

## Part 5. Extra-Galactic Astronomy

# WEAK GRAVITATIONAL LENSING—THE NEED FOR SURVEYS

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## 1. What is Weak Lensing, and What is it Good For?

Light rays from distant sources are deflected if they pass near an intervening matter inhomogeneity. This gravitational lens effect is responsible for the well-established lens systems like multiple-imaged QSOs, (radio) ‘Einstein’ rings, the giant luminous arcs in clusters of galaxies, and the flux variations of stars in the LMC and the Galactic bulge seen in the searches for compact objects in our Galaxy. These types of lensing events are nowadays called ‘strong lensing,’ to distinguish it from the effects discussed here: light bundles are not only deflected as a whole, but distorted by the tidal gravitational field of the deflector. This image distortion can be quite weak and can then not be detected in individual images. However, since we are lucky to live in a Universe where the sky is full of faint distant galaxies, this distortion effect can be discovered statistically. This immediately implies that weak lensing requires excellent and deep images so that image shapes (and sizes) can be accurately measured and the number density be as high as possible to reduce statistical uncertainties. Weak gravitational lensing can be defined as using the faint galaxy population to measure the mass and/or mass distribution of individual intervening cosmic structures, or the statistical properties of their mass distribution, or to detect them in the first place, independent of the physical state or nature of the matter, or the luminosity of these mass concentrations. In addition, weak lensing can be used to infer the redshift distribution of the faintest galaxies. After introducing the necessary concepts, I will list the main applications of weak lensing and discuss some of them in slightly more detail, stressing the need for very deep and wide-field images of the sky taken with instruments of excellent image quality.

## 2. The Mapping of Small Sources

A gravitational lens provides a map from the observer's sky to the undistorted sky. The properties of this map are determined by the surface mass density of the deflector (see, *e.g.*, Schneider, Ehlers & Falco 1992). Provided the angular size of a source is very much smaller than the typical angular scale of the deflector, the lens mapping can be linearized locally. This linearized map is then characterized by an isotropic focussing term and an anisotropic distortion, due to the tidal gravitational field. This term is also called shear; it causes a circular source to be mapped into an elliptical image.

From the basic assumption that the intrinsic orientations of an ensemble of galaxies taken from a large cosmic volume are randomly distributed one can statistically infer the local distortion from an ensemble of galaxy images.

The mapping of sources as described above also affects the flux of a galaxy image. This magnification effect can be observed either by its effect on the local number counts of galaxies, an effect called magnification bias (Broadhurst *et al.* 1995), or by effect on the mean image size at fixed surface brightness (Bartelmann & Narayan 1995).

The 'traditional' method to determine the shear and magnification uses the properties of (isolated) galaxy images; for each one has to determine a center and the tensor of second brightness moments, and the rest of the CCD is unused. Alternatively, one can use the two-point auto-correlation function (ACF) of the light distribution on the CCD,  $\xi(\vec{\theta})$ . This is related to the unlensed ACF  $\xi^s(\vec{\theta})$  by  $\xi(\vec{\theta}) = \xi^s(A\vec{\theta})$ . Since the unlensed ACF can be assumed to be isotropic, the anisotropy of the observed ACF immediately yields the (reduced) shear  $g$ . One can calculate the ACF locally and determine  $g$  locally. In addition, since the ACF is caused by very many faint galaxies per solid angle, one might suppose that it is a universal function (which can be determined from deep HST exposures); in that case, also the magnification can be determined locally. To avoid being dominated by just the brighter objects on the frame, they can be cut out, so that one works on a field with the topology of a Swiss Cheese. Eventually, if all objects are cut out which are significantly detected, one works in the noise limit. If the ACF of the noise is caused by faint high-redshift galaxies, the value of  $g$  determined from the noise should agree with that determined from the images, but gives independent information. This method was proposed and successfully tested both on synthetic images as well as on real data; in the latter case, the shear field obtained from individual galaxy images has been reproduced by the ACF of the noise (van Waerbeke *et al.* 1996). The ACF

method is also a sensitive diagnostics for testing image quality; improper data reduction shows up immediately as artificial features in the ACF.

### 3. Main Applications of Weak Lensing

In this section I will outline the main applications of weak gravitational lensing as currently known.

#### 3.1. RECONSTRUCTION OF CLUSTER MASS PROFILES

Though historically not the first application of weak lensing, the reconstruction of the two-dimensional mass distribution of clusters has been the major application of weak lensing up to now. Tyson *et al.* (1990) discovered a shear field in two clusters and determined the radial mass profiles from that. Kochanek (1990) and Miralda-Escudé (1991) investigated how shear data can be used to constrain the mass profiles of clusters. The pioneering paper by Kaiser & Squires (1993) paved the way for a non-parametric two-dimensional mass reconstruction: As in Newtonian gravity, the shear is given as a linear functional of the (surface) mass density. As was first shown by Kaiser & Squires, this relation can be inverted to express the surface mass density (up to an additive constant) in terms of the shear. Thus, if the shear can be measured, the surface mass density of the lens can be reconstructed.

In the weak lensing regime, the shear can be obtained from the local image ellipticities, as described above, and thus from an ensemble of images, the surface mass density can be evaluated. This method was first applied by Fahlman *et al.* (1994) to the cluster MS1224, and they obtained quite a large lower limit for the mass-to-light ratio of this cluster. Since then, several more clusters have been investigated with that method. The KS method has been modified to allow the inclusion of strong lensing (Schneider & Seitz 1995; Seitz & Schneider 1995; Kaiser 1995), to account for a finite region (*e.g.*, a CCD) on which observational data are given (Seitz & Schneider 1996a; see also Squires & Kaiser 1996), and to account for a broad redshift distribution of the galaxies (Seitz & Schneider 1996b). Using these generalizations, Seitz *et al.* (1996) have reconstructed the mass profile of the inner part of the cluster Cl 0939+4713 from a deep image taken with the HST. The resulting detailed two-dimensional mass map, when compared with the distribution of bright cluster galaxies, shows that the light traces the mass very well in this cluster. Also, the number density effect caused by the magnification has been discovered in this cluster. The mass-to-light ratio is only moderate ( $\sim 200$ , depending on the mean redshift of the galaxies), but that should be no surprise: Cl 0939 is the highest-redshift cluster in the Abell catalog (A851) and therefore expected to have a very

high optical luminosity. A low-resolution X-ray map (Schindler & Wambsganss 1996) indicates that also the X-ray emission traces the (dark) mass; this will be checked in more detail once a HRI map of this cluster becomes available.

The prospects of this method are simply excellent: deep images taking under good conditions will allow to study the dark mass distribution in clusters (*e.g.*, the radial density profile, detection of substructure and ellipticity), independent of assumptions about symmetries or dynamical or thermal equilibrium of the matter. It therefore provides the least prejudiced mass distributions, and can be used to calibrate other methods, *e.g.*, those using the X-ray profile and temperature (for example, see Squires *et al.* 1996). As stressed before, the accuracy of this method depends sensitively on the data quality, and on the available number density of galaxy images—thus on the depth of the observations. The combination of distortion and magnification effects, using maximum-likelihood techniques (Bartelmann *et al.* 1996), will increase the efficiency and accuracy of the reconstructions.

### 3.2. STATISTICAL PROPERTIES OF THE (DARK) MASS DISTRIBUTION IN GALAXIES

Individual galaxies are not massive enough to produce a significant shear signal, but statistically combining the signals from many (foreground) galaxies can yield a detectable ‘relative alignment’ of background images relative to the direction of the nearest foreground galaxy. First attempted by Tyson *et al.* (1984), this effect has now been discovered by Brainerd, Blandford & Smail (1996). Fitting a parametrized model to the alignment data, they have shown that the characteristic velocity dispersion (or rotational velocity) of galaxies is in the range expected from other investigations. In addition, they were able to obtain an interesting lower bound on the spatial extent of the dark halos in galaxies. This study was carried out with a relatively small number of galaxies; Schneider & Rix (1996) have shown that even with moderately-sized samples of galaxies, one can obtain very accurate determinations of model parameters such as  $\sigma_*$  or the characteristic size  $s_*$  of an  $L_*$  galaxy. In addition, the Tully-Fischer exponent can be probed, as well as the evolution of the mean redshift with apparent magnitude. All that is needed is a collection of wide-field images taking in excellent seeing conditions. Galaxy-galaxy lensing has also been detected in the HST MDS (Griffiths *et al.* 1996) and the HDF (Dell’Antonio & Tyson 1996).

### 3.3. DETECTION OF 'DARK' MASS CONCENTRATIONS

On wide-field images, one can search for (dark) mass concentrations by looking for statistically significant alignments of faint galaxy images. Based on the aperture densitometry developed by Kaiser (1995), I have investigated the statistical properties of the appropriately-defined aperture mass calculated from the image ellipticities in annular regions (Schneider 1996). The expectation is to detect isothermal halos with velocity dispersion in excess of  $\sim 600$  km/s, without any reference to the optical or X-ray luminosity of these halos. Depending on the cosmological model, one expects about 10 such halos per square degree for a standard CDM model, increasing by a factor of order 10 in a COBE-normalized CDM model. This method will thus allow for the first time to investigate the statistics of dark halos without any assumption about bias factors, so that these results can be directly compared to numerical LSS simulations. In fact, dark halos have already been discovered by their shear effects: the 'dark' lens in the double QSO 2345+007 was discovered by the shear field it creates (Bonnet *et al.* 1993), and significant shear fields have been discovered around several high-redshift radio-loud quasars (Fort *et al.* 1996), supporting the magnification bias hypothesis for the associations of these QSOs with foreground galaxies (*e.g.*, Bartelmann & Schneider 1994).

### 3.4. CONSTRAINTS ON THE REDSHIFT DISTRIBUTION OF VERY FAINT GALAXIES

The lensing strength of a given deflector increases with increasing source redshift. This yields the possibility to obtain information about the redshift distribution of the faintest detectable galaxies, as proposed by Smail, Ellis & Fitchett (1994), Bartelmann & Narayan (1995) and others. In particular, the fact that significant shear was observed in the high-redshift ( $z_d = 0.83$ ) cluster MS 1054-03 (Luppino & Kaiser 1996) shows that a large fraction of the galaxies used in this study ( $21.5 < I < 25.5$ ) must have a redshift significantly larger than 1. For a different study of source redshifts from weak lensing, using the magnification effect, see Fort, Mellier & Dantel-Fort (1996).

### 3.5. DETERMINATION OF THE POWER SPECTRUM OF COSMIC DENSITY FLUCTUATIONS

The density fluctuations of the mass inhomogeneities in the Universe distort light bundles from distant sources and can produce an observable effect. It has been shown in several papers (see, *e.g.*, Villumsen 1996 for references) that the statistical properties of the distortion field are directly related to

the power spectrum of the density fluctuations. For example, the two-point correlation function of the image ellipticity caused by the LSS is obtained by a convolution of its power spectrum with a known kernel function. Whereas the expected magnitude of the shear is quite small (of order 1%; however, including the non-linear evolution of the density field, the expected rms shear increases to about 4%; B. Jain, private communication), its detection and quantitative investigation will allow to study the statistical properties of the density field in the Universe, on (co-moving) scales much smaller than those achievable with CMB experiments, again without any assumption about bias factors.

### 3.6. PRACTICAL CONSIDERATIONS

In order to obtain good angular resolution and/or high accuracy on the local determination of the shear and the magnification, one has to take very deep exposures to be able to work with a high number density of galaxy images. The images are affected by any residual anisotropic PSF which mimics a shear. In order to correct for these instrumental effects, a stable PSF is needed, and good sampling of the PSF is required. It is also obvious that the seeing is crucial in this game: seeing circularizes small elliptical images and thus significantly reduces the shear signal. In order to regain the image ellipticities ‘before seeing,’ correction factors have to be applied, which are determined by simulating images with the same PSF and comparing the input shear with that estimated from the convolved image with pixelization and noise added. These correction factors can be quite large and reduce the accuracy with which the shear can be measured significantly (for a detailed discussion on these methods, see Bonnet & Mellier 1996; Kaiser, Squires & Broadhurst 1995).

## 4. The Need for Surveys

Weak gravitational lensing—because it is ‘weak’—requires statistical ensembles of galaxy images to infer properties of the intervening mass distribution. For relatively strong deflectors, such as a massive cluster of galaxies, a single large CCD frame provides sufficient area for the reconstruction of the central mass distribution. However, to investigate the outskirts of clusters, either wide-field images or mosaics have to be taken. For the search of dark mass concentrations, a deep, high-resolution map of a large consecutive area is needed—a few square degrees mapped to magnitudes of  $R \sim 26$  with seeing less than  $0.8''$  will certainly reveal a sample of mass concentration selected only by their mass properties. To study the large-scale structure shear, the requirements on image quality are tremendous—the residual anisotropies of the PSF have to be understood at a percent level!

However, it seems that such accuracy can be achieved with the SUSI camera at NTT (Fort *et al.* 1996). For an analysis of galaxy-galaxy lensing, one can compromise between depth of a survey and solid angle; in the latter case, the mass properties of relatively nearby galaxies are probed, and the relatively large angular separations of such potential lenses allows to investigate the spatial extent of the galaxy halos to large distances, whereas for deeper images, the cosmological evolution of galaxy halos can be probed. Concerning the first strategy, the imaging part of the SDSS will almost certainly allow the by far most detailed study of the statistical properties of the mass distribution of (low-to-medium-redshift) galaxies.

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