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EDITORIAL ANNOUNCEMENT

Special Issue of EJAM: The Mathematics in Renewable Energy

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The conversion, storage and distribution of energy from renewable sources drive some of the most innovative technologies with the goal to meet global energy demands and to mitigate climate change. For instance, research in photovoltaics, which combines materials science, device modelling and optics, has seen intensive growth in all areas and on all relevant scales during the past decades. The role of mathematics has been key to understand and develop novel optimised photovoltaic devices. However, while the global cumulative solar capacity is growing fast, the temporal variability of photovoltaic or wind-generated electricity from intra-day to seasonal scales constitutes a major obstacle for matching demand. Energy storage plays a major role in addressing this problem, for example via large-scale stationary battery systems, or photochemical hydrogen production.

Most importantly, the growing complexity of power distribution across coupled distribution grids constitutes perhaps the hardest current challenge. Apart from setting up the infrastructure, monitoring and planning, these increasingly complex networks pose difficult mathematical problems, relating to fluctuations in coupled energy networks, that are also impacted by market regulations.

The focus of research articles in this special issue of the *European Journal of Applied Mathematics* presents some of the mathematical challenges encountered in the prediction of power grid responses on different temporal and spatial scales and to various types of disturbances. In particular, contributions to the special issue report findings on coupled energy networks composed of gas and electric power networks of realistic size, that are coupled to stochastic fluctuations due to fluctuating demands and supplies. Others address the stability of electrical power grids, develop guiding principles for power grid operation, control and design, and perform case studies on optimisation of energy generation, taking into account sustainability goals for 2050 in Europe, under uncertain future market conditions. In addition, a modelling framework that enables researchers to quantify degradation effects of lithium-ion battery cells and a new approach to model parabolic trough power plants are presented.

Such complex problems require a broad spectrum of mathematical tools and perspectives as well as numerical methods that span from stochastic to deterministic and from particle-based to continuum descriptions. They include linear response theory for fluctuation-driven networked dynamical systems to provide analytical descriptions of the dynamical responses of networks, which allows for the interpretation of spatiotemporally distributed response patterns. Other tools include optimisation techniques under ambiguity using Wasserstein balls as ambiguity sets, the reduced basis method to efficiently investigate parameterised nonlinear system of partial differential equations, classical methods featuring optimisation problems with nonsmooth convex constraints, stability theory and rigorous mathematical studies in partial differential equations.

The first article (Zhang and Timme) in this issue addresses the response of networked dynamical systems. These are systems of dynamical units coupled via nontrivial interaction topologies and driven by often irregular and distributed fluctuating input signals modelling power inputs and outputs power fluctuations. The collective response of networked dynamical systems to such fluctuations depends on the location and type of driving signal, the interaction topology and other factors and is largely unknown to date. To predict, control or mitigate the equally fluctuating and distributed responses of such networks, the authors review linear response theory (LRT) that relates sufficiently small time-dependent driving signals to time-dependent responses. The theory approximates a resulting system dynamics near some operating point to first order in the strength of the driving signals.

To systematically characterise fluctuating responses that are distributed across meshed networks, and to understand qualitatively different response regimes, the article offers a framework of linear response theory for uncovering spatiotemporal response patterns in systems of units that simultaneously interact via intricate network topologies and evaluates it both for stationary and non-stationary responses for power grid models. The article employs asymptotic approximation to make such spatiotemporal response patterns interpretable as a function of parameters, such as the driving frequency. Moreover, similar asymptotic approaches yield qualitative statements regarding the first occurrence of large excursions during transient periods of the responses.

The second contribution (Schäfer et al.) investigates the stability of the electrical power system with the focus on voltage dynamics, which has largely been neglected in the literature, but, as shown here, is necessary for stable operation of the electrical power grids. Inspired by results in the literature that show that applying secondary frequency control to the well-known second-order electric network model can deeply modify the dynamics of a network, the authors couple the frequency secondary control to the electric network that involves the voltage dynamics. A mathematical model of the power grid is presented, where the power grid is considered as networks of synchronous machines each of which is controlled by frequency controllers. Based on linear stability analysis and bulk dynamics, the authors quantify the stability of the power grid. In particular, the linear stability analysis of the network shows that the frequency secondary control guarantees the stability of a particular electric network. Moreover, while only secondary frequency control is considered, numerical analysis shows a stabilising effect on the voltage dynamics.

Numerical simulations carried out for the perturbed network and in the presence of the secondary control show that the secondary control actually plays the role of a primary control for the voltage. The frequency secondary control after a perturbation dampens the variation of the voltage and stabilises it to a new steady value different than the original one. These results showcase how voltage stability and secondary frequency control needs to be considered in other power system stability analyses. In contrast, although including only the primary frequency control might stabilise the frequency, it cannot guarantee voltage stability. The research suggests to complement the secondary frequency control, which acts as an effective primary voltage control, by a secondary voltage control to bring the voltage back within its operational boundaries. Their model sets the stage to investigate how precisely network topology and heterogeneous parameters affect the voltage and frequency stability and return times, which are still open questions.

When designing capacity expansion models, while taking into account several sources of uncertainty such as wind and solar energy that entails more risky investment, stochastic programming models are not suitable for providing sound decision-making because uncertain future market conditions are not directly observable. To address this issue, Borges et al. use distributionally robust stochastic optimisation (DRSO), where the probability is considered as an additional decision variable, to be chosen among many distributions in a certain ambiguity set.

A further concern for the decision-maker is how to deal with random realisations that are influenced by the decisions. For capacity expansion problems, this issue is relevant for the case where the Government subsidises the cost of equipment related to wind or solar power. A large future investment on renewables could then result in the Government lowering its subsidised fraction.

With a DRSO model, endogenous probability distributions imply a variation of the ambiguity set with the decision variable. This is the model explored in this work, where the authors focus on decision-dependent distributionally robust two-stage stochastic optimisation (ddDR2SO). A ddDR2SO problem outputs the optimal value for the probability distribution together with the optimal decisions. The connection between the endogenous probability and the decision variable can be explicitly given through constraints in the ambiguity set, or indirectly, by the fact that the optimisation is performed jointly on both type of variables. Regarding ambiguity sets, one possibility is to look for probability distributions whose first- and second-order moments are close to those of some exogenous empirical estimation. Typical measures of closeness of measures are the phi-divergence distance and the Wasserstein balls.

In the spectrum of possible choices for the probability distribution that enters the optimisation problem, the modelling paradigms of stochastic programming and DRSO are seen as positioned at opposite extremes with respect to a certain distance to a nominal probability distribution. The latter takes the worst case over the ambiguity set of probabilities, while with the former there is only one possible choice. Borges et al. propose an in-between paradigm, which is both optimistic and pessimistic to certain degrees. For ambiguous two-stage stochastic programmes, the proposed ddDR2SO model defines a robustified expected recourse function using probabilities in a Wasserstein ball. The novelty is that, instead of taking a nominal probability, the ball centre is a considered variable. This additional variable is minimised in the first stage over a simple convex set, for example the convex hull of several nominal probabilities taken as a reference.

Finally, Borges et al. consider discrete distributions, with a finite number of scenarios, and reformulate the new model as a bilinear programming problem. This structure is suitable for decomposition: bilinearity appears only in the objective function, and the feasible set has as many convex nonsmooth constraints as scenarios in the problem. The interest of the decomposition in a ddDR2SO framework is that, instead of solving one large, difficult, problem with many bilinear terms, each iteration solves a much easier problem, having less bilinear terms, less variables and a polyhedral feasible set, and thus contributes to computational efficiency. The methodology is illustrated on a case study for planning investments in energy generation, under uncertain market conditions. The simplified model considers the whole of Europe over the horizon 2020-2050, taking into account the progressive decommissioning of thermal power plants and the increasing proportions of renewable technologies that are foreseeable for the power mix.

The combined energy networks are considered subject to changing energy demands and supplies. Here, the unpredictable and volatile energy sources are complemented with traditional means of production as well as possibly additional large-scale storage. Fokken et al. are interested in a full and efficient simulation of both the gas and the power network as well as the simulation of the stochastic demand, respectively supply. This allows for a prediction at all nodes as well for studying dynamic effects of changing supplies and demands in the network. A major concern when coupling gas and power networks is guaranteeing a stable operation even at times of stress due to fluctuating loads. The propagation of possible uncertain loads on the power network, and its effect on the gas network has been subject to recent investigations. The numerical approach presented here relies on underlying models with established power flow, gas flow and stochastic demand models.

Power flow is typically modelled through prescribing real and reactive power at nodes of the electric grid. Their values are obtained through a nonlinear system of algebraic equations. Supply and demand can be time-dependent, which requires having to frequently resolve the nonlinear system. While propagation of electricity is typically assumed to be instantaneous as in the power flow equations, the propagation of gas in networks has an intrinsic spatial and temporal scale. While following an approach based on hyperbolic balance equations, the description presented here allows for the prediction of gas pressure and gas flux at each point in the pipe as well as in the nodes of the network. Both quantities are relevant to assess possible stability issues as well to allow for coupling towards the electricity network. Recent results on modelling of the prediction of the electricity demand are used to simulate the uncertain power fluctuations. The numerical approach presented here has the advantage of being more easily extensible and includes stochastic power demands in the power flow network setting to investigate relevant scenarios.

In Gugat et al., a semilinear model for gas pipeline flow from the quasilinear isothermal Euler equations is derived. The corresponding stationary states for the case of pipes with constant slopes are presented, and it is shown that for any given finite time horizon if the continuous initial data are sufficiently small, a continuous transient solution of the semilinear system exists. Interestingly, the velocity of the gas remains below given a priori bounds and the pressure of the gas remains within a prescribed interval, which is typically the case in the operation of gas pipelines. Moreover, the authors show that under certain smallness assumptions, the continuous solution exists globally in time.

Continuous solutions are those where the system is controlled from a given stationary state to another desired stationary state in such a way that the state constraints are satisfied everywhere throughout the process. In terms of the physical variables, the state constraints are box constraints for the pressure and an upper bound for the absolute value of the Mach number. These state constraints can be transformed to linear constraints in terms of the Riemann invariants. A numerical scheme based on the midpoint rule to integrate the Riemann invariants on the characteristics is introduced and numerical simulations supporting the theoretical analysis are performed. While the analysis presented here covers models of ideal gases, the authors suggest that extensions to models of non-ideal gases would involve analysing quasilinear systems that occur in many applications.

The research focus in Tacke et al. concerns the development of a model framework for intercalation batteries and the application of homogenisation methods to ultimately simplify the model to enable efficient numerical simulations of many loading cycles for describing long-time degradation effects.

The mathematical model framework for an intercalation battery, consisting of multi-phase porous electrodes, is developed on the basis of non-equilibrium thermodynamics. Special emphasis is made on thermodynamic consistency of the transport equations and their respective reaction boundary conditions by employing the same chemical potential function throughout the model. Periodic homogenisation theory is applied to derive a general set of partial differential equations (PDEs) for the porous battery cell. Spherical symmetry of the intercalation particles is further employed, as well as a one-dimensional approximation of the macro-scale. This approach yields important characteristics of an intercalation battery, such as the cell voltage as a function of the status of charge. Moreover, battery degradation is considered in terms of cycle number-dependent parameters. To efficiently simulate degradation effects for the electrochemical battery model, a model reduction technique, known as the reduced basis method, was used together with an empirical operator interpolation technique. In this way, capacity curves over the number of cycles that exhibit a dramatic speedup, as compared to full numerical simulations of the same implementation, were achieved.

The final contribution considers one of the most promising types of solar power plants, the so-called parabolic trough power plants, which are used on an industrial scale for long-distance transportation of electrical energy. The article by Gasser et al. presents a mathematical model of such a power plant. It consists of a network of tubes for the heat transport. While the resulting mathematical problem involves a system of nonlinearly coupled PDEs on a network and presents mathematical and numerical challenges, the authors use several simplifications that allow for the optimisation of power output for realistic network settings.

The collection of contributions clearly demonstrates the interdisciplinary nature of the field of renewable energy networks as well as the many intricate mathematical, conceptual and methodological challenges it continues to uncover. We thank all contributors for showing us the way forward and for offering highlights for future research directions.