Book reviews

The final part, Part VII, collects together papers from the early 1970s on R/S analysis and its applications. This statistical procedure was introduced to analyse long-range dependence, although, as the author has noted more recently, there are processes, known as meso-diffusive processes, where R/S analysis fails to detect long-term correlations. Again, this work had its origins in hydrology, but the subsequent papers include applications to areas as diverse as sunspot activity and the Chandler wobble of the Earth's pole.

The book ends with a very substantial bibliography, of which over 10 pages detail the author's own publications.

Written in Mandelbrot's unique, thought-provoking style, these papers and commentaries contain a wealth of ideas which will appeal to mathematicians, statisticians, scientists and economists alike. It is a book both for dipping into and for detailed study, making readily accessible seminal papers which contain ideas that are as relevant now as when first published. The substantial new contributions provide ample evidence that that the subject is very much alive and that the father of fractal geometry continues to brim with ideas that will ensure the subject's vitality for many years to come.

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JAMES, G. AND LIEBECK, M. *Representations and characters of groups*, 2nd edn (Cambridge University Press, 2001), 0 521 00392 X (paperback), £24.95 (US\$35.95), 0 521 81205 4 (hardback), £80 (US\$120).

This book provides an introduction from a character-theoretic point of view to the representation theory of finite groups over the complex numbers. Starting from only the most basic group theory and linear algebra, it covers a wide range of topics, from pure representation and character theory, through abstract group theory, to the foreign field of molecular vibrations. The expected reader is either a relatively advanced undergraduate or a beginning graduate student.

The contents of the book fall roughly into three parts. Chapters 1–11 introduce the basic notions and theorems of group representation theory. After recalling a little preliminary material from abstract algebra, the authors present group representations, the group algebra FG, Maschke's theorem and Schur's lemma. The theory is very well illustrated throughout by a number of examples, and the reader is encouraged too by a liberal number of exercises at the end of each chapter. The difficulty of these exercises varies, but all reinforce or provide further examples to the material in each chapter. Complete answers are provided at the back of the book.

Chapters 12–24, which form the heart of the book, are concerned with character theory. After defining characters and their inner products, the character table of a group is introduced and orthogonality relations studied. The authors supply the reader with many techniques and methods for calculating characters of a given group. These range from lifting characters from normal subgroups, through restriction and induction, to using arithmetic properties of characters. A discussion of real representations and real characters is included not only as an adjunct to the complex theory, but to illustrate the use of character theory in abstract group theory through the Frobenius–Schur count of involutions and the Brauer–Fowler theorem. Again there are many examples throughout the text and exercises for the reader at the end of each chapter.

Chapters 25–32 illustrate and apply character theory in a variety of interesting examples. Character tables of groups of order pq and of some p-groups are calculated, as are those for the simple group of order 168 and the general linear groups GL(2,q) with entries in any finite field \mathbb{F}_q . By the end of these chapters, the reader has the character tables for all groups of order less than 32 and of all the simple groups of order less than 1000 (there are five non-abelian

Book reviews

simple groups in this list). A brief introduction to permutation characters is given, allowing the authors to present the beginnings of the representation theory of the symmetric group.

There follow two chapters on the applications of characters in abstract group theory: finding subgroups of a group through its character table; the Brauer programme to determine finite simple groups containing an involution with a prescribed centralizer; and Burnside's theorem on the solvability of groups of order $p^a q^b$.

Finally, the authors discuss the use of group representations and characters in determining the solutions of the equations of motion of a vibrating molecule. Each chapter concludes with a set of exercises for the reader. The above chapters on permutation representations and on the character table of GL(2,q) are new to the second edition. In addition, Clifford's theorem has been added to the chapter on normal subgroups and lifting characters, and the Brauer–Fowler theorem and Brauer programme to the chapters on real representations and applications in abstract group theory, respectively.

The reviewer found this to be a clearly and carefully written book, which he will recommend to all students seeking an introduction to group representation theory. The authors write in a down-to-earth style, provide a large number of examples throughout, and give very useful content summaries at the end of each chapter. Parts of the book certainly could form the basis for a one- or two-term undergraduate course in representation theory, whilst some of the later chapters might be ideal as the kernel of undergraduate projects. In summary, the additions to the second edition only improve what was already an excellent text.

I. G. GORDON