GROUPS WITH PRESCRIBED AUTOMORPHISM GROUP:
A CLARIFICATION

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In Theorems 1 and 2 of [1] necessary and sufficient conditions were given for a group $G$ to have a finite automorphism group $\text{Aut} G$ and a semisimple subgroup of central automorphisms $\text{Aut}_c G$. Recently it occurred to us, as a result of conversations with Ursula Webb, that these conditions could be stated in a much simpler and clearer form. Our purpose here is to record this reformulation. For an explanation of terminology and notation we refer the reader to [1].

**Theorem 1*. Let $G$ be a group such that $\text{Aut} G$ is finite and $\text{Aut}_c G$ is semisimple. Then one of the following holds: here $Q$ is a finite group with trivial centre and $q = |Q_{ab}|$.

(i) $G \cong \tilde{G}(Q, F, e)/\tilde{T}(Q)q$, where $F$ is a torsion-free abelian group, $e \in \text{Ext}(Q_{ab}, F)$ and $C_{\text{Aut}_F(e)} = 1$.

(ii) $G \cong G(Q, 1, 0)/K \times D$ where $K \leq M(Q)q$, $D$ is an elementary abelian 2-group of order different from 4 and

$$q \cdot |M(Q)q| \cdot K$$

is prime to $|D|$.

The point here is that if $G$ is infinite, then $G \cong \tilde{G}(Q, F, e)$, which is a central extension of $F$ by $Q$, the cohomology class being determined by $e$. If $G$ is finite, its structure is given by (ii); note that $G(Q, 1, 0)$ is a central extension of $M(Q)q$ by $Q$ whose cohomology class is determined by the canonical projection $M(Q) \rightarrow M(Q)q$.

**Proof of Theorem 1*. We know that $G$ has the structure described in Theorem 1 of [1]. Assume that $G$ is infinite, so that $F \neq 1$. Now $F$ is divisible by $l = |D| \cdot |M(Q)q:K|$. The mapping $x \mapsto x^l$ is an automorphism of $F$, say $\alpha$; this induces an automorphism in $E = \text{Ext}(Q_{ab}, F)$ which is just multiplication by $l$. Since $(q, l) = 1$, there is a positive integer $n$ such that $l^n \equiv 1 \mod q$. Also $qE = 0$. Hence $\alpha^n$ operates trivially on $E$. But $C_{\text{Aut}_F(e)} = 1$, so in fact $l = 1$. Therefore $D = 1$ and $M(Q)q = K$, as required.

Theorem 2 of [1] provides an immediate converse of Theorem 1*.

**Theorem 2*. 

(i) If $G = \tilde{G}(Q, F, e)$ as in Theorem 1* (i), then $\text{Aut} G \cong \text{St}_{\text{Aut} Q}(\epsilon^{\text{Aut} F})$ and $\text{Aut}_c G = 1$. 

59
(ii) If $G=(G(Q, 1, 0)/K) \times D$ as in Theorem 1*(ii), then $\text{Aut } G \cong N_{\text{Aut } Q}(K) \times \text{Aut } D$ and $\text{Aut}_e G \cong D$.

Finally, as a result of these simplifications we may refine Theorem 7 of [1] by replacing statement (v) by

(v)* An infinite group $G$ satisfies $\text{Aut } G \cong S_4$ if and only if $G \cong G(F, e)/\mathbb{Z}_3 \cong \Gamma(A_4, F, e)$ where $F$ is a non-trivial torsion-free abelian group, and $e$ in $F/F^3$ is such that $C_{\text{Aut } F}(e) = 1$.

REFERENCE