written and helps the reader to gain confidence in dealing with the whole subject of finite simple groups. Most libraries should have a copy and many algebraists will find it sufficiently useful and instructive to want to have their own copy. The standard of printing is high and the book is well-produced.

J. D. P. MELDRUM

Conway, J. B., *A course in functional analysis* (Graduate Texts in Mathematics 96, Springer-Verlag, 1985), xiv + 404 pp. DM 118.

For many years, the only general account of the basics of functional analysis could be found in the magnificent treatise *Linear Operators* by N. Dunford and J. T. Schwartz and, as a result, this work served both as the definitive source for the established researcher and as a text for beginning graduate students to cut their teeth on. Although "Dunford and Schwartz", as it is always affectionately referred to, remains the best reference—I recall being told of the apocryphal mathematician who needed three copies: one for the office, one for home and one for the car—at the same time, a number of less ambitious texts aimed more directly at the beginner have appeared in recent years. Conway's book is one of the latest of these and is to be recommended. As with many such books, it grew out of a year-long course given to graduate students over a number of years. The contents probably represent the union of the topics covered in these courses as they evolved and changed and so, by suitable selection, a number of different courses could be based on the book. Such courses would differ in detailed content, but all would have the same underlying theme, namely linear analysis.

When approaching the fundamentals of any subject, an author must choose whether to proceed from the particular to the general or vice versa. In the present text, the former approach is adopted. This inevitably leads to a certain amount of repetition, but this will probably be welcomed by the student reader and, as the author observed in the introduction, it is the way mathematics usually develops. The book starts with an account of Hilbert spaces and the basic associated operator theory (adjoints, projections, compact operators and so on) and then turns to Banach spaces (Hahn–Banach theorem, duality, the closed graph theorem and other applications of Baire's theorem etc.). A limited amount of locally convex space theory is included, primarily so that weak topologies can be discussed, and some basic Banach algebra theory is covered, to be applied in the approach to spectral theory. The remainder of the book is somewhat more specialized, reflecting the author's particular interests in Hilbert space operator theory. There is a good account of the basic properties of C^*-algebras and this is put to work in the analysis of normal operators. The book ends with chapters on unbounded operators and Fredholm theory.

Many applications and examples are included, together with references to further developments, so there is much to whet the appetite of the interested reader. There are also plenty of exercises at the end of each section. Some of these might be criticized as being a trifle on the dull side, though a few routine problems are probably helpful for the beginner, and there are a fair number of interesting ones to offset them. All in all, this is an excellent book which will prove invaluable to any graduate student wanting a groundwork in the principles of the subject.

T. A. GILLESPIE

Garling, D. J. H., *A course in Galois theory* (Cambridge University Press, 1986), pp. 167, cloth £22.50, paper £8.95.

As a course of study for undergraduates, Galois theory certainly has a lot going for it. Its clear purpose—finding the conditions for solubility of a polynomial equation using the usual arithmet-

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ical operators, together with extraction of roots ("solving by radicals") provides both a focus and a direction for the course. It does not have very demanding prerequisites: a little field and group theory, and a smattering of linear algebra. It seems in fact a convenient application for these topics, reinforcing the students' previous knowledge. Further, by showing that before we can decide whether we can solve a polynomial equation we must define exactly what we mean by "solve", a very useful lesson in precise mathematical modelling is set before the student.

Next, the fact that an "impossibility" proof (of solving by radicals equations with non-soluble Galois group) is produced adds for the student a new dimension to his view of what mathematics can do. It isn't only for proving facts about triangles or vector spaces. It can also be used to show the limitations of a particular constructive technique. Amazing!

Finally, of course, Galois theory has Galois. What a story: the boy genius, the revolutionary politics, the quarrel over a "coquette", the hurriedly written manuscript—"I haven't the time", the duel at dawn, the losses and rejection of his manuscript, its eventual rescue by Liouville,... What other scientific or mathematical topic can boast such a colourful founder?

Garling's book presents Galois theory in a style which is at once readable and compact. The necessary prerequisites are developed in the early chapters only to the extent that they are needed later. The proofs of the lemmas and main theorems are presented in as concrete a manner as possible, without unnecessary abstraction. Yet they seem remarkably short, without the difficulties being glossed over. In fact the approach throughout the book is down-to-earth and concrete. The final chapter, "The calculation of Galois groups", is a good example of this. It contains explicit examples of quintic equations having all possible Galois groups (of orders 5, 10, 20, 60, 120). Also the technique of obtaining information about the Galois group by factoring the polynomial modulo a prime p is explained simply there.

An important definition in Galois theory is that of an extension of a field "by radicals". There are, perhaps surprisingly, several variations of this definition in the literature. Garling (in common with I. Stewart, *Galois theory*, 1973) uses the following one: L is an extension of K by radicals if there are fields $K=K_0\subseteq K_1\subseteq K_2\subseteq\cdots\subseteq K_N=L$, where $K_i=K_{i-1}(\beta_i)$ $(i=1,\ldots,N)$, and some power $\beta_i^{n_i}$ of β_i lies in K_{i-1}. A polynomial in $K[x]$ is then soluble by radicals if it has all its zeros in some such L. According to this definition, however, x^n-1 is soluble by radicals simply because the field $L=\mathbb{Q}(\beta_1)$, β_1 a primitive nth root of unity, is obviously an extension by radicals, since $\beta_1^n=1\in\mathbb{Q}$. Thus, in this definition, a primitive nth root of unity is regarded as a radical. This definition is simpler than the others, which for instance require that $[K_i:K_{i-1}]=n_i$ (van der Waerden) or that n_i is prime and K_{i-1} already contains a primitive n_ith root of unity (W. M. Edwards, *Galois theory*, 1984). It is therefore probably the best one for an undergraduate text. It is not the classical one, however, and for instance renders Gauss's famous expression for the 17th roots of unity using square roots (*Disquisitiones arithmeticae*, Art. 365) irrelevant for solving $x^{17}-1$ by radicals (though not of course for the construction of the regular 17-gon).

Because of its clarity and economy of expression, I can heartily recommend this book as an undergraduate text. It would need, at least for background reading, to be supplemented by a historical perspective on the subject (e.g. from Edwards or Stewart) in order to motivate the subject. All right, we want solutions of polynomial equations, but why solution by *radicals*? Further, since the book presents the subject in a rather closed way, mention would also have to be made of other ways of solving polynomial equations. The same applies to extensions of Galois theory, and to unsolved problems on the subject, which receive only a passing reference in the book. For indeed, as the author says, "Galois theory has a long and distinguished history: nevertheless, many interesting problems remain".

C. J. SMYTH

MATSUMURA, HIDEYUKI, *Commutative ring theory* (Cambridge studies in advanced mathematics 8, Cambridge University Press, 1986), pp. 320, £30.

Professor Matsumura is already well known for his standard text *Commutative Algebra* (Benjamin/Cummings, Reading, Mass., 1969), reissued in 1980 with a number of substantial

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