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Some ordered topological groups

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This thesis is concerned with two topologies which are naturally connected with the order and algebraic structures of certain abelian groups. These topologies are the open interval topology defined in §1.2.2 and what we call the weak topology defined in §2.2.1.

Fuchs showed that in non-fully ordered groups the open-interval topology is discrete unless the group is an anti-lattice. To avoid dealing with the discrete case we restrict ourselves (when considering the open-interval topology) to what we call tight Riesz (2, 1) groups (§1.2.3), these having a modified form of a property introduced by Riesz [5].

Numerous results involving the open-interval topology, most of them valid in tight Riesz (2, 1) groups, have already been obtained by Fuchs [1], [2], Loy and Miller [3], Miller [4] under slightly different conditions.

The weak topology can be defined for any partially ordered abelian group, but may very well not be Hausdorff or even non-trivial unless certain quite stringent requirements are satisfied. However, almost all of the real or complex valued function spaces which are dealt with in analysis meet these requirements or may be embedded in groups that meet these requirements.

Chapter 1 introduces the basic terms and concepts of partially ordered abelian groups and establishes various fundamental results concerning the open interval topology.

In Chapter 2 the weak topology is introduced and developed to the extent needed for its applications in later chapters. A Hahn-Banach type

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theorem, important for the weak topology, is also proved, and yields a known representation theorem.

Chapter 3 explores some of the relationships between Riesz groups and lattice groups. A considerable amount of space is devoted to a completion of a Riesz group in the weak topology.

In Chapter 4 we find out more about the open interval topology for tight Riesz (2, 1) groups and see how it is related to the weak topology.

Chapter 5 contains a number of results on lattice ordered topological groups and closely related tight Riesz groups.

Using the weak topology it is possible to develop a measure theory on a certain (unfortunately small) class of ordered vector spaces. Chapter 6 gives an account of this measure which reduces to Lebesgue measure when the underlying vector space is finite dimensional.

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