low inclinations and perihelia near Jupiter's orbit (Everhart 1972), and for such orbits the probability of capture per unit time increases rapidly with the inverse semimajor axis z (Rickman and Vaghi 1976).

The work to be briefly described here is an attempt to explore the statistical effects of Jovian perturbations on low-inclination orbits typical for the observed Jupiter family and the neighboring ranges of the aphelion (Q) and perihelion (q) distances, and thereby to picture the final stages of cometary capture.

Six groups of comets have been defined according to the aphelion and perihelion distances as follows:

Group 1.	8.5 a.u. < Q < 13 a.u. ;	4 a.u. < q < 6 a.u.
Group 2.	8.5 a.u. < Q < 13 a.u. ;	q < 4 a.u.
Group 3.	6 a.u. < Q < 8.5 a.u.;	2.5 a.u. < q < 6 a.u.
Group 4.	6 a.u. < Q < 8.5 a.u.;	q < 2.5 a.u.
Group 5.	4 a.u. < Q < 6 a.u.;	2.5 a.u. < q < 6 a.u.
Group 6.	4 a.u. < Q < 6 a.u.;	q < 2.5 a.u.

This division reflects a concern for dynamical homogeneity of each separate group, and at the same time it makes possible certain useful identifications. Groups 1 and 2 may be identified with the Saturn family. In particular group 1 represents probable source comets for captures by Jupiter. Groups 3-6 may be called the Jupiter family. In particular groups 4 and 6 represent the observable part of the family, since q < 2.5 a.u. may be considered as an approximate condition for observability.

One random sample of 1000 comets with inclinations $i < 10^{o}$ has been chosen for each of the six (Q,q)-regions described above. The cometary motions have been calculated by a computer program for orbit integration (Stumpff and Weiss 1968) which is not extremely accurate, and in case of a close encounter the result for any particular comet may be considerably in error. The dynamical model is a nonplanar, elliptical restricted three-body model (Sun - Jupiter - comet), and for each comet the part of one orbital revolution within 6 a.u. from the sun has been considered, since the main perturbation by Jupiter occurs in this region.

The differences $\Delta\varepsilon$ between the final and initial orbital elements ε are the perturbations to be considered. *Univariate distributions* $f_j(\Delta\varepsilon)$ *for the different* elements and groups have been derived as well as some bivariate distributions, most often of the type $\phi_j(\Delta z,\varepsilon_o)$ giving the energy perturbations Δz in relation to the initial elements ε_o. All the statistical results cannot find place in the present report, and they will be published in a separate paper. However, a few general characteristics of the distributions may be anticipated.

1. The foremost characteristic of the perturbation distributions is their extreme peakedness - a property common to all different orbital parameters, whether referred to the whole sample of 6000 comets or to anyone of the six subgroups. The peak is always situated very near the zero perturbation.

It is obvious that in theoretical studies of planetary perturbations on cometary orbits normal distributions should, if at all, be used with great care. However, as a consequence of the peakedness, the true values of the second and higher moments of the distributions are difficult to determine with high precision.

2. Tails are usually observed on both sides of the peaks, but they are not always symmetrical. A small number of comets contribute to the tails. These are mostly comets which experienced close encounters with Jupiter (to within 0.41 a.u.). However, relatively large perturbations (*e.g.*, $\Delta Q > 2.1$ a.u. in group 1) may also affect comets with no such close encounters.

3. Examining group 1 as the "source group" for captures, 17 out of the 1000 comets experienced a close encounter with Jupiter in the above sense. However, in no case was the orbit so strongly modified as to transfer the comet directly into the observable Jupiter family. Hence no direct capture was observed.

4. Concerning Ω, the longitude of the ascending node, the results have so far been analyzed for groups 1 and 6. The distribution of $\Delta\Omega$ is sharply peaked, and more than 80% of the peak has $\Delta\Omega < 0$. The median value of $\Delta\Omega$ is about $-0\overset{\circ}{.}02$ for both groups. There is hence a general regression of the nodes, which

simply means precession caused by Jupiter's attraction toward the ecliptic plane.

5. Among the bivariate distributions we may accord to $\phi_j(\Delta z, i_o)$ a particular interest, because it provides an opportunity to measure quantitatively the rate of decrease of energy perturbations with inclination in the range $i < 10^o$.

It is worth mentioning that the statistical material thus collected may be used for the study of collective effects of orbital evolution for a population of comets by means of Monte Carlo simulation. Each comet then performs a random walk in the (Q,q)-plane, where the steps are chosen in accordance with the observed distributions $\phi_j(\Delta Q, \Delta q)$.

Such a technique has recently been employed in a study of the transfer of comets between intermediate-period orbits and the observable part of the Jupiter family (Rickman and Vaghi 1976). The method is very efficient when one wants to treat the orbital evolution of a large population of hypothetical comets over a long period of time, and its future development may offer the way to handle a more complete dynamical model including the perturbations by all major planets.

REFERENCES

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