

Section IV

Dust in Galaxies

FAR INFRARED EMISSION FROM GALACTIC AND EXTRAGALACTIC DUST

GEORGE HELOU

*Infrared Processing and Analysis Center,
California Institute of Technology,
Pasadena, CA 91125*

ABSTRACT. This review examines the question of how similar far infrared and submillimeter dust properties are among galaxies including the Milky Way, and within the Milky Way. The IRAS data are examined, and no evidence is found for variations in the broad-band dust properties between 10 and 100 μm . Submillimeter data are examined in a search for a power-law index describing the emissivity of dust at $\lambda > 100 \mu\text{m}$. The data are consistent with a universal value of the index between 1 and 2. The evidence for large amounts of cold dust (10 to 15 K) in galaxies is reviewed and found quite soft in light of conflicting data. In most spiral galaxies, the dust-to-gas ratios are comparable to their values in the Milky Way, with a few well-documented exceptions such as the Magellanic Clouds and Blue Compact Dwarf galaxies, which have a lower ratio. No compelling evidence is found for gradients in the dust-to-gas ratio within disks of galaxies. Very small grains with fluctuating temperatures appear to be present in all galaxies examined, and to behave just like those observed in the Milky Way.

1. INTRODUCTION

The poorly known properties of interstellar dust in the far infrared and submillimeter range often act as an extraneous variable, an unknown quantity adding scatter to the data and uncertainty to the calculations. There is not an object in which far infrared dust properties are known with the degree of accuracy available in the near infrared and the optical. Needless to say then, the question of object to object variability in these properties is still open. Faced with this situation, most authors have adopted some reasonable description of dust in the solar neighborhood, and have assumed that all dust in the observable universe behaves in the same fashion. It is this mediocre assumption that I will try to examine in this review.

Emission from dust is responsible for a very broad feature between 10 μm and 1 mm in the spectrum of galaxies endowed with a substantial interstellar medium (see Figure 6 below). The prominence of this feature is, to first order, a measure of the heating intensity to which the dust is exposed (Soifer, Houck, and Neugebauer, 1987; Telesco, 1988). The IRAS satellite was perfectly tuned in its spectral coverage on the infrared side of this feature, whereas the long wavelength side is the purview of submillimeter and millimeter astronomy. In what follows, the feature

is first examined on the infrared side (§ 2.1), then on the submillimeter side (§ 2.2), in a search for color variations between galactic and extragalactic objects, and among galaxies. The question of dust temperatures derived in the submillimeter is addressed in § 2.3. Attention then turns to the dust-to-gas mass ratio, with § 3.1 presenting the rigorous approach to its estimation, and § 3.2 a more loose but more easily applicable statistical test. § 3.3 reviews reports of gradients in the dust-to-gas ratio within galaxy disks. Finally, § 4 addresses the question of very small grains in galaxies and how to estimate their relative abundance.

2. COLORS OF DUST IN EMISSION

2.1. IRAS COLORS

The Infrared Astronomical Satellite (IRAS) mission has generated more than 95% of all of the data in existence in the 10–100 μm range, including, as a centerpiece, the remarkably uniform survey of 95% of the sky. The survey was made in four photometric bands nominally centered at 12, 25, 60, and 100 μm , with full widths at half maximum of 7, 11, 32, and 31 μm respectively (IRAS Explanatory Supplement, 1988). In spite of its low spectral resolution, the IRAS survey provides the best data for comparing the far infrared properties of dust in various astronomical sources. This section examines the IRAS color-color diagrams of several samples of sources, extragalactic and galactic, simply looking for differences in these diagrams, and deferring the full interpretation of the general appearance of the diagrams to § 4 below.

2.1.1. Extragalactic Sources

While several IRAS color-color diagrams of galaxies have been discussed (e.g. Rowan-Robinson and Crawford, 1986), I will concentrate on a combination involving all four IRAS bands, namely a plot of $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ vs. $f_\nu(12 \mu\text{m})/f_\nu(25 \mu\text{m})$. Galaxies where stars are the dominant power source have their colors in this diagram determined entirely by the dust properties and the mix of dust temperatures. Figure 1 shows such a diagram for three samples of galaxies with quite different properties. The first one (represented by a box in Figure 1) is based on the IRAS Point Source Catalog (1988), and consists of galaxies detected by the satellite but unresolved at all four wavebands, with the further constraint that $f_\nu(25 \mu\text{m})/f_\nu(60 \mu\text{m}) < 0.18$ to avoid Seyfert-type active nuclei (Helou, 1986a). This selection tends to favor high surface brightness objects, and a relatively narrow range of redshifts. The galaxies in this sample populate rather uniformly the box outlined in Figure 1, which contains 90% of the 149 objects. The color sequence along this box corresponds to increased heating of the same dust sample (Helou 1986a; § 2.1.2 and § 4 below), whereas displacements across the band could be due to mixing of components at different heating levels, or to variations in dust properties.

The second sample is represented by all of the individual data points shown as dots on the diagram. This is a flux-limited sample consisting of all galaxies (about 320) with $f_\nu(60 \mu\text{m}) > 5.40$ Jy within a 14,500 deg^2 area of the sky (Soifer *et al.*, 1987, 1989). It contains objects ranging in distance from the Local Group to 240 h^{-1} Mpc, where h is the Hubble constant normalized to 100 $\text{km s}^{-1}\text{-Mpc}^{-1}$, and in infrared luminosity from 10^8 to 2×10^{12} L_\odot . Except for extending the box at both

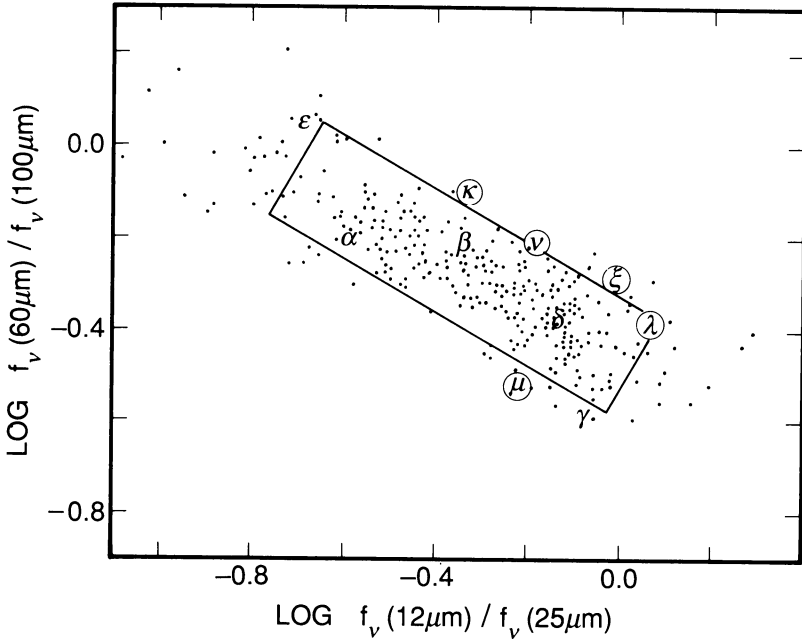


Fig. 1. IRAS color-color diagram for dust in galaxies. The box represents data from Helou (1986a), the dots data from Soifer *et al.* (1989), and the Greek letters data from Impey *et al.* (1989). Details on these three samples are given in § 2.1.1

ends along the major axis, this sample is statistically indistinguishable from the first one, confirming that the band-shaped distribution is not due to selection effects, and that the dust properties gauged by this color-color diagram are quite constant from galaxy to galaxy. At constant $f_{\nu}(60 \mu\text{m})/f_{\nu}(100 \mu\text{m})$, the interval holding 90% of values on the $f_{\nu}(12 \mu\text{m})/f_{\nu}(25 \mu\text{m})$ axis is 0.45 wide on the decimal log scale, while the interval holding 90% of values on the $f_{\nu}(60 \mu\text{m})/f_{\nu}(100 \mu\text{m})$ scale at constant $f_{\nu}(12 \mu\text{m})/f_{\nu}(25 \mu\text{m})$ is 0.27 wide. In the extreme and unlikely case where all dispersion on each of the color axes is due to variations in the exponent of a power-law emissivity, and assuming a Gaussian distribution on the exponent, the band widths above correspond to a dispersion of 0.43 on the exponent between 12 and 25 μm , and of 0.37 on the exponent between 60 and 100 μm .

Although most objects in the two samples above are late type galaxies, early type galaxies have also been shown by Thronson and Bally (1987) and Knapp (private communication) to be indistinguishable in Figure 1 from the samples shown, as long as their IRAS colors are determined by grains in the interstellar medium.

The third sample consists of ten radio galaxies initially selected from the B2 Survey for a follow-up study by Impey, Wynn-Williams and Becklin (1989). Whereas in the previous two samples, dust is heated predominantly by stellar radiation, the dust in these radio galaxies may be exposed to different heating processes such as far ultraviolet and X-ray radiation, or a flux of energetic particles, due to the pres-

ence of a compact nuclear source (e. g. Tytler, 1987). The unusual conditions of the dust in these objects are manifested in the wide range of infrared-to-radio ratios observed for the sample. The five radio galaxies represented on Figure 1 by the Greek letters α , β , γ , δ , ϵ have infrared-to-radio ratios typical of disks of galaxies and of star-bursts (Helou, Soifer and Rowan-Robinson, 1985), whereas the remaining five objects have ratios that are larger by a factor that ranges from a few to a few hundred, increasing in the following order: κ , λ , μ , ν , ξ . Such high ratios are more characteristic of compact nuclei and radio-loud quasars. The maximum ratio (of object ν) is consistent with a flat spectrum between 21 cm and 100 μm , implying that the broad feature peaking around 100 μm associated with dust emission might not be evident in the spectrum of this radio galaxy. In spite of the potentially exotic environment within these radio galaxies, the far infrared emission, presumably still dominated by dust, maintains colors within the range of what is observed in normal galaxies.

2.1.2. Galactic Sources

For comparison between extragalactic and galactic objects, Figure 2 shows the IRAS color-color diagram for three sets of data on galactic objects. As for most observations in the Milky Way, these data, in contrast with the integrated colors of galaxies, are subject to uncertainties due to choice of boundaries, background-subtraction or other sources of confusion.

The first data set derives from measurements by Boulanger *et al.* (1988) of the surface brightness of the California Nebula, an HII region of intermediate density (varying from 1 to 50 cm^{-3}) excited by ξ Per, an O7.5III star. The data points plotted as crosses represent the colors at eleven locations arranged in a sequence of increasing distance from ξ Per, which maps into a sequence of monotonically decreasing ratios of $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$. This sequence of colors thus typifies the IRAS emission from dust exposed to decreasing energy densities of heating radiation, with the boxed point corresponding roughly to an energy density of 5 eV cm^{-3} . This sequence of colors wanders through the band occupied by galaxies, confirming the interpretation that increased heating intensity moves an object to the left along the band.

The second data set consists of more surface brightness measurements by a number of authors (e. g. Low *et al.*, 1984; Leene, 1986; Weiland *et al.*, 1986; de Vries *et al.*, 1987; see also Boulanger *et al.*, 1988 and the review paper by Puget, 1989) in a variety of molecular and atomic clouds heated by the diffuse interstellar radiation field. These data are summarized on Figure 2 by two envelopes (circles of dashed lines), one for atomic and the other for molecular clouds, containing all of the observations. In addition, the colors of the atomic medium on the largest scales is shown separately, as derived from the $\text{cosec}|b|$ law fit to the galactic background (Boulanger and Pérault, 1988), as the open circle with error bars attached.

The third data set, adopted from Thronson and Bally (1987), is for a collection of "star-forming regions" in the Milky Way, ranging in infrared luminosity from 10^4 to $10^6 L_\odot$, and in mass from 10^3 to $10^6 M_\odot$. In spite of the difficulties in estimating total IRAS fluxes for these objects and the resulting uncertainties on the colors, these complexes essentially agree in their colors with galaxies. The tendency of these objects to fall below the band of galaxies may be in part due to greater optical depths, either in emission or in absorption, which affect colors in the way indicated

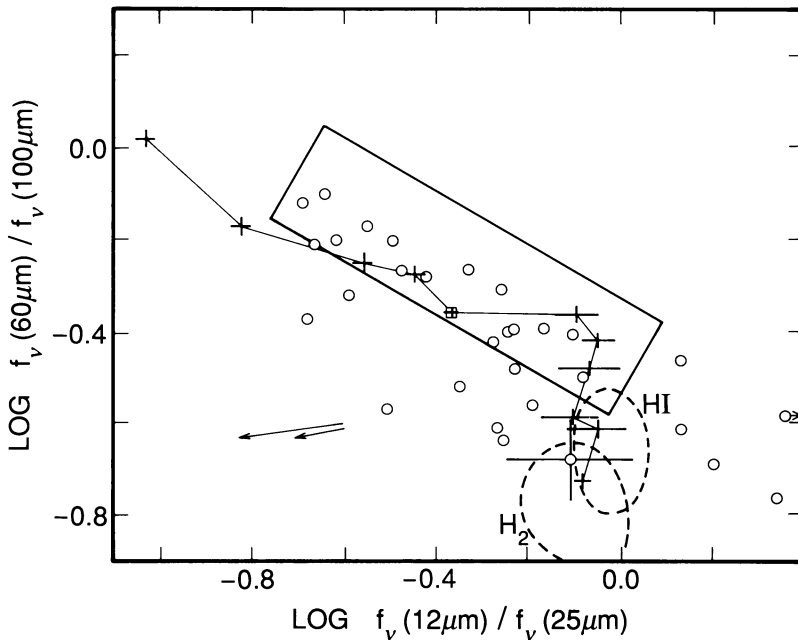


Fig. 2. IRAS color-color diagram for dust in the Milky Way. The box is the same as in Figure 1, and is drawn for comparison with galaxies. The connected crosses represent data from Boulanger *et al.* (1988), the circles data from Thronson and Bally (1987), and the one circle with error bars data from Boulanger and Pérault (1988). Details on these data and the envelopes for atomic and molecular media are given in § 2.1.2

by the arrows on Figure 2. Both arrows indicate the change in colors associated with the optical depth at 12 μm going from near zero to one, and assuming the optical depth scales as λ⁻¹ for 10 < λ < 100 μm. The longer arrow corresponds to a varying optical depth in absorption only between source and observer, and the shorter arrow to a varying optical depth in emission at the source. Much better agreement is obtained between galaxies and interstellar clouds containing young, moderately massive stars (Terebey and Fich, 1989) measured in the outer Milky Way where confusion and background subtraction are more manageable.

Thus no evidence is found for significant differences between the IRAS colors of dust in the Milky Way and those of dust in other galaxies. On the other hand, positive evidence is presented indicating that object to object variations in the colors of dust in emission are dominated by variations in heating, and that the broad-band properties of dust are essentially constant among the galaxies detected by IRAS.

2.2. LONG WAVELENGTH EMISSIVITY

Dust emissivity at λ > 10 μm is usually described as a power law ε(λ) = ε₀(λ/λ₀)^{-β}, with β increasing from near 1 at 10 μm towards an asymptotic value near 2 at the

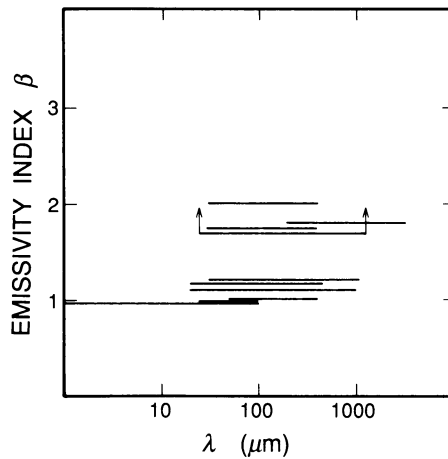


Fig. 3. Submillimeter emissivity indices derived from published model fits to observations of galactic objects. Each fit is represented by a line, which indicates the range of λ covered in the observations on the abscissa, and the value of the power-law index best describing the emissivity law on the ordinate. References and discussion are given in § 2.2

longest wavelengths, as expected for ideal dielectric grains (see also Draine and Lee, 1984). Laboratory measurements in the far infrared however yield values ranging from $\beta = 0.8$ to $\beta = 3$ (Koike, Hasegawa and Hattori, 1987, and references therein). In light of these measurements, the exact value of β allows only partial discrimination among various chemical or physical models of dust grains. In addition, Wright (1987) points out that β could be more affected by the shape of a conducting grain than by its composition. Still, β is needed for a correct derivation of temperature T and mass of emitting dust. If the spectrum of an object is measured at several wavelengths, then both β and T (or a modelled combination of temperatures) can be deduced from a best fit to the data. When such fits are carried out on objects in the Milky Way, they yield β values mostly between 1 and 2, as seen in Figure 3. The wavelength range covered and the uncertainties are such that the various determinations in Figure 3 are not necessarily in violent mutual contradiction. The data in Figure 3 were taken from Schwartz *et al.*, 1983; de Bernardis *et al.*, 1984; Chini *et al.*, 1986a; Rengarajan *et al.*, 1986; Rowan-Robinson *et al.*, 1986; and Gordon, 1987.

Because of their approach, the fits reported above are not necessarily optimized for evaluating β in the submillimeter. A fit however is not always needed, for if a source is warm enough for the Rayleigh-Jeans approximation to the blackbody curve to hold, then we have simply $\beta = n + 2$, where n is the power law index of the f_ν spectrum observed. Unfortunately, a source must be at least 170 K for the Rayleigh-Jeans approximation to hold to 0.1 in the power law index at $\lambda > 350 \mu\text{m}$. For cooler sources, an estimate of the temperature is needed for the non-linear derivation of β from n . Using published observations, I tried to estimate β directly at $\lambda > 300 \mu\text{m}$. The results are shown in Figures 4 and 5. The data were taken from Becklin and Wynn-Williams, 1987; Chini *et al.*, 1986b; Cunningham *et al.*, 1984;

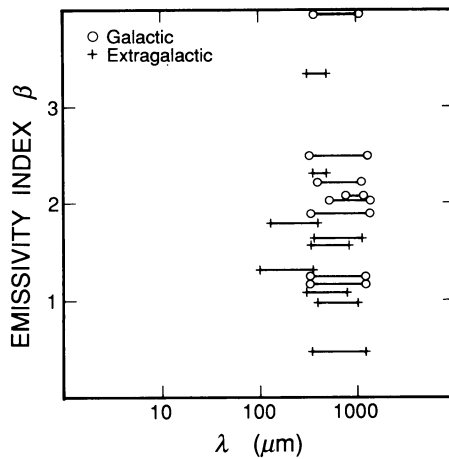


Fig. 4. Submillimeter emissivity indices computed from published observations as a function of the wavelength, for both galactic and extragalactic objects. Each line joins the two wavelengths at which the measurements were made, and has, for the ordinate, the value of the power-law index computed from the data. References and discussion are given in §2.2.

Eales *et al.*, 1989; Elias *et al.*, 1978; and Jaffe *et al.*, 1984 for extragalactic sources (crosses), and from Gear *et al.*, 1988; Loughran *et al.*, 1986; Mezger *et al.*, 1986; Mezger *et al.*, 1987; Mezger *et al.*, 1988; and Richardson *et al.*, 1985, for galactic sources (open circles).

In Figure 4, the estimate of β in each source is represented by a horizontal bar whose ordinate is the value of β and whose ends have for abscissae the two wavelengths between which the spectral slope was measured. The uncertainties on these estimates of β are substantial: if the two flux densities used are uncertain by 20% each (an optimistic number), and are taken at wavelengths separated by a factor 3, then β is uncertain by 0.6. In addition to this, there are uncertainties associated with the choice of T to correct for the departure from the Rayleigh-Jeans approximation. Figure 5 illustrates this last problem well, for it shows an “anticorrelation” between T and β . This trend is due to the fact that the same measured spectral slope n will yield a larger β if a smaller T is assumed; typical loci of (T, β) pairs corresponding to the same n measured between 300 and 1000 μm are shown as the broken lines on Figure 5. Galaxies tend to yield slightly lower values of β , but that is insignificant especially since galaxies often have a broad distribution of dust temperatures such that the colder dust emission mitigates the drop in the spectrum at long wavelengths.

Given the uncertainties, it may be concluded that there is no evidence for systematic differences in β between galactic and extragalactic dust, that there is no evidence for β variations with wavelength, and that β is in the range 1 to 2, with a median estimate of 1.5. The large dispersion of values plotted reflects mostly the measurement uncertainties. Submillimeter continuum observations are still a difficult enterprise, as is illustrated in the next section.

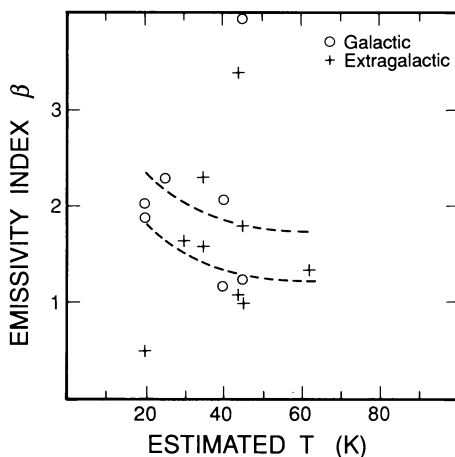


Fig. 5. Submillimeter emissivity indices computed from published observations as a function of the assumed temperature. The broken lines are a typical track along which plotted points would move if different temperatures were assumed for the same set of measurements. References and discussion are given in §2.2.

2.3. HOW COLD ARE GALAXIES?

The derivation of the properties of dust in the far infrared and submillimeter range is closely linked to our ability to model the radiative environment in which the dust is observed. A simple example is the interplay between β and T (or distribution of T) mentioned in § 2.2 above. The last decade has seen much interest in the question of typical temperatures of dust in various environments, and the relative contributions of various phases of the interstellar medium to the total dust emission from a galaxy (Ryter and Puget, 1977; Mathis, Mezger and Panagia, 1983; Caux *et al.*, 1984; Hauser *et al.*, 1984; de Muizon and Rouan, 1985; Cox, Krugel and Mezger, 1986; Persson and Helou, 1987; Sodroski *et al.*, 1987; Pérault *et al.*, 1988). One particular question in this area, namely the prediction (e.g. Mezger, Mathis and Panagia, 1982) that the Milky Way and other galaxies contain large amounts of very cold dust at 10 to 15 K, has received considerable interest since the advent of sensitive submillimeter and millimeter wave telescopes and detectors. Very cold dust would dominate the far infrared to millimeter wave spectrum if most of the luminosity of a galaxy in this range was provided either by dust in dark quiescent molecular clouds, or by silicate grains or large graphite grains (several microns) that reach equilibrium temperatures on the order of 10 to 15K. The amount of submillimeter data available to date is still insufficient to settle the question of very cold dust, but sufficient to generate new controversy, this time of a purely empirical nature.

The controversy is illustrated in Figure 6, adopted from Eales, Wynn-Williams and Duncan, 1989, which shows the average spectrum (normalized to 1 at 100 μm) of two samples of galaxies, together with the spectrum of the Milky Way. All filled symbols refer to the average of five galaxies (NGC 3079, NGC 3627, NGC

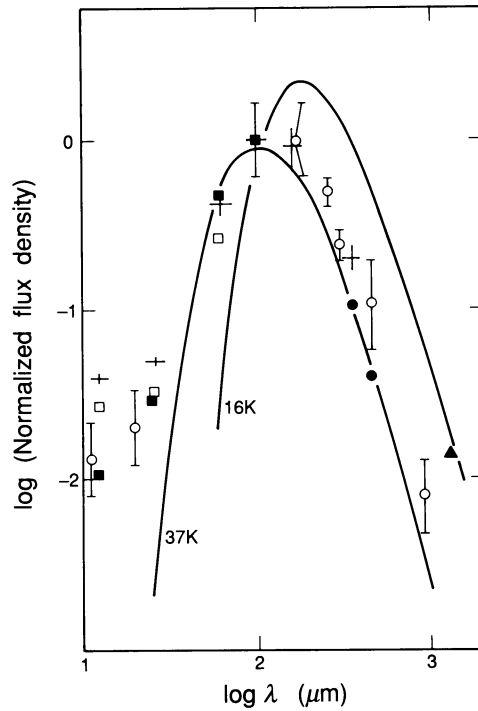


Fig. 6. Far infrared to submillimeter spectrum of two galaxy samples and of the Milky Way. Averaged flux densities for five galaxies are taken from IRAS (filled squares), from Eales *et al.*, 1989 (filled circles); and Chini *et al.*, 1986*b* (filled triangle). Data on Virgo Cluster galaxies are from Stark *et al.*, 1988 (crosses), and on the Milky Way are from Pérault *et al.*, 1988 (open squares); and Pajot *et al.*, 1986 (open circles). Details and discussion are given in § 2.3

4254, M51, and Arp 200) selected to be rather active based on IRAS data. The filled squares represent IRAS measurements, the filled circles 350 μm and 400 μm measurements by Eales, Wynn-Williams and Duncan (1989) at the UKIRT, and the filled triangle 1300 μm measurements by Chini *et al.* (1986*b*) at the NASA IRTF. The 16 K blackbody curve is the Chini *et al.* (1986*b*) fit to the combination of their data with IRAS data, whereas the 37 K curve is proposed by Eales, Wynn-Williams and Duncan (1989). The conflict between the two sets of measurements corresponds to an order of magnitude on the flux density scale, but becomes a factor of several hundred in the implied mass of emitting dust. The conflicting data sets cannot be reconciled without appealing to contrived distributions of temperature or surface brightness. More observations are clearly needed.

Shown on the same graph as crosses are data by Stark *et al.* (1988), who have used the KAO and IRTF to map four Virgo Cluster spirals (NGC 4254, 4321, 4501, and 4654). The spectrum of each galaxy is again normalized at 100 μm , and the size of the vertical bar on each cross indicates the total range of values covered by

the four galaxies. Even though the Virgo spirals are cooler and less active than the Chini objects, the Stark *et al.* (1988) data do not support the hypothesis of very cold dust in large amounts.

The Pajot *et al.* (1986) data on the emission from the Milky Way are shown on the same plot as the open circles with error bars, whereas the IRAS data on the Milky Way from Pérault *et al.* (1988) are shown as the open squares. The Milky Way is again cooler than the Virgo spirals, yet its submillimeter spectrum falls off too sharply to be compatible with the Chini *et al.* (1986*b*) picture. (see also Lange *et al.*, 1989)

Until this observational conflict has been settled, the conclusion is that the Milky Way is a typical galaxy in the submillimeter properties of its dust, and that the evidence for large amounts of very cold dust in galaxies is at best soft.

3. DUST-TO-GAS RATIOS

3.1. STANDARD TEST

By contrast to the complexities of comparing the composition and intrinsic properties of dust, the dust-to-gas mass ratio is a simple quantity to interpret, since it can be related directly to the metallicity and therefore to the history of the interstellar medium. Unfortunately, the estimation of this simple ratio involves several steps and is sensitive to the uncertainties encountered above in discussing dust properties (Hildebrand, 1983). The mass of emitting dust M_d in a source at a distance D may be written $M_d = D^2 f_\nu / \kappa_\nu \Omega B_\nu(T)$, where f_ν is the observed flux density, κ_ν is the emissivity per unit mass of the dust, Ω the solid angle in the source, and $B_\nu(T)$ the blackbody intensity at the source's temperature T . Since this formula assumes optically thin emission at a known T , it is best applied in the far infrared or the submillimeter; but that is precisely where temperatures are hard to estimate (see §2.2 above). Once M_d is computed using some "universal" κ_ν (e. g. Draine and Lee, 1984), and the mass of gas M_g is obtained independently, the dust-to-gas ratio is derived. Alternatively, the result of the measurements can be expressed in the form $\kappa_\nu M_d / M_g$, eliminating uncertainties in the choice of κ_ν .

A quick survey of the literature reveals that in most objects studied, galactic and extragalactic, M_d / M_g is consistent with the Hildebrand (1983) numbers ($M_d / M_g \sim 10^{-2}$) to within the uncertainties, which are typically a factor of two or more. The following is an incomplete sampling of recent relevant studies: Jaffe *et al.* (1983), the core of the W3 molecular cloud complex; Nordh *et al.* (1984), the S235 molecular cloud; Gee *et al.* (1984), the cold dust halo of NGC 7027; Hauser *et al.* (1984) and Sodroski *et al.* (1989), various large scale components of the Milky Way; Rengarajan *et al.* (1986), IRC+10216; Eales *et al.* (1989), a sample of seven IRAS bright galaxies.

In a remarkable exception however, Piet Schwering finds in his thesis (1989) that M_d / M_g in the Large Magellanic Cloud is about one third the ratio in the Milky Way, in good agreement with the deficiency obtained by comparing HI and E_{B-V} measurements (e. g. Koorneef, 1982). This agreement confirms the similarity between dust properties in the two galaxies from 0.5 to 100 μm , and rules out large amounts of very cold dust in the Large Magellanic Cloud (see § 2.3 above). On the other hand, Schwering finds M_d / M_g in the Small Magellanic Cloud deficient with

respect to the Milky Way by a factor of 55, which is several times more severe than the values derived from optical observations.

In a similar vein, Gondhalekar *et al.* (1986) have derived very low M_d/M_g values in three Blue Compact Dwarf galaxies, reaching down to 10^{-3} the value in the Milky Way. The three objects, Tol 1924–416, Mrk 36, and IZw18, were already known from optical spectroscopy to be severely metal deficient (Kunth and Sèvre, 1986).

3.2. GLOBAL RATIOS IN GALAXIES

In connection with the work of Eales *et al.* (1989), it should be pointed out that the approach to M_d/M_g outlined in § 3.1 is not easily applicable to a whole galaxy, because of the difficulty of defining a single temperature for the emitting dust, and because the global measures of dust emission and gas content give different sets of weights to the various phases of the interstellar medium within the same galaxy. Still, except for a handful of nearby galaxies, one is constrained to use global parameters (F_{IR} , F_{HI} , F_{CO}) in studying the variations of M_d/M_g among galaxies. The strategy then is to trust that simple ratios of global parameters such as F_{IR}/F_{CO} will respond to changes in M_d/M_g in spite of their large intrinsic dispersion. This dispersion is due, among other things, to variations in excitation conditions in the relative amounts of atomic, molecular, and ionized gas, and the fact that all three phases contribute to the infrared emission.

The ratio L_{IR}/M_{CO} (far infrared luminosity per carbon monoxide mass) is a natural gauge of the dust-to-gas ratio in a statistical sense. Its main drawback is the dispersion in the relation between CO line fluxes and CO masses (e.g. Rickard and Harvey, 1983; Maloney and Black, 1988; Bazell, Blitz, and Désert, 1989). In the plane of the Milky Way and in individual molecular clouds, L_{IR}/M_{CO} is in the range of 3 to 30 L_\odot/M_\odot (Scoville and Good, 1987). In galaxies, the same ratio taken globally ranges from 3 to 100 L_\odot/M_\odot (Sanders *et al.*, 1987; Solomon *et al.*, 1987). The dispersion in L_{IR}/M_{CO} among galactic and extragalactic objects primarily reflects variations in the heating intensity, and does not rule out a value of M_d/M_g which is similar, on average, in the Milky Way and in other galaxies. This point is well illustrated by Young *et al.* (1986) who apply to the global parameters of galaxies a treatment similar to the approach in § 3.1 above, and find that the data are consistent with a constant M_d/M_g among galaxies and a dust temperature derived from the global $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ ratio.

Similarly, the ratio L_{IR}/M_{HI} in galaxies can be examined for evidence of variations in M_d/M_g . The major drawback here is that a large fraction of M_{HI} often resides in the outer parts of the galaxy disk, an area known to contribute very little to the global infrared emission. Garwood *et al.* (1987) find that a sample of IRAS selected galaxies are indistinguishable from Virgo Cluster spirals in their L_{IR}/M_{HI} distribution, and that both samples are compatible with the corresponding observations in galactic cirrus given the uncertainties and systematics mentioned above. A similar result for dwarf irregular galaxies has also been reported by Helou (1986b).

In a combination of the two preceding approaches, Knapp *et al.* (1987) showed that $L_{IR}/(M_{HI} + M_{CO})$ shows less dispersion than either L_{IR}/M_{HI} or L_{IR}/M_{CO} , and that the inter-relations between L_{IR} , M_{HI} , and M_{CO} in Virgo Cluster spiral galaxies are indistinguishable from the same inter-relations in the Milky Way, given available data.

There are cases, however, where the ratio L_{IR}/M_{HI} can be made to reveal a difference in M_d/M_g . Hoffman *et al.* (1989) compared a sample of bright spirals to a sample of Blue Compact Dwarfs in the Virgo Cluster using HI and IRAS data, and observed three statistical trends: (i) both $\langle L_{IR}/L_{Opt} \rangle$ and $\langle L_{UV}/M_{HI} \rangle$ are about the same in both samples; (ii) the color temperature between 60 and 100 μm is lower in the spirals on average; and (iii) $\langle L_{IR}/M_{HI} \rangle$ is two or three times larger in the spirals. They conclude, based on this evidence, that M_d/M_g is two to three times larger on average in the spirals, which is a small enough difference to rule out the possibility that Blue Compact Dwarfs are now undergoing their first or second round of star formation. The critical assumption needed in the inference of a low M_d/M_g is that spirals and Blue Compact Dwarfs have the same relative amounts of atomic and molecular gas in their interstellar medium.

In a similar exercise, Thronson *et al.* (1988) find $\langle L_{IR}/M_g \rangle$ larger in S0 than in later type galaxies, which may be (among other things) a result of higher $\langle M_d/M_g \rangle$. This possibility is interesting for it would then imply that interstellar gas in lenticulars is mostly due to recently returned material from evolved stars.

3.3. GRADIENTS WITHIN GALAXIES

Boulanger and Pérault (1988) used the IRAS data to study the dust-to-gas ratio in the interstellar medium in the solar neighborhood. They concluded that M_d/M_g was uniform on the 100 pc scale in the atomic medium. They also found however that the color temperature obtained from $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ was independent of the 100 μm emissivity per H atom, which is very surprising if one admits that warmer dust will radiate more power (see also Laureijs *et al.*, 1989). This peculiar result would have been no more than an unpleasant curiosity except that it manifests itself on larger scales in both the Milky Way and M31.

In two independent studies of the IRAS emission from the Milky Way (Sodroski *et al.*, 1987), and from M31 (Walterbos and Schwering, 1987), $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ was found to be constant as a function of radius whereas $I_\nu(100 \mu\text{m})/N_{HI}$ dropped at larger radii by factors of three or so. Each of these concluded that a negative radial gradient in M_d/M_g exists in the particular galaxy studied. The conclusion was based on the expectation that, at constant temperature, the emissivity per dust mass should be constant, regardless of the unpleasant curiosity mentioned above. In more recent work, Sodroski *et al.* (1989) find a smaller drop in M_d/M_g associated with the atomic medium, but none associated with the molecular or the ionized media. By contrast, Hauser *et al.* (1984), using observations of the Milky Way at 160 and 260 μm , found no gradients in M_d/M_g .

The resolution of this conflict lies almost certainly in the fact (already hinted at by Walterbos and Schwering, 1987) that the ratio $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ is not an accurate measure of the temperature of the grains emitting at 100 μm . The reason is that the 60 μm emission includes a contribution from very small grains (down to $10^{-3} \mu\text{m}$ in size) transiently heated to very high temperatures, as opposed to larger classical grains (typically 0.1 μm in size) which dominate the 100 μm band and are maintained at an equilibrium temperature (see § 4 below). This contribution is evidently sufficient in the case of the cool atomic medium to render meaningless the color temperatures derived from $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m})$ (see also Wesselius *et al.*, 1989 and Caux *et al.*, 1989). Thus the M_d/M_g gradients derived from IRAS

data within galaxies should be regarded with caution until more detailed analysis is available which takes into account the contribution from very small grains at 60 μm . More generally, any dust temperature naively derived from a ratio $f_\nu(60 \mu\text{m})/f_\nu(100 \mu\text{m}) \leq 0.3$ is potentially suspicious. This warning entails questions of how wide-spread very small grains are, and how constant their abundance is, both of which will be addressed in the next section.

4. VERY SMALL GRAINS

Since this topic is covered by Jean-Loup Puget in a separate review (Puget, 1989), the following discussion will assume the interpretation and results advanced in that review, and will focus on extragalactic data in that context.

As soon as an interpretation is sought for the IRAS color-color diagram in Figure 1, it becomes clear that very small grains must play an important role in determining its appearance (Helou, 1986a), for the observed colors cannot be reconciled with populations of grains in thermal equilibrium (see also Pajot *et al.*, 1986). It follows from Helou (1986a) that for all objects within the box in Figure 1 the 12 μm flux is dominated by fluctuating emission from very small grains, whereas the 25 μm flux is a changing mixture of fluctuating (dominant at the quiescent end of the box) to equilibrium emission from large "classical" grains (dominant at the active end). Thus it seems that very small grains are a universal presence, an essential component of grain populations in every spiral galaxy we have so far examined with the IRAS data. That does not however rule out galaxies whose 12 μm emission is not solely due to very small grains: stellar photospheres and dust shells around evolved stars have been shown by Jura *et al.* (1987) to contribute substantially to the 12 μm emission of Shapley-Ames elliptical galaxies. Marston and Dickens (1988) examined Centaurus A in detail and found evidence for the same kind of contribution in the fact that Cen A lies well above the box of galaxies in Figure 1.

Besides the statistical argument based on Figure 1, detailed analysis of the IRAS data in individual objects also points to very small grains as a universal presence. Walterbos and Schwering (1987) find that the 12 and 25 μm emission of M31 requires the existence of a dust component at about 380 K, and favor the explanation that this component consists of very small grains transiently heated to high temperatures. Similarly, Schwering (1988) concludes that very small grains are the most probable explanation for the 12 and 25 μm excess in the Magellanic Clouds.

While the presence of very small grains is easily demonstrated in systems as diverse as bulge-dominated quiescent spirals and Magellanic type irregulars, the question of variations in the relative abundance of small and large grains is harder to address. The simplest estimator of this abundance may be the ratio of 12 μm to far infrared (60 and 100 μm) emission, introduced by Ryter *et al.* (1987) and used by Boulanger *et al.* (1988). Its use is based on the assumption that in spiral galaxies the 12 μm emission is dominated by very small grains whereas the far infrared is dominated by large grains. At constant excitation conditions this estimator is probably reliable, except that it cannot be easily related to a mass ratio without a detailed model of the emission spectrum of very small grains, or an estimate of their emissivity at 12 μm . More specifically, we have no way to estimate the fraction of the total emission by very small grains intercepted by the 12 μm band of IRAS. As

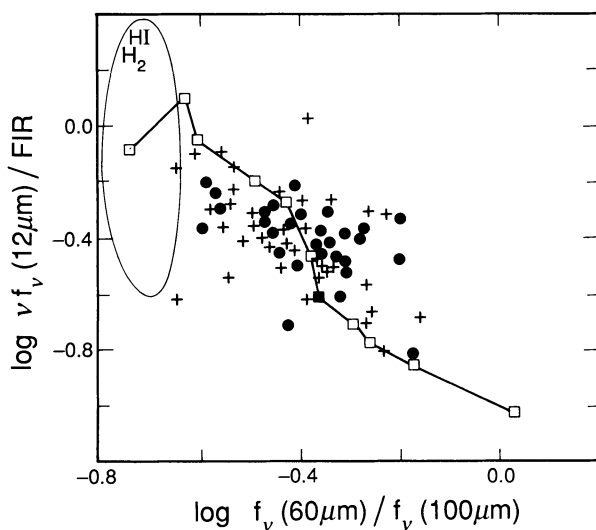


Fig. 7. IRAS color-color diagram used in estimating the abundance of very small grains compared to “standard” grains. Data from the Milky Way are represented by the oval envelope (Puget, 1989), and by squares (Boulanger *et al.*, 1988). Crosses and circles are for Virgo Cluster galaxies (Helou *et al.*, 1988). Details and discussion are given in §4.

excitation conditions change, this fraction certainly changes, as does the fraction of large grain emission in the 60 and 100 μm bands. The result is that this ratio between 12 μm and the far infrared is unlikely to be simply proportional to the mass ratio of small to large grains, and is in particular meaningless without an indication of the excitation conditions at which it is measured.

With that caveat in mind, Figure 7 plots this indicative ratio of small to large grains as a function of $f_\nu(60\mu\text{m})/f_\nu(100\mu\text{m})$, which is a reasonable index of the heating intensity (plot adopted from Ryter *et al.*, 1989). The oval to the left of Figure 7 delineates an area containing most colors measured in the Milky Way in low excitation media, both atomic and molecular (see the review by Puget (1989) for more details). The squares connected by a thin line are derived from the California Nebula measurements in Boulanger *et al.*, 1988 (see § 2.1.2 above for more details). The rest of the symbols represent the colors of spiral galaxies in the Virgo Cluster (Helou *et al.*, 1988), with the crosses corresponding to more uncertain data (mean error on each color greater than 0.1 in the log), and the filled circles to less uncertain data. This diagram argues in favor of a similar ratio of small to large grains in the Milky Way and in the Virgo Cluster spirals. It would also appear from Figure 7 that this ratio varies from galaxy to galaxy by factors of three or so, which is a smaller variation than that observed within molecular cloud complexes on a scale of a few parsecs (Boulanger *et al.*, 1988; and Puget, 1989).

Boulanger *et al.* (1988) argue that the drop in the 12 μm to far infrared ratio observed in the California Nebula requires that the 12 μm emitters be destroyed as the heating intensity increases. Using the Soifer *et al.* (1989) IRAS selected sample

of galaxies (see § 2.1.1 above); Ryter *et al.* (1989) also find that the colors of galaxies in this diagram track the California Nebula color sequence. This finding reinforces the case for interpreting both data sets as reflecting the same physics, namely the interplay between emission mechanisms from small and large grain populations, and suggests that very small grains in other galaxies are similar to those observed in the Milky Way.

5. SUMMARY

The starting point of this review was the question: Is it right to assume that far infrared and submillimeter dust properties are the same everywhere? The answer suggested by available data appears to be: It is not wrong!

The IRAS color-color diagram was examined for three samples of galaxies and a collection of galactic objects. No evidence was found for singular dust properties in the Milky Way as compared to other spirals, or for significant variations in these properties among individual or classes of late-type galaxies. The far infrared broadband properties of dust are essentially the same in all objects examined.

Submillimeter data were reviewed in a search for a power-law index describing the emissivity of dust at $\lambda > 100 \mu\text{m}$. Within the uncertainties, no variations in such an index could be found among objects in the Milky Way, or among these and dust in other galaxies. The determination of this power-law index depends on how well the temperature of the emitting dust is known; it was found that evidence for the existence in galaxies of very cold dust (10 to 15 K) in large quantities is quite soft, especially with the Chini *et al.* (1986*b*) measurements still controversial.

Dust-to-gas ratios seem comparable in most galaxies to the values observed in the Milky Way (about 10^{-2} in mass), except for some notable exceptions: The Magellanic Clouds are found by Schwering (1988) to have a low dust content, in agreement with determinations at optical wavelengths; and Blue Compact Dwarf galaxies are shown plausibly by two different approaches to be dust deficient (Gondhalekar *et al.*, 1986; Hoffman *et al.*, 1989). Rather than confirming reports of radial gradients in the dust-to-gas ratio within the Milky Way and M31, their review concludes to a warning that very small grains could affect color temperatures in the cool interstellar medium where $f_{\nu}(60 \mu\text{m})/f_{\nu}(100 \mu\text{m}) \leq 0.3$.

Finally, very small grains containing less than a few hundred atoms are identified as an essential ingredient of dust in all galaxies examined in the IRAS data. The galaxy-wide average ratio of very small to "classical" grains is hard to quantify, but seems to vary less from galaxy to galaxy than it does within the Milky Way on the scale of a few parsecs.

ACKNOWLEDGEMENTS. I would like to thank Helen Knudsen, Rosanne Hernandez and the IPAC Library for help with the literature search and collection of references, Mary Ellen Barba for help with the manuscript, and François Boulanger, Chas Beichman and Tom Soifer for helpful comments on the manuscript. This work was supported as part of the IRAS Extended Mission by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National

Aeronautics and Space Administration.

REFERENCES

- Basell, D., Blitz, L., and Désert, F. X. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Becklin, E. E., and Wynn-Williams, C. G. 1987, in *Star Formation in Galaxies*, ed. C. J. Persson (Washington, D.C.: Government Printing Office), 643.
- Boulanger, F., Beichman, C., Désert, F. X., Helou, G., Pérault, M., and Ryter, C. 1988, *Ap. J.*, **332**, 328.
- Boulanger, F., and Pérault, M. 1988, *Ap. J.*, **330**, 964.
- Caux, E., Serra, G., Gispert, R., Puget, J.-L., Ryter, C., and Coron, N. 1984, *Astr. Ap.*, **137**, 1.
- Caux, E., Solomon, P. M., Mooney, T. J. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Chini, R., Kreysa, E., Mesger, P. G., and Gemund, H.-P. 1986a, *Astr. Ap.*, **157**, L1.
- Chini, R., Kreysa, E., Krugel, E., and Mesger, P. G. 1986b, *Astr. Ap.*, **166**, L8.
- Cox, P., Krugel, E., and Mesger, P. G. 1986, *Astr. Ap.*, **155**, 380.
- Cunningham, C. T., Ade, P. A. R., Robson, E. I., and Rodostitz, J. V. 1984, *M. N. R. A. S.*, **211**, 543.
- de Bernardis, P., Masi, S., Melchiorri, B., Melchiorri, F., and Moreno, G. 1984, *Ap. J.*, **278**, 150.
- de Muisson, M., and Rouan, D. 1985, *Astr. Ap.*, **143**, 160.
- deVries, H. W., Heithausen, A., and Thaddeus, P. 1987, *Ap. J.*, **319**, 723.
- Draine, B. T., and Lee, H. M. 1984, *Ap. J.*, **285**, 89.
- Eales, S. A., Wynn-Williams, C. G., and Duncan, W. D. 1989, *Ap. J.*, in press.
- Elias, J. H., et al. 1978, *Ap. J.*, **220**, 25.
- Garwood, R. W., Helou, G., and Dickey, J. A. 1987, *Ap. J.*, **322**, 88.
- Gar, W. K., Robson, E. I., and Griffin, M. J. 1988, *M. N. R. A. S.*, **231**, 55.
- Gee, G., Emerson, J. P., Ade, P. A. R., Robson, E. I., and Nolt, I. G. 1984, *M. N. R. A. S.*, **208**, 517.
- Gondhalekar, P. M., Morgan, D. H., Dopita, M., and Ellis, R. S. 1986, *M. N. R. A. S.*, **219**, 505.
- Gordon, M. A. 1987, *Ap. J.*, **316**, 258.
- Hauser, M. G., Silverberg, R. F., Stier, M. T., Kelsall, T., Gezari, D. Y., Dwek, E., Walser, D., Mather, J. C., and Cheung, L. H. 1984, *Ap. J.*, **285**, 74.
- Helou, G., Soifer, B. T., and Rowan-Robinson, M. 1985, *Ap. J. (Letters)*, **298**, L7.
- Helou, G. 1986a, *Ap. J. (Letters)*, **311**, L33.
- . 1986b, in *Star Forming Dwarf Galaxies and Related Objects*, ed. D. Kunth, T. X. Thuan, and J. Tran Thanh Van, (Gif-sur-Yvette: Editions Frontieres), p. 319.
- Helou, G., Khan, I. R., Malek, L., and Boehmer, L. 1988, *Ap. J. Suppl.*, **86**, 151.
- Hildebrand, R. H. 1983, *Quart. J. R. A. S.*, **24**, 267.
- Hoffman, G. L., Helou, G., Salpeter, E. E., and Lewis, B. M. 1989, *Ap. J.*, in press.
- Impey, C., Wynn-Williams, L. G., and Becklin, E. E. 1989, in preparation.
- Infrared Astronomical Satellite (IRAS) Catalogs and Atlases: Explanatory Supplement 1988*, eds. C. A. Beichman, G. Neugebauer, H. J. Habing, P. E. Clegg, and T. J. Chester, (Washington, DC:NASA).
- Infrared Astronomical Satellite (IRAS) Catalogs and Atlases: Point Source Catalog*, 1988, Joint IRAS Science Working Group (Washington, DC:NASA).
- Jaffe, D. T., Becklin, E. E., and Hildebrand, R. H. 1984, *Ap. J.*, **285**, L31.
- Jaffe, D. T., Hildebrand, R. H., Keene, J., and Whitcomb, S. E. 1983, *Ap. J. (Letters)*, **273**, 189.
- Jura, M., Kim, D. W., Knapp, G. R., and Guhathakurta, P. 1987, *Ap. J. (Letters)*, **312**, L11.
- Knapp, G. R., Helou, G., and Stark, A. A. 1987, *Ap. J.*, **94**, 54.
- Koiike, C., Hasegawa, H., and Hattori, T. 1987, *Ap. Space Sci.*, **134**, 95.
- Koorneef, J. 1982, *Astr. Ap.*, **107**, 247.
- Kunth, D., and Sèvre, F. 1986, in *Star Forming Dwarf Galaxies and Related Objects*, ed. D. Kunth, T. X. Thuan, and J. Tran Thanh Van, (Gif sur Yvette: Editions Frontieres), p. 331.
- Lange, A. E. et al. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Laureijs, R. J., Chlewicki, G., Clark, F. O., Wesselius, P. R. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Leene, A. 1986, *Astr. Ap.*, **154**, 295.

- Loughran, L., McBreen, B., Fazio, G. G., Rengarajan, T. N., Maxson, C. W., Serio, S., Sciortino, S., and Ray, T. P. 1986, *Ap. J.*, **303**, 629.
- Low, F. J., et al. 1984, *Ap. J. (Letters)*, **278**, L19.
- Maloney, P., and Black, J. H. 1988, *Ap. J.*, **325**, 389.
- Marston, A. P. and Dickens, R. J. 1988, *Astr. Ap.*, **193**, 27.
- Mathis, J. S., Mezger, P. G., and Panagia, N. 1983, *Astr. Ap.*, **128**, 212.
- Mezger, P. G., Chini, R., Kreysa, E., and Gemund, H.-P. 1986, *Astr. Ap.*, **160**, 324.
- Mezger, P. G., Chini, R., Kreysa, E., and Wink, J. E. 1987, *Astr. Ap.*, **182**, 127.
- Mezger, P. G., Chini, R., Kreysa, E., Wink, J. E., and Salter, C. J. 1988, *Astr. Ap.*, **191**, 44.
- Mezger, P. G., Mathis, J. S., and Panagia, N. 1982, *Astr. Ap.*, **105**, 372.
- Nordh, H. L., van Duinen, R. J., Fridlund, C. V. M., Sargent, A. I., Aalders, J. W. G., and Beintema, D. 1984, *Astr. Ap.*, **131**, 221.
- Pajot, F., Boissé, P., Gispert, R., Lamarre, J. M., Puget, J.-L., and Serra, G. 1986, *Astr. Ap.*, **157**, 393.
- Péroult, M., Boulanger, F., Puget, J.-L., and Falgarone, E. 1988, preprint.
- Persson, C. J. L., and Helou, G. 1988, *Ap. J.*, **314**, 513.
- Puget, J. L. 1989 in *IAU Symposium 185, Interstellar Dust*, eds. L. J. Allamandola and A. G. G. M. Tielens, (Dordrecht: Kluwer), p. 119.
- Rengarajan, T. N., Fazio, G. G., Maxson, C. W., McBreen, B., Serio, S., and Sciortino, S. 1986, *Irish Astron. J.*, **17**, 313.
- Rickard, L. J., and Harvey, P. M. 1983, *Ap. J. (Letters)*, **268**, L7.
- Richardson, K. J., White, G. J., Gee, G., Griffin, M. J., Cunningham, C. T., Ade, P. A. R., and Avery, L. W. 1985, *M. N. R. A. S.*, **216**, 713.
- Rowan-Robinson, M., Lock, T. D., Walker, D. W., and Harris, S. 1986, *M. N. R. A. S.*, **222**, 273.
- Rowan-Robinson, M., and Crawford, J. 1986, in *Light on Dark Matter*, ed. F. P. Israel, (Dordrecht: Reidel), p. 421.
- Ryter, C. E., and Puget, J.-L. 1977, *Ap. J.*, **215**, 775.
- Ryter, C., Puget, J.-L., and Péroult, M. 1987, *Ap. J.*, **186**, 312.
- Ryter, C., Helou, G., and Soifer, B. T. 1989, in preparation.
- Sanders, D. B., et al. 1987, in *Star Formation in Galaxies*, ed. C. J. Persson, (Washington, D.C.: Government Printing Office), p. 411.
- Schwartz, P. R., Thronson Jr., H. A., Lada, C. J., Smith, H. A., Glaccum, W., Harper, D. A., and Knowles, S. H. 1983, *Ap. J.*, **271**, 625.
- Schwering, P. B. W., 1988, Doctoral Thesis, University of Leiden.
- Scoville, N. Z., and Good, J. C. 1987, in *Star Formation in Galaxies*, ed. C. J. Persson, (Washington, D.C.: Government Printing Office), p. 3.
- Soifer, B. T., Sanders, D. B., Madore, B. F., Neugebauer, G., Danielson, G. E., Elias, J. H., Lonsdale, C. J., and Rice, W. L. 1987, *Ap. J.*, **320**, 238.
- Soifer, B. T., Houck, J. R., and Neugebauer, G. 1987, *Ann. Rev. Astr. Ap.*, **25**, 187.
- Soifer, B. T., Boehmer, L., Neugebauer, G., and Sanders, D. B. 1989, in preparation.
- Sodroski, T. J., Dwek, E., Hauser, M. G., and Kerr, F. J. 1987, *Ap. J.*, **322**, 101.
- Sodroski, T. J., Dwek, E., Hauser, M. G., and Kerr, F. 1989, *Ap. J.*, in press.
- Solomon, P. M., Rivolo, A. R., Mooney, T. J., Barrett, J. W., and Sage, L. J. 1987, in *Star Formation in Galaxies*, ed. C. J. Persson, (Washington, D.C.: Government Printing Office), p. 37.
- Stark, A. A., Davidson, J. A., Platt, S., Harper, D. A., Pernic, R., Loewenstein, R., Engargiola, G., and Casey, S. 1988, preprint.
- Telesco, C. M. 1988, *Ann. Rev. Astr. Ap.*, **26**, 343.
- Terebey, S. and Fich, M. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Thronson, H. A., and Bally, J. 1987, *Ap. J. (Letters)*, **319**, L63.
- Thronson, H. A., Tacconi, L., Greenhouse, M. A., Kenney, J., Tacconi-Garman, L., and Young, J. 1988, preprint.
- Tytler, D. 1987, preprint.
- Walterbos, R. A. M., and Schwering, P. B. W. 1987, *Astr. Ap.*, **180**, 27.
- Weiland, J. L., Blitz, L., Dwek, E., Hauser, M. G., Magnani, L., and Rickard, L. J. 1986, *Ap. J. (Letters)*, **306**, L101.
- Wesselius, P. R., Chlewicki, G., Laureijs, R. J. 1989, in *Interstellar Dust Contributed Papers*, eds. A. G. G. M. Tielens and L. J. Allamandola, NASA CP-3036.
- Wright, E. L. 1987, *Ap. J.*, **320**, 818.
- Young, J. S., Schloerb, F. P., Kenney, J., and Lord, S. D. 1986, *Ap. J.*, **304**, 443.