

CORRESPONDENCE

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Sirs,
A Budget of Paradoxes

May I offer the following *jeu d'esprit* without intention of offence. I claim that each of the following statements is incorrect:

- (1) l_x is a cumulative distribution function.
- (2) It is the oldest form of distribution function known.
- (3) The normal distribution function originated with De Moivre.
- (4) Birmingham, England, is not in Europe.
- (5) The function $1 - l_x/l_0$ was recognized as a cumulative distribution function long before 1933.

All of these assertions were made or implied in a single paragraph printed in Vol. 11 of this *Journal*.

The disproof of (1) is provided in the statement (5). The oldest known cumulative distribution function is provided by Pascal's solution to the Problem of Points (1654); Graunt published his *Observations* in 1662. The notion of a continuous probability distribution was apparently unknown to De Moivre (1733). It is to Simpson (1757) that we owe the first formulation of a continuous (symmetric triangular) law, and it was Laplace himself (1778) who first used the normal law as an independent entity. Finally, the earliest explicit treatment of $l_x \mu_x / l_0$ as a probability distribution seems to be due to Cramér (1930).

In conclusion, I mention that Laplace made use of a function $\phi(x/\omega)$, the probability of a newly-born infant dying at precise age x , to obtain the probability distribution of the mean expectation of life in n infants. However, it would not be correct to argue from this that Laplace's contemporaries, or his successors, thought in

terms of l_x/l_0 as $\int_x^\omega \phi\left(\frac{z}{\omega}\right) dz$.

Yours faithfully,
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