7

The Higgs mechanism in the Glashow–Salam–Weinberg model

7.1 Masses for gauge bosons

In order to give masses to the gauge bosons and the fermions, we follow the method described in Section 5.3. We introduce a complex scalar doublet

$$\phi = \begin{bmatrix} \phi_+ \\ \phi_0 \end{bmatrix}. \tag{7.1}$$

From the relation $Q = T_3 + Y/2$, it follows that $Y = 1$ for ϕ. Each of the fields has a real part and an imaginary part, so there are four independent scalar fields. The Lagrange function for the scalar sector is given by

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^+ \phi), \tag{7.2}$$

with the covariant derivative defined by

$$D_\mu = \partial_\mu + \mathrm{i}g' B_\mu + \mathrm{i}g \frac{\tau^i}{2} W^i_\mu \tag{7.3}$$

and the potential by

$$V(\phi^\dagger \phi) = -\mu^2 \phi^+ \phi + \lambda(\phi^\dagger \phi)^2. \tag{7.4}$$

We have been gradually enlarging the Lagrangian and so far it consists of three terms:

$$\mathcal{L} = \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{B}} + \mathcal{L}_\phi. \tag{7.5}$$

It contains fermions, vector bosons, and scalar fields and is invariant under gauge transformations of the group SU(2) $\times$ U(1).

https://doi.org/10.1017/9781009402378.008 Published online by Cambridge University Press

Classically, the potential $V(\phi)$ has a locus of minima at

$$\frac{\partial V}{\partial \phi_+^*} = -\mu^2 \phi_+ + 2\lambda\big(|\phi_+|^2 + |\phi_0|^2\big)\phi_+ = 0, \tag{7.6}$$

$$\frac{\partial V}{\partial \phi_0} = -\mu^2 \phi_0 + 2\lambda\big(|\phi_+|^2 + |\phi_0|^2\big)\phi_0 = 0; \tag{7.7}$$

that is, at

$$|\phi_+|^2 + |\phi_0|^2 = \frac{\mu^2}{2\lambda}. \tag{7.8}$$

We can choose the ground state (vacuum state) at the minimum of the potential. Since we wish to conserve charge, the field must carry vacuum quantum numbers

$$|\phi_0| = \frac{\mu}{\sqrt{2\lambda}} \quad \text{and} \quad \phi_+ = 0. \tag{7.9}$$

In the quantum theory the symmetry is broken by introducing

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad \text{with} \quad v = \frac{\mu}{\sqrt{\lambda}}.$$

In other words, one of the neutral scalar fields acquires a vacuum expectation value at the minimum of the potential.

The next step is to try to rewrite the Lagrangian in terms of fields displaced relative to the minimum of the potential and arrive at a physical interpretation. The selection of a vacuum expectation value chooses a direction in the potential, thus breaking the symmetry. We define four scalar fields, ξ_1, ξ_2, ξ_3, and η, by

$$\phi = U^{-1}(\vec{\xi})\begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}, \tag{7.10}$$

where $U^{-1}(\vec{\xi})$ is the unitary transformation

$$U^{-1}(\vec{\xi}) = \exp\left(-\frac{\mathrm{i}\vec{\xi}\cdot\vec{\tau}}{2v}\right). \tag{7.11}$$

This is very similar to the discussion concerning Eqs. (5.38) and (5.39). Again, we define new fields through a gauge transformation

$$\phi \to \phi' = U(\vec{\xi})\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}, \tag{7.12}$$

$$\psi_{\mathrm{L}} \to \psi'_{\mathrm{L}} = U(\vec{\xi})\psi_{\mathrm{L}} \quad \text{and} \quad \psi'_{\mathrm{R}} = \psi_{\mathrm{R}}, \tag{7.13}$$

$$\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu \to \frac{1}{2}\vec{\tau}\cdot\vec{W}'_\mu = \frac{1}{2}U(\vec{\xi})\vec{\tau}\cdot\vec{W}_\mu U^{-1}(\vec{\xi}) + \frac{\mathrm{i}}{g}\Big[\partial_\mu U^{-1}(\vec{\xi})\Big]U(\vec{\xi}). \tag{7.14}$$

This transformation has the form described in Chapter 5, with a new feature: the fields themselves occur in the transformation. Upon substitution into the Lagrangian, the terms $\mathcal{L}_{\mathrm{F}}$ and $\mathcal{L}_{\mathrm{B}}$ retain the same form when expressed in terms of the new fields, but $\mathcal{L}_\phi$ is modified. In fact, as we show next, several of the scalar fields disappear and the Lagrangian has a new physical interpretation.

Consider the $\mathcal{L}_\phi$ term and set

$$\phi' = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix} = \frac{v+\eta}{\sqrt{2}}\chi \quad \text{with} \quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{7.15}$$

On substituting for the ϕ field in terms of the new field, it appears as if we are making a gauge transformation. The covariant derivatives become

$$D_\mu \phi' = \left[\partial_\mu \eta + \frac{\mathrm{i}}{2}(v+\eta)\big(g' B_\mu + g\tau^i W_\mu^i\big)\right]\frac{\chi}{\sqrt{2}}, \tag{7.16}$$

$$\begin{aligned}(D_\mu\phi')^\dagger(D_\mu\phi') + \text{h.c.} &= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \\ &\quad + \frac{1}{8}(v+\eta)^2\chi^+\big\{\big(g'B_\mu + g\tau^i W_\mu^i\big)(g'B^\mu + g\tau^i W^{i,\mu})\big\}\chi.\end{aligned} \tag{7.17}$$

The cross-term is purely imaginary and does not appear in the product. We study in detail the structure of the second term,

$$(g'B_\mu + g\vec{\tau}\cdot\vec{W}_\mu)(g'B^\mu + g\vec{\tau}\cdot\vec{W}^\mu) = \left[g'^2 B_\mu B^\mu + g^2\vec{W}_\mu\vec{W}^\mu + 2gg'B_\mu\vec{\tau}\cdot\vec{W}^\mu\right], \tag{7.18}$$

and between the χ states

$$\begin{aligned}\chi^+[\ldots]\chi &= g'^2 B_\mu B^\mu + g^2 W_\mu^i W^{i,\mu} - 2gg'B_\mu W^{3,\mu} \\ &= \big(g'B_\mu - gW_\mu^3\big)^2 + 2g^2 W_\mu^+ W^{-\mu} \\ &= (g^2 + g'^2)Z_\mu Z^\mu + 2g^2 W_\mu^+ W^{-\mu}.\end{aligned} \tag{7.19}$$

The evaluation of the term linear in $\vec{\tau}$ is most easily done using $\vec{\tau}\cdot\vec{W}_\mu = \sqrt{2}(\tau^+W_\mu^- + \tau^-W_\mu^+) + \tau^3 W_\mu^3$ and properties of $\tau^\pm\chi$. New fields were also introduced:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\big(W_\mu^1 \pm \mathrm{i}W_\mu^2\big), \quad Z_\mu = \frac{-gW_\mu^3 + g'B_\mu}{\sqrt{g^2+g'^2}},$$

and

$$A_\mu = \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2+g'^2}}. \tag{7.20}$$

https://doi.org/10.1017/9781009402378.008 Published online by Cambridge University Press

We note that the fields $W^{\pm}$ and Z are now massive with

$$M_W = \frac{1}{2} g v \quad \text{and} \quad M_Z = \frac{1}{2}\left(g^2 + g'^2\right)^{1/2} v, \tag{7.21}$$

but the field A_μ remains massless. The physical correspondence for the fields is evident. A_μ represents the photon and the other three the intermediate gauge bosons of the weak interaction. An interesting property is the disappearance from the Lagrangian of the ξ_1, ξ_2, and ξ_3 fields. These three degrees of freedom were transformed into longitudinal states of the massive vector mesons. This form of the theory, with its clear physical interpretation, is referred to as the unitary gauge.

To sum up, we have constructed a theory with vector, scalar, and spin-$\frac{1}{2}$ particles based on the symmetry group SU(2) $\times$ U(1). The symmetry was broken in the Higgs mode by introducing a non-zero vacuum expectation value for the neutral field ϕ_0. Then it was shown that a judicious choice of the gauge eliminates three scalar fields. In this gauge the physical interpretation is clear, with the states $W^{\pm}$ and Z^0 being massive. They also have three longitudinal degrees of freedom.

In the quantum theory, the breaking of the symmetry by Eq. (7.9) implies that the vacuum is not the empty state but a complicated superposition of states, as demonstrated for the simple Hamiltonian in Problem 3 of Chapter 5. Condition (7.9) does not break the symmetry completely, because the charge generator annihilates the vacuum and the law for charge conservation is preserved. The other three generators are broken and to each of them there corresponds a massive gauge boson. Their masses satisfy the relation

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2}. \tag{7.22}$$

They also couple to fermions through charged and neutral currents, which satisfy the SU(2) $\times$ U(1) algebra. These and other couplings will be studied in the following chapters.

Finally, the simple mass relations (7.21) depend on the fact that the field ϕ was a weak isodoublet. It survives even if ϕ is replaced by a finite number of isodoublet fields. It fails, however, when Higgses belonging to other representations are introduced. Consider, for instance, a theory that contains, in addition to the doublet, a triplet of Higgs fields,

$$\vec{\Sigma} = \begin{bmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{bmatrix}, \tag{7.23}$$

with $\langle \Sigma^0 \rangle = \sigma \neq 0$. Then, by repeating the steps (7.16)–(7.21) and using the SU(2) matrices for the three-dimensional representation, the reader can verify the new mass relations

$$M_W = \frac{1}{2}g(v^2 + \sigma^2)^{1/2} \quad \text{and} \quad M_Z = \frac{1}{2}\left(g^2 + g'^2\right)^{1/2} v. \tag{7.24}$$

7.2 Masses for leptons

The standard model based on the group SU(2) × U(1) allows us to make some of the simplest choices. It is the simplest group which contains charged, neutral, and electromagnetic currents. This is at the expense of introducing two coupling constants, g and g', which are related through the Higgs mechanism to other parameters of the theory (masses of gauge bosons, structure of neutral currents, ...).

It has the simplest multiplet assignment for the fermion multiplets which is consistent with parity violation. The left-handed particles are SU(2) doublets and the right-handed components singlets,

$$\psi_e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e_R^-, \tag{7.25}$$

with the same pattern repeated for the other two families:

$$\psi_\mu = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \mu_R^- \quad \text{and} \quad \psi_\tau = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad \tau_R^-. \tag{7.26}$$

We note that there are no right-handed neutrinos because it was thought that they are massless. This attitude changed with the discovery of neutrino oscillations, which require small and finite masses. The subject of neutrino masses is covered in Chapter 13.

Masses for leptons are generated through Yukawa couplings. A Yukawa interaction invariant under SU(2) × U(1) is given by

$$\mathcal{L}_y = g_e \bar{\psi}_e \phi e_R + \text{h.c.} \tag{7.27}$$

As mentioned earlier, the symmetry is broken by giving a vacuum expectation value to ϕ^0:

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{breaking}} \begin{pmatrix} 0 \\ (1/\sqrt{2})(v + \eta) \end{pmatrix}, \tag{7.28}$$

which gives the mass $m_e = (1/\sqrt{2})g_e v$. Similarly, masses are generated for the mu and tau leptons. The lepton masses remain arbitrary parameters without any relation among them.

The theory has the simplest symmetry-breaking mechanism. The Higgs particles are in the fundamental representation of SU(2) containing just enough fields to make $W^\pm$ and Z^0 massive and leave one neutral Higgs as a physical particle. This pattern of symmetry-breaking provides a consistent way to control the higher-order terms by absorbing infinities into the masses and couplings of the theory. ’t Hooft (1971) derived the correct Feynman rules in a class of gauges and constructed gauges for which the theory is manifestly renormalizable. Detailed studies of renormalization and unitarity followed (Lee and Zinn-Justin, 1972; ’t Hooft and Veltman, 1972). This remarkable success opened the road for many investigations and predictions that have been confirmed by experiments.

Problems for Chapter 7

1. Use the matrices

$$\lambda_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} \quad \text{and}$$

$$\lambda_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

for the $I = 1$ representation of SU(2) and the Pauli matrices for the $I = 1/2$ representation in order to prove Eq. (7.24).

2. Generally, it is possible to construct SU(2) × U(1) theories with several multiplets of scalar fields. We denote them by ϕ_i and they carry weak isospin I_i and have a neutral component I_{3i}. If each neutral component develops a vacuum expectation value $v_i/\sqrt{2}$, show that the W and Z masses satisfy

$$M_W^2 = \frac{1}{2} g^2 \sum_i \left[I_i(I_i + 1) - I_{3i}^2 \right] v_i^2 ,$$

$$M_Z^2 = \sec^2\theta_W \, g^2 \sum_i I_{3i}^2 v_i^2 .$$

Find the first values (I, I_3) for which the relation

$$M_W = M_Z \cos\theta_W$$

is maintained.

References

Lee, B. W., and Zinn-Justin, J. (1972), *Phys. Rev.* **D5**, 3121
’t Hooft, G. (1971), *Nucl. Phys.* **B33**, 173; **B35**, 167
’t Hooft, G., and Veltman, M. (1972), *Nucl. Phys.* **B50**, 318

Select bibliography

The following references cover the Higgs mechanism, the electroweak theory, and renormalization in the order in which they appeared.

Higgs, P. W. (1966), *Phys. Rev*. **145**, 1156
Kibble, T. W. B. (1967), *Phys. Rev*. **155**, 167
Weinberg, S. (1967), *Phys. Rev. Lett*. **19**, 1264
Salam, A., and Strathdee, J. (1972), *Nuovo Cimento* **11A**, 397
’t Hooft, G. (1971), *Nucl. Phys*. **B33**, 173; **B35**, 167
Lee, B. W. (1972), *Phys. Rev*. **D5**, 3121
Lee, B. W., and Zinn-Justin, J. (1972), *Phys. Rev*. **D5**, 3121
’t Hooft, G., and Veltman, M. (1972), *Nucl. Phys*. **B50**, 318