RANDOM VARIATIONS IN THE EARTH ROTATION

G. E. O. GIACAGLIA*

The University of Texas at Austin, Austin, Texas

Abstract. Researches on random variations in the Earth rotation, published in the last decade, are analyzed. The basic conclusions are that both mechanical and electromagnetic interactions between mantle and core have definite and measurable influence in the rate of rotation; turbulent motion within the core and non-periodic flows in the mantle must also be taken into account; atmospheric and irregular tidal effects have possible influence. A statistical approach based only upon observations and a simple description of the above phenomena is proposed as an extension of previous works.

The observations of the fluctuation in the rate of the Earth were discussed and explained by Spencer Jones (1939). Classically they are put into three categories: secular, periodic and irregular. In this summary, the irregular variations are dealt with assuming largely the hypotheses put forward by Brouwer (1952). Assuming second differences in the irregular fluctuations to be uncorrelated random variables, he obtained some agreement with the observed fluctuation curve. It might be of interest to note that he also considered the possibility that a change in moment of inertia C of the Earth, of the order of $\delta C/C \simeq 2.2 \times 10^{-9}$ could also explain the cumulative observed change. Much later, Kozai (1970) found

$$2\delta J_2 = \delta C/C = 5.0 \times 10^{-9} \cos(2\pi t + 151°) + \\ + 3.6 \times 10^{-9} \cos(4\pi t + 310°), \quad t \text{ in years}.$$

Later work by van Woerkom (1953) proved that the hypotheses were not quite completely satisfied and differences in the fluctuation curve did show a high correlation for periods of about 10–20 yr. A time of about 25 yr was necessary for the hypothesis of non-correlation to be verified. At the same epoch other theories developed such as the suggestion made by Holmberg (1952) that there was not even a secular decrease in the length of the day (l.o.d.), against all past and future results. He ascribed the acceleration due to atmospheric forces completely to nullify the tidal retardation. Munk and Revelle (1952) observed that fluctuations in l.o.d. do not seem to come from atmospheric, oceanic or mantle dynamics, but from core-mantle electromagnetic coupling. Also, the geomagnetic westward drift indicated that the mantle rotates faster than the core and the observed variations obtained from geomagnetism were consistent with observed variations in the length of day. Further, Vestine (1953) made analyses showing that the core-mantle coupling was magnetic rather than mechanical. A full detailed description of the situation up to 1954 was given by Spencer Jones (1954). He concluded that nothing definitive could be decided. Runcorn (1955) estimated the electrical conductivity of the lower mantle and produced effects in agreement with the observed spectrum of secular variations of the geomagnetic field

* On leave from the University of Sao Paulo, Sao Paulo, Brazil.

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and also explained, to the right order of magnitude, the irregular fluctuations in l.o.d. Along the same lines Elsasser and Takeuchi (1955) showed that the magnetic flux through the top layers of the core and the associated toroidal field fluctuations were adequate to explain the observed variations in the length of day. Later, Melchior (1959) showed some skepticism, well founded, about any simple model which could explain both the Chandler wobble and the variations in l.o.d., because of the many geophysical processes involved of which he gave a very good account. Numerical values of the irregular variations compared with atomic clock standards were given by Stoyko (1959) in three different periods covering from June 1955 to May 1959 and he deduced the effective existence of random changes of the order of -2×10^{-5} s/mo to $+4\times10^{-5}$ s/mo, as compared with the few milli-seconds per decade now generally accepted. In 1959, Danjon fitted a cubic to the fluctuations and found good agreement although the process might be questionable if one assumes the presence of correlated random fluctuations. Nemiro and Pavlov (1959) concluded that, as is still the opinion of many, the major problem was the questionable precision of star catalogues. Markovitz (1959) gave a detailed and accurate description of the fluctuation using Brouwer's explanation of random effects on the mantle. In a very long entirely theoretical work, Rochester (1960) analyzed the time variations of the westward drift of the Earth magnetic field due to motions in the core and could estimate consistent changes of 1 μs/10 yr in l.o.d. He made use of Bullard's (1949) model of a geomagnetic dynamo mechanism in the core, with a toroidal field of hundreds of Gauss in the core interior and high conductivity in the deep layers of mantle. He produced, according to different assumptions, changes in l.o.d. from 0.3 to 1.2 μs/10 yr. Up to 1960, Munk and Macdonald (1960) gave again the best available description of the problem. They tentatively agreed with the theory of random processes in the mantle. Following in Brouwer's steps, van der Waerden (1961) opposed the idea of minimizing the sum of the squares of the irregular variations because of the year-to-year correlation shown by van Woerkom (1953). He introduced frictional forces between core and mantle and also assumed the presence of random torques on the mantle. These are basically the hypotheses assumed in the present work without the assumption that the moments of inertia of both the Earth and the core are constant. Van der Waerden could improve agreement with observations and satisfy more closely the relation obtained by Jeffreys for the coefficients of tidal effects. At the same time, Takeuchi (1961) concluded that in order to obtain information on the core mantle coupling, observations of free periods of torsional oscillations would need to have an accuracy better than 1 min. Munk and Hassan (1961) also concluded that excitation of the Chandler wobble cannot be due to irregular variations in atmospheric inertia or to motions within the core; the electromagnetic coupling seemed too weak to account for this, but was good enough to explain an increase of 0.43 μs/yr in l.o.d. between July 1955 and January 1958. If the core were responsible for both wobble and change in l.o.d., the equatorial torque ought to be several hundreds times larger than the estimated values, with a consequent proportional increase in l.o.d. This seems to indicate that in fact, the two problems may be treated independently, at least for a good first order evaluation. In

1961, Iijima and Okasaki objected to Danjon's findings of sudden changes in the Earth's rotation around July 1955 which was supposed to be correlated with solar eruptions. They again agreed on the inadequacy of the polynomial fittings (used by Danjon) because of the potential presence of fluctuations of purely random character. New discussions and theories on the variability of the moment of inertia of the Earth, in relation to the observed fluctuations and secular changes in l.o.d., were presented by Runcorn (1964) with no particular emphasis in favor of one or another. Along the lines taken here, Kakuta (1965) used a model with impulsive mechanical actions and magnetic restoring torque, and arrived at a relaxation time of about 69 yr, still of the right order of magnitude of the time necessary to show that the differences in the curve of fluctuations in l.o.d. were not correlated (uncorrelate differences in the curve of fluctuations in l.o.d.). Again Rochester (1965) showed that geomagnetic core-mantle interaction cannot excite the Chandler wobble but can explain both the westward drift and the irregular variations of length of day. Sudden changes in the acceleration were observed by Markowitz (1967) and he concluded that UTI–A1 could well be represented by arcs of parabolas (thus supporting the random hypothesis) but such variations were not correlated with changes in motion of the mean pole. The same type of study was performed by Fliegel and Hawkins (1967) but they concluded that UT1–A1 could as well be represented by fourth degree polynomials using Washington PZT observations from 1956 to 1964. They used a fitting by least squares which might be a bad approach as already discussed and got good agreement with Stoyko's classical value of seasonal variations. Their results show nevertheless large variances of 5.8 μs in annual and 3.3 μs in the semiannual terms. At the same time, Runcorn (1967) described the flow of matter by convection in the mantle as related to Continental Drift. Important relations were obtained with coefficients of spherical harmonics in the geopotential and zones of stress in the crust. Such relations could be important in describing the geodynamic processes from satellite observations and possible relations with irregularities in the Earth's rotation. Theories of artificial satellites are at the present much more precise than measurements of lunar and solar longitudes and laser tracking should make it possible to obtain variation in harmonics of higher degree in the geopotential (Kozai, 1970). Again in 1968, Vestine and Kahle showed good correlation between fluctuations in the westward drift (associated with motions within the core) and the irregular variations of l.o.d. Both show random character. The law of conservation of angular momentum of the Earth plus the atmosphere was shown to be a possibly bad approximation by Sidorenkov (1968) owing to large solar influences. At the same time, Rochester (1968) indicated that the electromagnetic restoring torque on the mantle was proportional to the angular velocity of the mantle relative to the core. The coupling affects diurnal nutation very little but gives changes in l.o.d. of the right order of magnitude. At this point one may conclude that variations in l.o.d. and in the westward drift of the eccentric magnetic field are basically correlated and come almost certainly from core-mantle interaction whether mechanical or electromagnetic or both; this is the most probable assumption. Good use of artificial satellite observations was made by Newton (1968) who derived variations in the angular velocity of the Earth in good agreement with the adopted values. In this respect, Melchior (1968) concluded that rotation and deformation of the Earth cannot be treated independently and the over-all problem from the point of view of artificial satellite geodesy has recently been discussed by Kozai (1965). The idea of mechanical friction between mantle and core was again introduced by Aoki (1969) in giving a possible mechanism for the secular change of obliquity. He obtains a westward drift of 0°.29/yr of core relative to mantle, in good agreement with observations. The secular decrease in the length of day comes out wrong but the difference could be assigned to anisotropy of friction with respect to the axis of rotation and the neglect of the electromagnetic core-mantle couple. Kaula (1969) indicated that irregularities in the Moon's longitude could be due to asymmetric tidal effects, so that again the importance of the use of artificial satellites to track the Earth' motion becomes evident. In spite of all previous work, Hide (1969) showed that an electromagnetic torque alone could account for the observed variations of rotation. On the contrary Sidorenkov (1969) showed that atmospheric circulation can produce a substantial contribution to seasonal and irregular variations. Finally, in the most recent account of the problem, Markowitz (1971) showed that great uncertainty is still present in all results, especially as far as random fluctuations are concerned.

The conclusions one can draw at the present time are that:

(1) Possible mechanical and electromagnetic interactions exist at the core-mantle boundary layer. Frictional or magnetic coupling cannot be excluded.

(2) Turbulent motion within the core may have direct influence on the irregular variations of l.o.d. Non-periodic flows in the mantle with irregular or organized changes in the moment of inertia may also be responsible.

(3) Periodic variation in the inertia of the Earth should be included in a consistent theory of rotation.

(4) Atmospheric currents and irregular tidal effects cannot yet be ruled out as part of the origin in irregular variations.

The quantities of interest here, which are used in a model similar to that of Brouwer, van Woerkom and van der Waerden are defined as follows: K: constant tidal retarding couple; C_m: moment of inertia of mantle; Ω_m: angular velocity of mantle; L: angular moment of the Earth; C: moment of inertia of the Earth; Ω: defined by $L = C\Omega$; K_1: frictional and or electromagnetic core-mantle couple $= f(C\Omega - C_m\,\Omega_m)$; $K_2 = \psi(t)$: random couple acting on mantle due to several causes (core, atmosphere, tides, etc.).

The following properties are assigned to $\psi(t)$: $\langle\psi\rangle = 0$, $\langle\psi^2\rangle = \sigma^2$, $\psi(t_1)$ and $\psi(t_2)$ correlated if $t_2 - t_1 \leqslant \Delta t$ ($\sim$20–40 yr). The equations can be simply written as:

$$(\mathrm{d}/\mathrm{d}t)\,(C\Omega) = -\,K$$

$$(\mathrm{d}/\mathrm{d}t)\,(C_m\Omega_m) = K_1 + K_2 - K$$

and defining $\phi(t) = C\Omega - C_m\,\Omega_m$, it follows that irregular variations in $\phi(t)$ are given by

$$\phi(t) = -\int_0^\infty e^{-f\tau}\psi(t-\tau)\,\mathrm{d}\tau$$

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including the all past history. Of course, for τ large enough, the damping factor $e^{-f\tau}$ will have no influence. The corrections in time due to irregular variations introduced by $\psi(t)$ are given, at any time T, by

$$\mathrm{d}T = \frac{1}{\langle C_m \rangle} \int_0^T \phi(t)\,\mathrm{d}t,$$

where $T=0$ is any arbitrary epoch. The corresponding variations in longitude of Moon and Sun are evidently proportional to δT, $F=k\int_0^T \phi(t)\mathrm{d}t$. We define the quantities $\theta(t)=k\phi(t)$, $Q(t)=k\psi(t)$ so that $\langle Q \rangle=0$. The equations for F and θ are

$$\dot{F} = \theta(t)$$
$$\dot{\theta} = -f\theta + Q(t)$$

and if $i=0,1,2,\dots$ correspond to $0,\Delta t, 2\Delta t,\dots$, we assume $\langle Q_i \rangle=0$, $\langle Q_i^2 \rangle=\sigma_q^2$ for all i and assign to Q_i a Gaussian distribution. It follows that

$$F_n = \sum_{i=0}^{n-1} \frac{1}{f}\left[1 - e^{-f(n-i-1)\Delta t}\right] Q_i$$
$$\theta_n = \sum_{i=0}^{n-1} e^{-f(n-i-1)\Delta t}\, Q_i$$

so that $\langle F_n \rangle=\langle \theta_n \rangle=0$ and $\langle F_n^2 \rangle=\sigma_F^2=(\sigma_q/f)^2 T$ for large T. Also $\langle \theta^2 \rangle=\sigma^2= =\Delta t\sigma_q^2/2f$ for large T. One can write

$$\theta(\tau)=\sum_j p_j,$$
$$p_j=e^{-jf\Delta t}\, q_j\, \Delta t$$

where q_j are defined, according to the relaxation time Δt, as

$$\Delta t q_j = \int_{j\Delta t}^{(j+1)\Delta t} \psi(t-\tau)\,\mathrm{d}\tau.$$

From the classical theory of Markov processes it follows that the probability function of θ is

$$p(\theta;T) = \frac{1}{\sqrt{2\pi\varrho_\theta^2}} \exp\left[-\frac{\theta^2}{2\varrho_\theta^2}\right]$$

where

$$\varrho_\theta^2 = \frac{1}{2f}\sigma^2\Delta t(1-e^{-2fT}).$$

The same procedure applies to the fluctuation curve F and one finds

$$p(F;T) = \frac{1}{\sqrt{2\pi\varrho_F^2}} \exp\left[-\frac{F^2}{2\varrho_F^2}\right]$$

where

$$\varrho_F^2 = \Delta t \frac{\sigma^2}{f^2}\left[T - \frac{2}{f}(1-e^{-fT}) + \frac{1}{2f}(1-e^{-2fT})\right].$$

It is seen that for large values of T both distributions are Gaussian. One can also obtain the composite probability function $p(F;\theta;T)$ which could help in the direction of correlating fluctuations in Moon's longitude and Earth' rotation due to irregularities in the latter. Applications of this sort and extensive theoretical and numerical developments will be published elsewhere.

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DISCUSSION

S. Débarbat: What do you think about the differences between the paper of Markowitz (1967) and the paper of Fliegel and Hawkins (1967) you mentioned at page 167.

G. E. O. Giacaglia: The paper by Fliegel and Hawkins (1961) shows that UT1-A.1 can be fitted well by parabolic arcs or fourth degree polynomials (Washington PZT observations, 1956–1964). They use a fitting by least squares which might be bad if variations are random correlated variables. Their seasonal variations agree with Stoyko's classical values but show too large a variance in the annual term (5.8 μs) and in the semiannual term (3.3 μs). The work by Markowitz (1967) shows the presence of sudden changes in acceleration and proves that UT1-A.1 can well be represented by arcs of parabolas. The hypothesis of random variations is well justified therefore, but they seem to indicate no correlation whatsoever with changes in the mean pole. The papers are basically equivalent and the apparent inconsistency of the curve fitting is probably due to what has become clear in this Symposium, i.e., personal taste, preference and traditions are very important factors in the actual conclusions obtained from observations. Again we should also not forget that random variations make a simple least square fitting meaningless unless we allow for a relaxation time.

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