ON THE ADOPTION OF A TERRESTRIAL REFERENCE FRAME

Dennis D. McCarthy
U. S. Naval Observatory
Washington, D.C. 20390

ABSTRACT. The report of the IAU Working Group on Nutation endorsed by Commissions 4, 8, 19 and 31 at the 1979 General Assembly points out that "... the complete theory of the general nutational motion of the Earth about its center of mass may be described by the sum of two components, astronomical nutation, commonly referred to as nutation, which is nutation with respect to a space-fixed coordinate system, and polar motion, which is nutation with respect to a body-fixed system ...". Unlike the situation for the space-fixed frame, there is not an adequate, formally accepted, body-fixed system for this purpose. The Conventional International Origin (CIO) as it is presently defined is no longer acceptable because of recent improvements in observational techniques. The effective lack of this type of terrestrial reference frame limits the complete description of the general nutational motion of the Earth. In the absence of a terrestrial reference frame suitable for specifying the orientation of the Earth, it is suggested that a body-fixed system could be represented formally in a manner analogous to that used to represent the space-fixed frame. This procedure would be quite similar to methods employed currently by the International Polar Motion Service and the Bureau International de l'Heure, and would allow for the use of observations from new techniques in the definition of a terrestrial reference frame to be used to specify the complete nutational motion of the Earth.

1. INTRODUCTION

The theories of precession and nutation along with the observed polar motion and rotation angle of the Earth are required to describe the orientation of users' local, body-fixed reference directions in the space-fixed frame. These directions are reproducible reference vectors used to make observations. Examples include local plumb lines, baseline vectors of radio interferometers, vectors directed from the Earth's center of mass to the observer's location, etc. The mathematical models and observations seek to describe the orientation of a terrestrial frame with respect to the space-fixed frame in such a

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way that an observer may account for the changing orientation of the local reference direction.

Commissions 4, 8, 19 and 31 of the International Astronomical Union have recently specified the theory of nutation to be used to model the motion of the Celestial Reference Pole with respect to a space-fixed reference frame (IAU Commission 4 Report; Kinoshita, et al, 1979). The space-fixed frame is realized in practice by the adopted precession along with the positions and proper motions of a set of fundamental stars (FK4 or FK5) or by positions and proper motions of stars which can be considered to be "in the fundamental system" (Fricke, 1975). Other realizations of a space-fixed system are considered to be a set of positions of distant radio sources or the dynamic reference frame defined by the ephemerides of solar system objects or Earth satellites (Kovalevsky, 1979). Important considerations regarding the rigorous definition of a space-fixed or non-rotating system remain to be discussed. However, for practical purposes we may consider the directions to the stars of the FK4 or its successor, FK5, to be the realization of the space-fixed system implied in the theories of precession and nutation.

To complete the description of the Earth's orientation to meet users' needs, then, we require a terrestrial system which can be related to the space-fixed frame through observations using the theories of precession and nutation. We assume that the individual reference vectors may be related to a global reference frame. If the orientation of this frame is determined, the observer can account for the changing orientation of the local reference. Traditionally, the Earth-fixed vectors are specified by the adoption of numerical constants at some epoch and are generally related to a terrestrial coordinate system defined by the Conventional International Origin (CIO) and a longitude reference (e.g., Bureau International de l'Heure zero meridian). In view of the advent of new, more precise observational techniques and the need for higher accuracy, it is important to re-examine the concept of the terrestrial reference system, particularly in conjunction with the IAU precession and nutation theories to be adopted in 1984.

2. OBSERVATIONAL PRACTICE

Presently we assume that the reference directions are "fixed to the Earth" and that these directions do not change in time with respect to some Earth-fixed reference frame. These directions are defined at a reference epoch, T_O, in the space-fixed system. This is done by making astronomical observations of the reference direction at an epoch, T, and rotating the observed direction vector through assumed rotation angles to account for the change in orientation of the Earth in the space-fixed system from T_O to T. The orientation at T with respect to T_O is determined using the adopted positions and proper motions of the fundamental stars, the adopted precession and nutation theories, and an assumed knowledge of the polar motion and the change

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in rotational speed from T to T_O. Since the reference directions are fixed to the Earth, observations of the changes of the direction of the vectors in the space-fixed system as a function of time may be used to determine the changing orientation of the terrestrial reference system with respect to the space-fixed system. In this process the Celestial Ephemeris Pole or the rotational pole in Woolard's (1953) nutation serves as an intermediate reference direction. Note that the Celestial Ephemeris Pole is not Woolard's rotational pole.

Because classical astro-geodetic instruments have been used to monitor the orientation of the Earth, the reference directions until now have generally been specified by the astronomical latitudes and longitudes of observing sites. This has been done to make use of the plumb line, or gravity vector, as the local reference direction. Astronomical coordinates, however, in general do not specify the location of a site with respect to a particular origin, but they do provide an orientation reference useful to measure changes in the Earth's orientation in space. To determine the changing orientation of the Earth a set of coordinates specifying station location with respect to an origin (e.g., Earth's center of mass) is not required. Only the specification of local reference directions is needed (Fedorov, 1979; Grafarend, et al, 1979).

2.1. DEFINITION OF THE CIO

To define the equator of the currently adopted terrestrial system, the astronomical latitudes of the five International Latitude Service (ILS) visual zenith telescopes are specified. Since the astronomical latitude defines the angle between the local vertical and the equatorial plane, the equator is defined and the CIO is the direction perpendicular to that plane (Fig. 1). Note that this does not define a unique pole on the surface of the Earth (Fedorov, 1979), but rather a direction perpendicular to the plane of the equator. The definition of this equator (and the direction of the CIO) contains the effects of any errors in the adopted astronomical latitudes of the rather limited set of the five ILS instrumental reference vectors. This definition is extended informally through the adoption of observatory "coordinates" for a number of observing locations. These coordinates are related to the CIO by the IPMS and the BIH.

2.2. DEFINITION OF THE ZERO MERIDIAN

Analogously, we define a reference direction lying in the plane of the equator by the adopted astronomical longitudes of a number of stations which observe the times of transit of stars (Fig. 2). The stars have an assumed angular relationship to the fiducial point defining UT1 through their adopted positions and proper motions. The procedure by which the longitudes are defined is based on the combination of observations from a large number of instruments which is done in an effort to eliminate systematic errors. Operationally this function is performed by the BIH. This procedure, while adequate to determine UT1-UTC until the present time, is not clearly defined in terms of a for-

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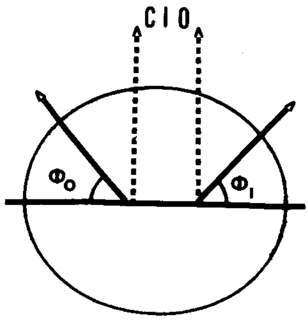

Fig. 1. Relationship between astronomical latitudes and the CIO.

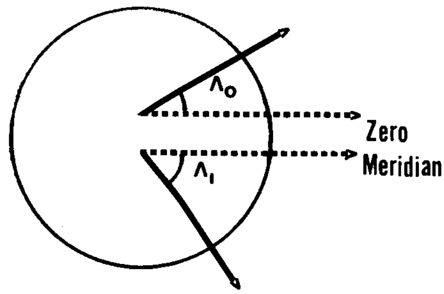

Fig. 2. Relationship between astronomical longitude and the zero meridian.

mally adopted terrestrial reference frame.

2.3. NEED FOR IMPROVEMENT

Recently, new techniques and improvements in classical techniques have made it possible to determine Earth orientation parameters with unprecedented precision. At the same time new demands have been made on the accuracy required by users of Earth orientation information. The precision already achieved and the accuracy demanded exceeds the precision with which the present formal realization of a terrestrial system is defined. Because of the age of the instruments, the ILS visual zenith telescopes may no longer be capable of providing polar motion data. In addition, the present definition does not formally incorporate the possibility of including new techniques in the definition of the terrestrial frame. Therefore, it is important to reconsider the realization of the terrestrial frame in order to make use of the precision currently available and to produce a clearly defined system for users of Earth orientation information that may continue to be useful in the future. Any useful reference system must also be easily accessible operationally.

3. SUGGESTED IMPROVEMENT

A possible improvement is to define a Conventional Terrestial System by a set of reference directions similar to the way the Fundamental Star System is defined by the directions to fundamental stars. This is quite similar to what is done now in practice by the BIH and the IPMS through the adoption of the astronomical coordinates of contributing optical observatories. However, as we have seen above, the adopted astronomical coordinates are suitable only for those techniques which utilize the gravity vector for the local reference direction. The present situation could be improved and generalized by the adoption of reference vectors suitable for each technique and by including a larger number in the formal definition of the system. This would be

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analogous to the Fundamental Star System where the directions to a number of stars are specified in order to define it. The direction perpendicular to the Conventional Earth System equator could continue to be called the CIO. The adopted reference vectors can be chosen so that there would be no discontinuity in the polar coordinates and UT1-UTC. This procedure would allow the formal definition of a terrestrial system to incorporate a larger number of reference directions with improved precision and accuracy. It would also involve continued dedicated observations of the defining reference directions.

3.1. IMPLEMENTATION

Consider a body-fixed reference frame with unit base vectors E_1, E_2 E_3 defined by the direction of reference vectors observed in the space-fixed frame at an epoch, T_o. These reference vectors include plumb lines, interferometer baselines, vectors directed from the center of mass of the Earth to the observer, etc. Assume that the reference frame is rotating about E_3 with an angular speed of Ω, and that E_3 is the direction of the pole implied in the theory of precession and astronomical nutation. E_1 and E_2 are orthogonal to E_3 and to each other (Fig. 3).

Assuming that we know Ω and the theory of nutation and precession, we can predict the orientation of (E_1, E_2, E_3) as a function of time $[E_1(T), E_2(T), E_3(T)]$ in the space-fixed system. However, if the observed effects of the Earth's variable rotation rate and polar motion are not accounted for, this will not accurately specify the orientation of the "Earth-fixed" reference vector, R, in the space-fixed frame. To do this we define the Terrestrial Reference Frame (unit base vectors e_1, e_2, e_3) to be identical with (E_1, E_2, E_3) at T_o.

This system is defined in reality by the definition of the reference vectors,

$$R(T_o) = R_1 e_1(T_o) + R_2 e_2(T_o) + R_3 e_3(T_o) = R_1 E_1(T_o) + R_2 E_2(T_o) + R_3 E_3(T_o).$$

We assume that the (e_1, e_2, e_3) system is rotating with angular speed $\omega(T) = \Omega + \delta\omega(T)$, $\delta\omega(T)/\Omega$ being small. Let us also assume that polar motion is represented by a counterclockwise rotation about E_2 by an angle $x(T)$ and a counterclockwise rotation about E_1 by an angle $y(T)$. Also let

$$\theta = \int_{T_o}^{T} \delta\omega(t)\, dt$$

represent the counterclockwise rotation of (e_1, e_2, e_3) about E_3. Then :

$$e_1(T) = E_1(T) + \theta(T)\, E_2(T) + x(T)\, E_3(T),$$
$$e_2(T) = E_2(T) - \theta(T)\, E_1(T) - y(T)\, E_3(T),$$
$$e_3(T) = E_3(T) - x(T)\, E_1(T) + y(T)\, E_2(T).$$

At an epoch, T, we can repeat the observations of R in the space-fixed frame and obtain

$$R(T) = [R_1 + \delta R_1(T)]\ E_1(T) + [R_2 + \delta R_2(T)]\ E_2(T) + [R_3 + \delta R_3(T)]\ E_3(T).$$

We also have the expression:

$$R(T) = [R_1 + m_1(T)]\ e_1(T) + [R_2 + m_2(T)]\ e_2(T) + [R_3 + m_3(T)]\ e_3(T)$$

where the terms m_1, m_2, m_3 represent possible effects of the relative motion of the reference vectors. For a simple rigid-Earth model such as is used currently, these terms are zero. Combination of the above expressions leads to:

$$\begin{pmatrix} \delta R_1(T) \\ \delta R_2(T) \\ \delta R_3(T) \end{pmatrix} = \begin{pmatrix} m_1(T) \\ m_2(T) \\ m_3(T) \end{pmatrix} + \begin{pmatrix} -R_3 & 0 & -R_2 \\ 0 & R_3 & R_1 \\ R_1 & -R_2 & 0 \end{pmatrix} \begin{pmatrix} x(T) \\ y(T) \\ \theta(T) \end{pmatrix}$$

The solution for all three components of the Earth orientation parameters using observations from one reference vector is indeterminate. If more than one suitably conditioned reference vector is available the solution is possible. It can be shown that the use of the above matrix expression leads to the expression for the variation in interferometer baseline vector components as a function of the Earth orientation (McCarthy, et al, 1979) as well as the familiar expressions for the classical "variation of latitude" and UT1-UT0 which depend on the orientation of the Earth.

The definition of the reference vectors can be accomplished in an internal adjustment to reduce the internal errors of a solution involving many possible observing instruments. The solution could be constrained to have no systematic deviation from the past IPMS and BIH solutions.

In practice, we use a local Earth-fixed reference frame at the location of the observer (longitude λ and latitude ϕ). Let this frame with unit base vectors j_1, j_2, j_3 be oriented so that j_3 is parallel to e_3, j_1 is in the plane of the local meridian, and j_2 is oriented to the west. Note that this is a left-handed system. If the components of the reference vector in this system at T_o are r_1, r_2, r_3, and if a number of such reference directions are to be used to determine Earth orientation parameters, the system may be solved using a least--squares solution:

$$\begin{pmatrix} a_1 & a_4 & a_7 \\ a_2 & a_5 & a_8 \\ a_3 & a_6 & a_9 \end{pmatrix} \begin{pmatrix} x \\ y \\ \text{UT1-UTC} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix},$$

$$a_1 = \Sigma r_3^2 + \Sigma r_1^2 \cos^2\lambda + \Sigma r_2^2 \sin^2\lambda - 2\Sigma r_1 r_2 \sin\lambda \cos\lambda,$$

$$a_2 = a_4 = \Sigma r_1^2 \sin\lambda \cos\lambda + \Sigma r_1 r_2 \cos^2\lambda - \Sigma r_1 r_2 \sin^2\lambda - \Sigma r_2^2 \sin\lambda \cos\lambda,$$

$$a_3 = a_7 = -\Sigma r_2 r_3 \cos\lambda - \Sigma r_1 r_3 \sin\lambda,$$

$$a_5 = \Sigma r_3^2 + \Sigma r_1^2 \sin^2\lambda + \Sigma r_2^2 \cos^2\lambda + 2\Sigma r_1 r_2 \sin\lambda \cos\lambda,$$

$$a_6 = a_8 = \Sigma r_3 r_1 \cos\lambda - \Sigma r_2 r_3 \sin\lambda,$$

$$a_9 = \Sigma r_1^2 + \Sigma r_2^2,$$

$$c_1 = \Sigma\delta r_1 r_3 \cos\lambda - \Sigma\delta r_2 r_3 \sin\lambda - \Sigma\delta r_3 r_1 \cos\lambda + \Sigma\delta r_3 r_2 \sin\lambda,$$

$$c_2 = \Sigma\delta r_1 r_3 \sin\lambda + \Sigma\delta r_2 r_3 \cos\lambda - \Sigma\delta r_3 r_1 \sin\lambda - \Sigma\delta r_3 r_2 \sin\lambda,$$

$$c_3 = \Sigma\delta r_2 r_1 - \Sigma\delta r_1 r_2.$$

Preliminary results found from the application of this procedure to data from two photographic zenith tubes, Doppler satellite data and connected element interferometer observations show that the daily values of polar coordinates can be determined with an accuracy of $\pm 0\overset{''}{.}012$ while daily values of UT1-UTC can be obtained with an accuracy of $\pm 0\overset{s}{.}0016$.

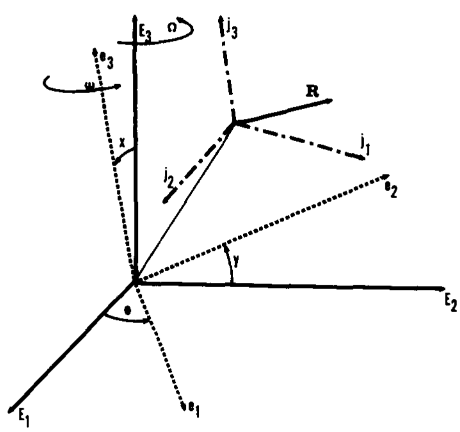

Fig. 3. Relationship among terrestrial reference frames.

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3.2. RELATIONSHIP TO GEODETIC SYSTEMS

For the purpose of determining the changing orientation of the Earth it is not strictly necessary to base a terrestrial system on a geodetic system of locations. The Conventional Earth System outlined above can be related to Earth-centered geodetic systems if the location of the center of mass of the Earth with respect to a sufficient number of observing stations were determined. This is not necessary to describe the motion of the Earth about its center of mass, however. The Conventional Earth System is not a geodetic system just as the CIO and the BIH zero meridian do not presently form a geodetic reference system suitable for station location. It does not provide geodetic positions but directions to be used only to define the orientation of the Earth and Earth-centered geodetic systems.

3.3. SYSTEMATIC ERRORS IN $R(T_o)$

Since observations of $\delta r_i(T)$ are dependent on local conditions and observing procedures, most current observations contain systematic differences among themselves. The BIH and the IPMS allow for these errors by applying empirical corrections to contributed data. In the development of a Conventional Earth System the systematic differences could be evaluated to provide the best estimates of the $R_i(T_o)$ available at that time. Systematic errors, both periodic and aperiodic, in the observations will continue to require the application of error models to the observations. These, however, should be treated as corrections to the observations and not to the $R\ (T_o)$. Periodically, estimates of the systematic differences should be evaluated to determine improvements to the individual $R\ (T_o)$ in order to refine the definition and maintain the system just as is done with the Fundamental Star System. This may necessitate the introduction of models which describe the relative motion of the reference directions to allow for true geophysical motion (i.e. station proper motions).

4. CONCLUSION

In view of the need for an improved terrestrial reference system suitable for use in determining the orientation of the Earth with respect to a space-fixed frame, it appears that the presently adopted formal definition of the CIO and the generally accepted BIH zero meridian are not adequate. It is suggested that a more general definition capable of incorporating observations from a number of techniques be formally accepted. The Conventional Earth System presented here appears to meet these needs and experimental use of this system shows it is a viable solution to this problem.

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