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Filippo Calderoni, ***A Descriptive View of the Bi-embeddability Relation***, Università degli Studi di Torino, Italy, 2018. Supervised by Luca Motto Ros. MSC: 03E15. Keywords: bi-embeddability, analytic equivalence relations, Borel reducibility, torsion-free abelian groups.

Abstract

In this thesis we use methods from the theory of Borel reducibility to analyze the bi-embeddability relation.

We continue the work of Camerlo, Marcone, and Motto Ros investigating the notion of invariant universality, which is a strengthening of the notion of completeness for analytic equivalence relations. We prove invariant universality for the following relations: bi-embeddability between countable groups, topological bi-embeddability between Polish groups, bi-embeddability between countable quandles, and bi-embeddability between countable fields of a fixed characteristic different from 2. Our work strengthens some results previously obtained by Jay Williams; Ferenczi, Louveau, and Rosendal; and Fried and Kollár.

Then, we analyze the bi-embeddability relation in the case of countable torsion-free abelian groups, and countable torsion abelian groups. We obtain that the bi-embeddability relation on torsion-free abelian groups is strictly more complicated than the bi-embeddability relation on torsion abelian groups. In fact, we prove that the former is a complete analytic equivalence relation, while the latter is incomparable up to Borel reducibility with the isomorphism relation on torsion groups. Furthermore, we argue that the bi-embeddability relation between countable torsion abelian groups is strictly below isomorphism up to $\mathbf{\Delta}^1_2$-reducibility.

In the end, we analyze the bi-embeddability relation on torsion-free abelian groups in the framework of generalized descriptive set theory. We use a categorical construction to prove that bi-embeddability on κ-sized graphs Borel reduces to bi-embeddability on torsion-free abelian groups of size κ, for every uncountable cardinal κ which satisfies $\kappa^{<\kappa} = \kappa$. It follows that the bi-embeddability relation on torsion-free abelian groups of size κ is as complicated as possible among analytic equivalence relations.

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Raphaël Carroy, ***Functions of the first Baire class***, Université Paris 7 - Denis Diderot & Université de Lausanne, France, 2013. Supervised by Olivier Finkel & Jacques Duparc. MSC: Primary 26A21, 54C05. Keywords: Continuous functions, infinite games, well-quasi-orders, continuous reducibility.

Abstract

We aim at starting an analysis of definable functions similar to the Wadge theory for definable sets, focusing more specifically on Baire class 1 functions between 0-dimensional Polish spaces. To parallel Wadge's analysis, we break this study in two parts. The first concerns subclasses of the first Baire class characterisable by infinite games, while the second looks at the quasi-order of continuous reducibility on continuous functions.

Here X, Y, X', and Y' are variables for Polish 0-dimensional (P0D for short) spaces, considered as closed subspaces of the Baire space of infinite sequences of natural numbers.

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Playing in the first Baire class. [1] We consider various infinite games with a function $f : X \to Y$ as parameter. The simplest one sees Player *I* choosing a natural number x_i and Player *II* a natural y_i at the i-th round, thus building two infinite sequences x and y. Player *II* then wins the game if and only if $f(x) = y$. In this game, Player *II* has a winning strategy if and only if f is Lipschitz; we say that the game *characterises* Lipschitz functions.

New rules giving more and more power to Player *II* will then characterise larger and larger classes of functions. For instance, allowing Player *II* to skip her turn as often as she wants characterises continuous functions. These first two games were defined by Wadge in his Ph.D. thesis. We consider three others.

In the *backtrack* game, defined by van Wesep in his Ph.D. thesis, Player *II* can erase what she did at the previous round, but only finitely many times during a run. It characterises functions that are σ-continuous with closed witnesses.

Duparc in his Ph.D. thesis defined the *eraser* game by allowing Player *II* to erase infinitely often. This one characterises Baire class 1 functions.

The third one is a refinement of the first one, defined by Motto Ros in his Ph.D. thesis. It is called the *α-bounded backtrack* game, because the erasing ability of *II* is bounded by some countable ordinal α.

We prove that all three games are determined. Observe that instead of considering only Borel functions and use Martin's result on Borel determinacy, we show a stronger determinacy result.

Theorem 1.1. *For all functions $f : X \to Y$, the eraser game, the backtrack game and the α-bounded backtrack game with parameter f are determined.*

We define the finer notion of an *aggressive* strategy for Player *I* to get a stronger theorem; namely if *II* has no winning strategy in the backtrack game with parameter $f : X \to Y$, then *I* has an aggressive winning strategy.

As a corollary, we give a new—purely game-theoretical—proof of the Baire Lemma on pointwise convergence.

A quasi-order for continuous functions. [2] Given two functions $f : X \to Y$ and $g : X' \to Y'$, we write $f \leq g$ and say that g *continuously reduces* f if there are two continuous functions $\sigma : X \to X'$ and $\tau : \mathrm{Im}(g \circ \sigma) \to Y$ satisfying $f = \tau \circ g \circ \sigma$. We write $f \equiv g$ when both $f \leq g$ and $g \leq f$ hold.

Computable versions of this quasi-order, introduced by Weihrauch, have become central in computable analysis.

We first prove that any two continuous functions $f : X \to Y$ and $g : X' \to Y'$ with uncountable ranges are continuously bireducible. We then study more specifically functions with countable image; we let both C_∞ denote the class of all continuous functions between P0D spaces and C denote the subclass of C_∞ of functions with countable range.

We define the *Cantor–Bendixson rank* $\mathrm{CB}(f)$ *of a function* f in C. This rank generalises the usual Cantor–Bendixson rank on closed sets, in the sense that $\mathrm{CB}(\mathrm{Id}_F) = \mathrm{CB}(F)$ for all closed F. This rank stratifies C in classes C_α of all functions in C of Cantor–Bendixson rank α, for α countable.

We isolate two infinitary operations on P0D spaces, called the *gluing* and the *pointed gluing*. We give universal properties for these, and prove that they generate, from the empty set, all countable P0D spaces up to homeomorphism.

We then define the gluing and pointed gluing on sequences of functions so that the operations on sets and those on functions commute with the identity functor.

Letting $\mathsf{C}^\star$ be the subclass of C of functions with compact domain, we prove

Theorem 1.2. *The relation $\leq$ is a well-order on $\mathsf{C}^\star/\equiv$ of order type ω_1.*

Using $\mathsf{C}^\star$ as a leverage point in C, we describe the general structure of $(\mathsf{C}, \leq)$.

[1]This was published in ***Mathematical Logic Quarterly***, vol. 60 (2014), no. 1–2, pp. 118–132.
[2]This was published in the ***Journal of Symbolic Logic***, vol. 78 (2013), no. 2, pp. 633–648.

THEOREM 1.3. *Given f, g in C, $\lambda < \omega_1$ a limit ordinal and n a natural, we have*

1. If $\mathrm{CB}(f) = \mathrm{CB}(g) = \lambda$ then $f \equiv g$,
2. If $\mathrm{CB}(f) = \lambda + n$ and $\mathrm{CB}(g) = \lambda + 2n + 1$, then $f \leq g$.

In particular, if $(\mathsf{C}_\alpha, \leq)$ is a well-quasi-order (wqo) for all $\alpha \in \omega_1$ then $(\mathsf{C}_\infty, \leq)$ is a wqo. The question of whether C_∞ is a wqo or not remains open.

Applying Theorem 1.3 to the subclass of identity functions, we obtain an alternative proof that topological embeddability on P0D spaces is a wqo. This result is also obtained as a corollary of Laver's celebrated result on labelled trees.

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EVANDRO LUÍS GOMES, ***Sobre a história da paraconsistência e a obra de da Costa: a instauração da Lógica Paraconsistente [On the history of paraconsistency and da Costa's work: the establishment of paraconsistent logic]. (December, 2013). 535p + appendixes***. Philosophy Ph.D., Institute of Philosophy and Human Sciences and Centre for Logic, Epistemology and the History of Sciences, The University of Campinas, Campinas, São Paulo, Brazil, 2013. Supervised by Itala M. Loffredo D'Ottaviano. MSC: 01A85, 01–02, 03A05, 03B53. Keywords: logic, history and philosophy of logic, nonclassical logics, paraconsistent logic and history of philosophy.

Abstract

The establishment of paraconsistent logic, the quest for and the description of its historical background, by means of an analysis of their philosophical foundations are here presented by the way of contemporary historiography of logic. Nowadays the mature development of paraconsistent logic helps to write the history of its instauration and, in a quite archeological way, to rebuild the prehistory of this approach as well as its theoretical schemata. We especially study the effects of contradiction on rational contexts and the logical and theoretical tools for its suppression, handling or absorption. Broad and strict paraconsistency are also confirmed by the level of refusal of the *ex falso*. Such a logical law affirms that every formula follows from a contradiction and is related to the formal trivialization of the theories whenever their underlying theories are, e.g., classical or intuitionist. In paraconsistent theories, however, the *ex falso* does not hold in general. Such theories can be inconsistent without being trivial after all. We consider texts, contexts, and historical landmarks from analytical, descriptive and historical point of view in order to understand its stages of development. In Part I, we tell the history of the antecedents of paraconsistent approach. In Chapter 1, *Paraconsistent logical elements in ancient authors*, we identify and gather meaningful texts to the prehistory of paraconsistency in Western thought. From those elements, we outline an interpretation that matches paraconsistent elements with the theoretical achievements of Heraclitus of Ephesus, Aristotle, and by the Stoics. In Chapter 2, *Paraconsistent logical elements in medieval authors*, we study the growing process concerning of handling with contradiction in the rational thought, most of the latter being inspired by the treatment of the issue in the former tradition. There are paraconsistent grounds, for instance, in the work of Peter Abelard, Peter of Spain, and William of Ockham. Such authors, by their own efforts, could elevate such elements into sophisticated arguments found in their discussion whether the *ex falso* is or not admissible. In Part II, we introduce the history of the strictly paraconsistent positions properly said together with its prelude. In Chapter 3, *The dawn of contemporary paraconsistency*, we study statements concerning inconsistency in some studies of Leibniz, Hume, Kant, and Hegel, as well as we analyze other authors' contribution to that debate such as Jan Łukasiewicz and Nicolaj Vasiliev. Those thinkers were forerunners in the defense of a theoretical possibility of nonclassical logics, in the sense that some of their ideas has lead to paraconsistent approaches thereafter. In Chapter 4, *The stage of paraconsistency*, we focus on rebuilding the historical and theoretical context that occurs before the

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