

# CONDITIONS FOR INDEXABILITY OF RESTLESS BANDITS AND AN $O(K^3)$ ALGORITHM TO COMPUTE WHITTLE INDEX – CORRIGENDUM

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#### Abstract

This note corrects an error in the formula to obtain the Whittle index using the Sherman– Morrison formula in Akbarzadeh and Mahajan (2022). Also, some other minor typos are highlighted.

*Keywords:* Multi-armed bandits; restless bandits; Whittle index; indexability; stochastic scheduling; resource allocation

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### 1. Introduction

The following corrections need to be applied to [1]:

1. On p. 11, Equation (19) is missing a term and should be updated as follows:

$$D^{(h_{d,y})} = D^{(h_d)} - \frac{(1-\beta)\Phi^{(h_d)}(c^{(h_d)} - c^{(h_{d,y})}) + \beta\rho_y^{\mathsf{T}}\Phi_{\cdot y}^{(h_d)}D^{(h_d)}}{1+\beta\rho_y^{\mathsf{T}}\Phi_{\cdot y}^{(h_d)}}$$
$$N^{(h_{d,y})} = N^{(h_d)} - \frac{(1-\beta)\Phi^{(h_d)}(h_d - h_{d,y}) + \beta\rho_y^{\mathsf{T}}\Phi_{\cdot y}^{(h_d)}N^{(h_d)}}{1+\beta\rho_y^{\mathsf{T}}\Phi_{\cdot y}^{(h_d)}}.$$

These formulas are derived by combining Equations (8) and (18).

- 2. On p. 15, the numbers and indices in Part 4 are incorrect and should be updated as follows:
  - For y = 2,  $h_{2,2} = [0, 0, 0]$ ,  $N^{(h_{2,2})} = [0, 0, 0]$ , and  $D^{(h_{2,2})} = [-0.21, -0.22, -0.37]$ . Therefore,  $\Lambda_{2,2} = \{x \in \mathcal{X} : N^{(\bar{g}^{(W_2)})}(x) \neq N^{(h_{2,2})}(x)\} = \{1, 2, 3\}$ . Now, for each  $x \in \Lambda_{2,2}$ , we compute  $\mu_{2,2}(1) = \mu_{2,2}(2) = \mu_{2,2}(3) = 0.8$ . Therefore,  $\mu_{2,2}^* = 0.8$ .

Now  $\mu_{2,2}^* = 0.8$ . Therefore,  $W_3 = \{1, 2, 3\}$  and w(2) = 0.8.

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Some other grammatical errors and typos are highlighted below:

1. On p. 6, Figure 1: The lower  $D^{(h_2)}(x)$  should be changed to  $D^{(h_1)}(x)$ . The updated figure is as follows:



2. On p. 6, Lemma 1: The term 'piecewise linear' is repeated and should be removed from the first sentence. The updated lemma should read as follows:

**Lemma 1.** For any  $x \in \mathfrak{X}$ ,  $V_{\lambda}(x)$  is continuous, increasing, and concave in  $\lambda$ . Furthermore, when  $\mathfrak{X}$  is finite,  $V_{\lambda}(x)$  is piecewise linear in  $\lambda$ .

- 3. On p. 18, proof of Lemma 5: The statement starting with 'Now (D5) implies that ...' should be changed to 'The assumption that  $N^{(g^{(\ell)})}(x)$  is non-increasing in  $\ell$  implies that ....'
- 4. On p. 20, line 4: The second  $Z^{[g]}$  should be  $\tilde{Z}^{[g]}$ . So the line after the displayed equation should read as follows: '... then  $Z^{[g]} = \begin{bmatrix} 6 & 8 \\ 14 & 16 \end{bmatrix}$  and  $\tilde{Z}^{[g]} = \begin{bmatrix} 5 & 8 \\ 13 & 15 \end{bmatrix}$ .'
- 5. On p. 22, line 3: Where we say 'relative (percentage) performance improvement', we should instead say 'relative (percentage) performance'.

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## **Competing interests**

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#### Reference

[1] AKBARZADEH, N. AND MAHAJAN, A. (2022). Conditions for indexability of restless bandits and an  $\mathcal{O}(K^3)$  algorithm to compute Whittle index. *Adv. Appl. Prob.* **54**, 1164–1192.