ACCRETION DISK BOUNDARY LAYERS: THEIR STRUCTURE AND STABILITY

G. Q. G. STANLEY and J. C. B. PAPALOIZOU
Theoretical Astronomy Unit, School of Mathematical Sciences,
Queen Mary College, London, U.K.

ABSTRACT.
The importance of the boundary layer has been known for some time, (see Lynden-Bell and Pringle, 1974). Yet this region of the disc has never been studied in great depth. We present here some calculations which are undertaken in order to explore some of the complex processes which can go on in this region. It is shown how the structure of the boundary layer is affected by viscosity, how oscillations can occur in the outer disk and boundary layer regions. We also show how they disperse and dissipate.

1. INTRODUCTION.

This problem is so complex that we have had to simplify it substantially to make it tractable. The most obvious omission from this work is that of thermal effects. We feel that this can be justified to some extent in certain situations by reference to the work of Pringle and Savonije (1979) and King and Shaviv (1984). We use a modified form of the Shakura and Sunyaev (1973) viscosity model, this takes into account the change in scale lengths found in the inner regions of the disk. A more detailed explanation of some of the theory can be found in Papaloizou and Stanley (1986).

2. VISCOSITY AND SCALE LENGTHS.

Shakura and Sunyaev proposed a turbulent model for the shear viscosity, ν, present in a disk. It is effectively due to turbulence acting over a length l with a speed αC_S. l has been generally accepted to be on the order of the disk thickness H. However, in the boundary layer region, the pressure, P, varies rapidly as material accretes onto the central star. The scale length over which this pressure acts is given by H_p, where:

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$$H_p = -P\left[\frac{dP}{dr}\right]^{-1}$$

In the boundary layer turbulence is taken to work over the scale length H_p as we have $H_p << H$. Taking this into account, we propose that l is given by:

$$l = \frac{1}{\frac{1}{H} + \frac{\beta}{H_p}}$$

where if $\beta = 0$ we revert to the *'normal'* form of ν. In the outer regions of the disc $H_p \sim R$, where R is the radius of the disk. Thus for large values of $\beta \sim 7.5$, this parameter can effect the outer disk, as we find $H/r \sim 0.1$. We have also assumed non-isotropic turbulence, which means that we can take ν_r/ν as an extra free parameter, which is held constant throughout the disc. (ν_r is the viscosity present in the radial direction.)

Taking the maximum in Ω, the angular rotation rate, to occur at $r = r_m$, we find that the associated scale length is given in the steady state by:

$$r_L^2 = -\Omega(r_m)\left[\frac{d^2\Omega}{dr^2}\right]^{-1}_{r = r_m}$$

Neglecting ν_r we find that in the steady state,

$$r_L \sim (r_m H_m)^{1/2}$$

This length scale is generally larger than the disk thickness at this point, (i.e H_m), which agrees with our calculations, (see Section 4).

The thickness of the boundary layer, r_D, can be obtained by equating the rate of loss of kinetic energy to that of the dissipation present in the layer. This result reduces to:

$$r_D \sim \left[\frac{H_p}{\beta}\right]_B$$

where B indicates evaluation in the middle of the layer.
In general we have:

$$r_D \lesssim H_p < H_m < r_L < r_m$$

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Using local approximations around r_m, the equations for the rotational and radial motion, we find the change in V_r. For a sub-sonic boundary layer this change is:

$$\sim \alpha C_S / \beta$$

where C_S is the sound speed.

3. OSCILLATIONS, DISPERSION AND DISSIPATION.

The dispersion relationship can be obtained by using a local linear perturbation analysis on the equations of motion. We have taken the perturbed quantities, denoted by a prime, to vary like:

$$e^{i\sigma t + ikr}$$

We consider a disk which obeys the polytropic equation of state, for which the vertically integrated pressure $\alpha\ \Sigma^{1+1/n}$. When $\beta = 0$, ν and ν_r are functions of the surface density, Σ, and r. Writing:

$$(\nu\Sigma)' = \frac{d(\nu\Sigma)}{d\Sigma}\Sigma' = q\Sigma' \quad ; \ q = (1 + 1/n)\ \nu$$

we obtain:

$$\sigma^2 = \Omega^2 + C_s^2 k^2 + i\sigma k^2 \left[\frac{4}{3}\nu_r + \nu - 3\frac{\Omega^2}{\sigma^2}q \right] + \frac{4}{3}\nu\nu_r k^4 - i\frac{C_s^2}{\sigma}\nu k^4$$

For long wavelength oscillations we can ignore terms of order k^4. If, then, σ and Ω are not too disimilar from the Keplerian value we can see that instability occurs for:

$$\frac{4}{3}\nu_r + \nu - 3q < 0 \quad \Rightarrow \quad \frac{\nu_r}{\nu} < \frac{3}{4}\left[2 + \frac{3}{n}\right]$$

Hence we expect instabilities to occur in the models given below.

Firstly, consider large r and fixed σ, the dispersion relationship shows that at the point where $\sigma = \Omega$, $k = 0$. Because Ω decreases with increasing r, internal to this point there is an evanescent region and waves can only propagate outwards from this point with $|k|$ increasing. Eventually this results in initially unstable waves dissipating as they travel onwards. The value of k, k_M say, for which this first occurs is approximately given by: $k_M \sim H^{-1}$. Note that if a rigid boundary is present, then reflection may occur before the waves are dissipated. In this case unstable waves can be trapped at the outer region of the disc. This is seen in the figures given below.

The inner region is more complex due to the high radial inflow velocities present, which may not be ignored. When viscosity may be neglected, the dispersion relationship is modified to be

$$(\sigma + kV_r)^2 = \Omega^2 + C_s^2 k^2$$

As propagation to distant regions eventually causes damping of an initially long wavelength and hence unstable wave, waves with zero group velocity should be the most favoured. These have

$$\sigma = \left[1 - \frac{C_s^2}{V_r^2} \right]^{1/2} \Omega$$

We see from this that periods longer than the local Keplerian period may be generated.

4. NUMERICAL RESULTS.

Two sets of calculations for vertically averaged equations are presented, both with $\nu_r/\nu = 1$, $\alpha = 1$, and $n = 3$. However, for model A, $\beta = 7.5$, and model B, $\beta = 0.25$. The unit of time is the reciprocal of the Keplerian angular velocity at the outer boundary; length the discs radius, and mass that of the central object. For more details of the numerical scheme and boundary conditions see Papaloizou and Stanley (1986).

Model A has a longer inner oscillation period than model B. This ties in with the latter having a much larger velocity in the boundary layer (figures 1 and 2). Figures 3 and 4 show that these waves travel outwards from the point where $V_r = C_s$. Eventually dissipating further out.

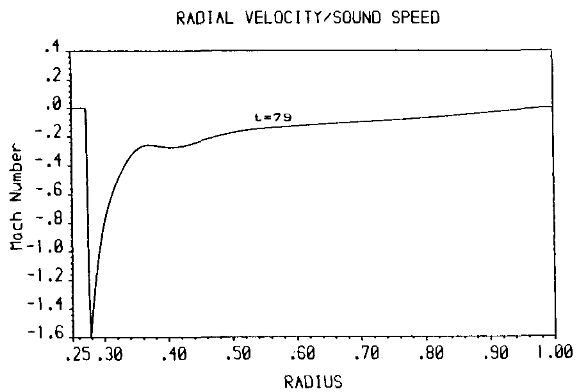

FIGURE 1. Model A. Mach number against radius at t = 79.

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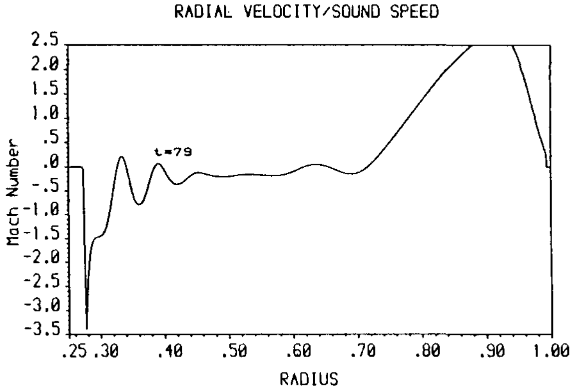

FIGURE 2. Model B. Mach number against radius at t = 79.

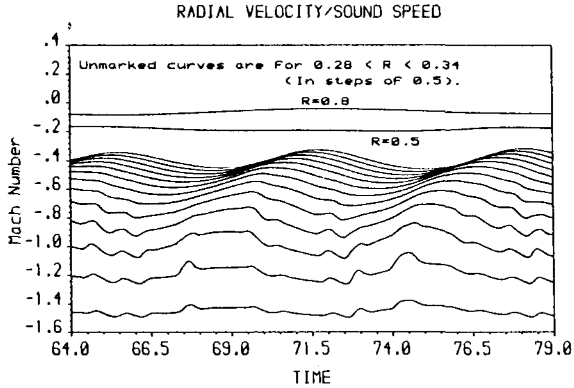

FIGURE 3. Model A. Mach number against time at the radii marked.

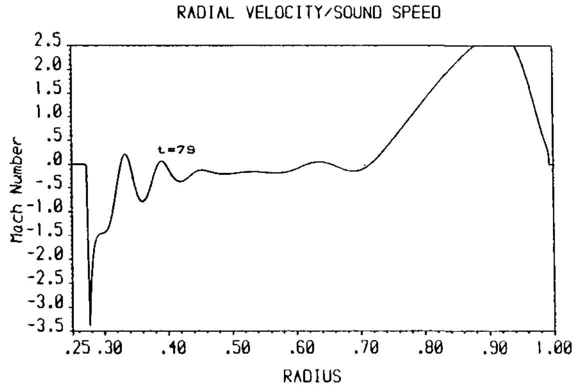

FIGURE 4. Model B. Mach number against time at the radii marked.

For the small disks modelled, waves are trapped at the outer boundary. The value of β affects the instabilities occuring in this region. For model B, with a strong viscous instability, velocities exceed the sound speed, and non-linear dissipative effects occur. Figures 3 and 4 show that two oscillatory regions are separated by an evanescent region between r = 0.5 and r = 0.6.

CONCLUSION.

Oscillations are shown to be present in both the inner and outer disc regions. The outermost oscillations are trapped near the outer boundary with the period depending on its location. Behaviour in the inner region is found to be very susceptible to changes in the viscosity. Small α or large ν_r/ν favours a stable disc. The value of β affects the amplitude and period of the oscillations that occur.

Modulation in the light curves from such discs is expected to be only small. However, these results show that a mechanism exists for luminosity variations with a period longer than the Keplerian period near the inner boundary.

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