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Overview and Preview

The beginnings of nuclear physics can be identified quite precisely with the experiments by Rutherford and his coworkers on the elastic scattering of α-particles [19]. While the techniques of measurement and indeed the particles used as projectiles have undergone change many times in the ensuing decades, the fundamental purpose of these elastic scattering measurements remains the same. They have provided much of our understanding of the size and structure of nuclei and of the interactions of elementary particles with them. As the energies of the particles have been raised, these experiments have become more elaborate in technique, but the importance attached to them has grown as well. The fact that the wavelength associated with the incident particles decreases with increasing energy means that measurements made at higher energies are implicitly capable of conveying more detailed information than those made at lower energies. Their resolving power, in other words, increases steadily with energy.

Making measurements at high collision energies characteristically requires the use of instrumentation that is large in scale and ponderous of mass. We need only recall the massive electromagnets required for calibration and momentum analysis of charged particle beams. The reduction and interpretation of the data gathered in these experiments have usually involved analytical problems of corresponding magnitude and weight. The more detailed the experimental measurements have become, the more computation seems to be required in order to extract fundamental data from them. While the most straightforward approaches have often been productive, they have tended to become lengthier with increasing particle energies, and to offer less direct insight into the meaning of even the more prominent features of the accumulated data.

When the kinetic energies of the projectile particles begin to exceed the energies of their interactions with target nuclei, the particles usually suffer only small deflections during collision processes. Since the particle wavelengths, at such energies, are considerably smaller than nuclear dimensions, the angular distributions

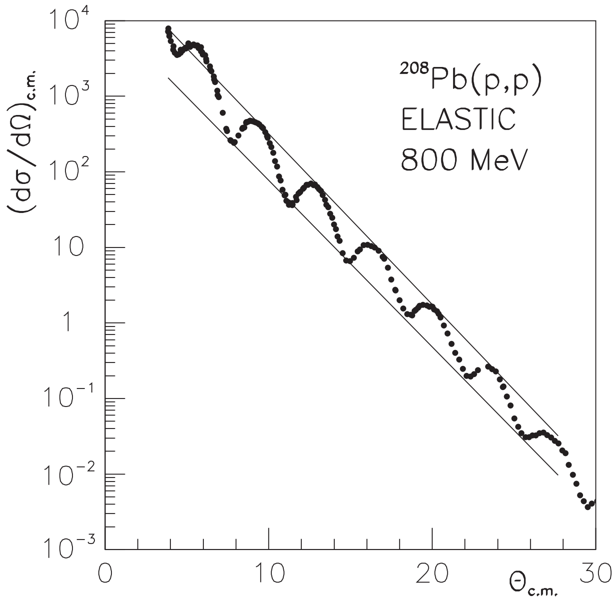

Figure 1.1 Differential cross section for elastic proton-lead scattering at 800 MeV [13].

of elastically scattered particles share many of the familiar properties of optical diffraction patterns. They tend to be strongly peaked for scattering in the forward direction, and to concentrate the scattered intensity within a fairly narrow cone. While the intensity tends to fall rapidly, on the average, with increasing scattering angle, it is also found often to oscillate more or less periodically. These features are quite evident, for example, in Fig. 1.1, which shows a succession of measured values of the differential cross section for the elastic scattering of 800 MeV protons by ^{208}Pb [13]. The measurements, which intentionally omit the high intensity of Coulomb scattering at small angles, show a decrease of the differential cross section by about seven orders of magnitude over the angular range from 3° to 30°. At the same time they show a fairly regular oscillation of the falling intensity which is surely a wave-mechanical interference effect of some sort.

To make accurate measurements of scattering over such a large range of interactions has meant overcoming many experimental challenges [13]. But once they are overcome, the theoretical analysis of the results presents formidable problems as well, and these call for innovative solutions. Our intention is to discuss the means of analyzing such data in the pages that follow by introducing an approximate approach to diffractive scattering, one which deals compactly with broad ranges of intensity. This approach should apply as well to other experiments in which diffracted intensities are studied over a broad range of magnitudes.

We have drawn two straight lines on the differential cross section graph of Fig. 1.1 to show that the average logarithmic slope of the curve is remarkably constant. The successive maxima and minima of the cross section both tend to follow a simple exponential law of decrease. The period of oscillations and their amplitudes both tend to remain surprisingly constant. Such observations immediately lead one to question the value of extending such measurements to still larger scattering angles. Will the angular dependencies to be found there reward the formidable labors of making measurements at extremely low intensities, or will they simply repeat the features of the differential cross section already seen at smaller angles? It would obviously be of value to develop a theory of the scattering process that allows direct insight into the reasons for the behavior noted and says clearly what can be determined by extending the range of measurements.

We shall show that there are indeed certain sorts of changes to be anticipated in the angular distributions at larger angles, and that it may indeed be worth the extensive effort of measuring them. We propose to do this by developing methods of approximation that are related quite closely to those of optical diffraction theory. The time-honored forms of optical diffraction theory are approximations based on integrals that can be evaluated in ways that are straightforward if fairly lengthy, but their results are not always transparent. We do not propose here to improve the accuracy of this approach. What we shall do is to discuss further approximations that give qualitatively clear insights into the nature of the results, and ones that often present no great sacrifice of accuracy. These methods furnish a considerable amount of analytic as well as numerical insight, we find, and engender qualitative understandings that have not been provided by the lengthier and less accessible, if more exact, methods of solving the scattering problem.

There are at least two traditional methods of calculating angular distributions of nuclear scattering. The most familiar depends on the separation of angular and radial variables. If the scattering interaction is spherical, the orbital angular momentum of the incident particle is conserved, and the Schrödinger equation can be separated into a sequence of radial differential equations, one for each integer value of the angular momentum. In that case integrating each of the radial equations leads to the evaluation of a complex phase shift for that angular momentum, and these in turn contribute to an explicit expression for the scattering amplitude as a sum of spherical harmonic functions (see Appendix A).

Though this procedure for evaluating the scattering amplitude is implicitly exact, it encounters serious difficulties in treating the problems we are considering. Measurements of scattering are most informative when the wavelength λ of the incident particle is much smaller than the radius R of the scatterer. In that case the number of radial equations to be integrated must be at least as large as R/λ, the minimal number of partial waves affected by this scattering. In practice it must often be much larger, particularly if the nuclear interaction does not cut off sharply at radius R. So there is no alternative to calculating a great many phase shifts. But there is a further difficulty implicit in the summation of the partial wave contributions. As we can see in the seven orders of magnitude by which the cross section drops in Fig. 1.1, there must be wholesale cancellation of the amplitudes contributed by the different partial waves. In the actual experiments, nature provides that cancellation effortlessly, but for calculational purposes, it places extreme demands on the accuracy with which the individual phase shifts must be determined. If compactly interpretable results are desired, this exact route would not be the one to follow.

An alternative approach, one that deals well with sums over large numbers of partial waves, corresponds closely to optical diffraction theory. If we are only interested in scattering through relatively small angles, we can express the scattering amplitude as an integral over the plane of impact parameters very much as it is done in Fraunhofer diffraction theory [21]. Evaluation of the scattering amplitudes is thus reduced to numerical evaluation of certain Fourier integrals. But here too the numerical approach encounters prohibitive problems. It would not be easy to maintain the accuracy required to reach a meaningful result given all the cancellation that must be implicit in the drop of the cross section by many orders of magnitude.

We conclude that it is a better strategy to isolate, if we can, the asymptotic behaviors of these scattering amplitudes, and to study them more directly, in order to interpret what goes on at larger momentum transfers. This approach, as we shall see, will involve further approximations, but with small and controllable errors. It will lead us gradually away from the analogies with classical diffraction theory with which our analysis begins, and introduce us to the discussion of particle trajectories in a space in which the particle coordinates become complex, rather than the familiar one in which they remain real-valued. It will reveal particle behaviors that depart significantly from those of classical mechanics and describe new behaviors of scattering amplitudes. Our hope is thereby to be able to find expressions for the amplitudes that are both compact and suggestive of new interpretations.

As a way of illustrating some of the properties of diffraction theory, let us consider the transmission of a plane light wave of propagation vector $\boldsymbol{k}$, $(k = 2\pi/\lambda)$, through an aperture in a flat opaque screen. We take the screen to define the plane of coordinate vectors $\boldsymbol{b}$ and assume that the amplitude of the scalar wave at any point in the plane of the aperture is $A(\boldsymbol{b})$. Then, within the approximations of Fraunhofer diffraction theory, the amplitude $f(\boldsymbol{k}',\boldsymbol{k})$ of the wave diffracted in the direction $\boldsymbol{k}'$ $(k' = k)$ is proportional to the two-dimensional Fourier integral [8],

$$f(\boldsymbol{k}',\boldsymbol{k}) \sim \int e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{b}}\, A(\boldsymbol{b})\, \mathrm{d}^2 b, \tag{1.1}$$

taken over the plane of the aperture, with $\mathrm{d}^2 b$ the element of area.

The slits and pinholes of physical optics have usually been made of metal. Their hard-edged character has meant that $|A(\boldsymbol{b})|$ is approximated as decreasing abruptly to zero at the edge of the aperture. We could, in principle however, fabricate (for example, by photographic means) an aperture such as a slit of continuously varying transparency, which transmits a wave with a smooth intensity profile. Let us consider, for example, a straight slit parallel to the y-axis, which transmits the smoothly varying wave amplitude

$$A(x) = \frac{\beta}{\pi} \frac{1}{x^2 + \beta^2}, \tag{1.2}$$

where x is the coordinate transverse to the slit and β is a measure of its width. Then the amplitude of the diffracted wave can be written as the integral

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim \frac{\beta}{\pi} \int_{-\infty}^{\infty} \frac{e^{-iqx}}{x^2 + \beta^2} \, \mathrm{d}x \tag{1.3}$$

$$\sim \frac{1}{2\pi i} \int_{-\infty}^{\infty} e^{-iqx} \left\{ \frac{1}{x - i\beta} - \frac{1}{x + i\beta} \right\} \mathrm{d}x, \tag{1.4}$$

where $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$. The integrand obviously has two poles in the complex x-plane. For $q < 0$ the contour can be closed by a semicircle in the upper half-plane, and then the pole at $x = i\beta$ contributes the residue $e^{q\beta}$. For $q > 0$ the contour can be closed in the lower half-plane and the residue is $e^{-q\beta}$, but the contour is closed in a clockwise sense. The result for both cases may be stated in the form

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim e^{-\beta|\boldsymbol{q}|}. \tag{1.5}$$

Since $|\boldsymbol{q}| = |\boldsymbol{k}' - \boldsymbol{k}| = 2k \sin \frac{\theta}{2}$, and at small angles this is just $k\theta$, we already have a hint of how exponential decreases of intensity, such as that shown in Fig. 1.1, can arise.

There are in fact two lessons to be learned here. The first is that a smooth absorption profile leads to a much more rapid decrease of intensity with increasing scattering angle than is familiar in optical experiments with sharp-edged apertures. The second is somewhat more abstract. For $q > 0$, for example, a single, complex value of x, the pole at $x = -i\beta$, furnishes a scattering amplitude equivalent to the whole range of real values of x that span the width of the slit. This is an elementary illustration of a technique of using complex trajectories that we shall explore further.

The scattered intensity $|f(\boldsymbol{k}', \boldsymbol{k})|^2$ for our diffuse slit drops off exponentially, according to Eq. (1.5), but it does not show any of the oscillations evident in Fig. 1.1. To secure such oscillations we need only let the waves transmitted by two such slits interfere [3,15]. Let us assume, for example, that we have two such slits that are parallel to each other and centered at $x = \pm c$. Then the wave amplitudes transmitted by the screen can be written according to the integral Eq. (1.1), which represents the approximation of Fraunhofer diffraction theory, as

$$A(x) = \frac{\beta}{2\pi}\left\{\frac{1}{(x-c)^2+\beta^2} + \frac{1}{(x+c)^2+\beta^2}\right\}, \tag{1.6}$$

and the corresponding diffraction amplitude, according to Eqs. (1.5) and (1.6), must then be

$$f(\boldsymbol{k}', \boldsymbol{k}) \sim e^{-\beta|\boldsymbol{q}|} \cos(cq). \tag{1.7}$$

The angular separation of the successive minima in the intensity is then fixed and varies inversely with the distance between the slits. There are several ways in which the diffraction pattern corresponding to Eq. (1.7) still differs from the angular distribution of Fig. 1.1. It has periodic zeros while the experimental intensity oscillates between positive bounds. Furthermore, the phase of its oscillations is somewhat different. Both of these differences can be addressed by assuming that some weak refraction takes place as the wave penetrates the aperture, and therefore adding appropriate phase shifts to the amplitude $A(\boldsymbol{b})$.

This set of ad hoc assumptions can readily be expanded to account for more of the features of the angular distributions measured in elastic nuclear scattering, but to proceed further by such means would risk omitting a number of interesting physical phenomena. It would furthermore leave the geometrical basis of the method quite unclear. Instead we shall begin in the next chapter the development of a systematic analysis that yields a more comprehensive version of the double-slit paradigm for proton scattering by lead and for a broad range of other cases that have been measured experimentally. But the two-slit paradigm is not universal in its application and, as we shall see, there are many cases in which the systematic analysis yields amplitudes that correspond to single-slit patterns or to those of multiplicities of dissimilar slits. The key to our analysis will be the application of a simple asymptotic approximation to the theory of Fraunhofer diffraction. We shall show, through this approach, that it is possible to give simple explanations for a considerable variety of behaviors of the differential cross section and shall discuss explicitly many of the more interesting behaviors that have been, or are likely to be, observed. We hope that the various cases we have treated can thus provide something akin to an atlas of the simpler diffractive angular distributions.

Regularities similar to those we have noted in elastic scattering are also present in the angular distributions of inelastic scattering. Three examples that occur in the scattering of protons by ^{144}Sm at 800 MeV [7] are shown in Fig. 1.2. These distributions too show a nearly constant average logarithmic slope and more or less regularly spaced oscillations. These features are present in inelastic scattering, in fact, for nearly the same reasons that cause their presence in elastic scattering.

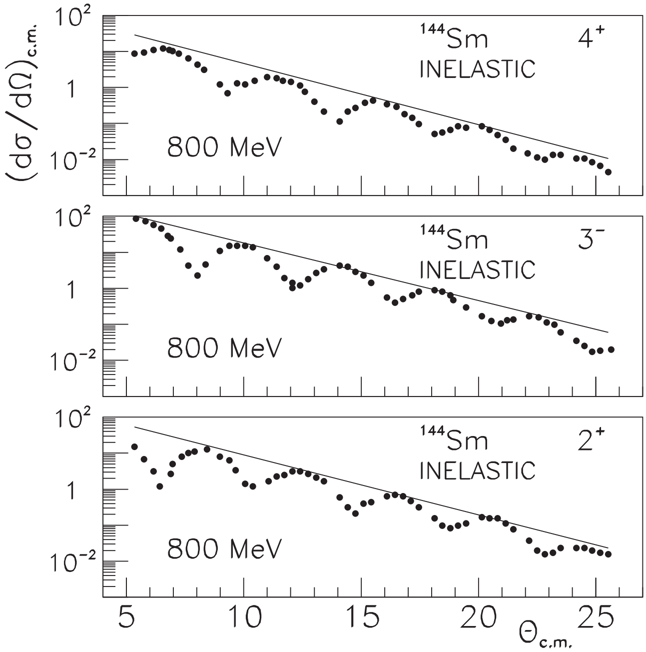

Figure 1.2 Differential cross section for inelastic proton-samarium scattering at 800 MeV [7].

Whatever rearrangement of nucleons an incident particle may produce in an inelastic collision, the collision process may be pictured as taking place against a background of elastic scattering by the nucleus as a whole. It is important therefore to concentrate on the description of elastic scattering, before moving on to the consideration of inelastic scattering.