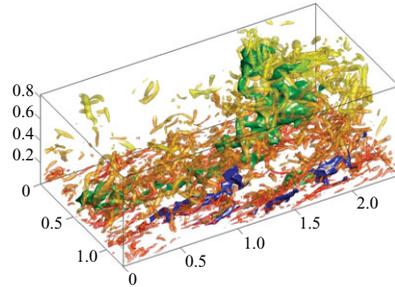


# Revisiting Taylor's hypothesis

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Taylor's hypothesis, relating temporal to spatial fluctuations in turbulent flows is investigated using powerful numerical computations by del Álamo & Jiménez (*J. Fluid Mech.*, 2009, this issue, vol. 640, pp. 5–26). Their results cast doubt on recent interpretations of bimodal spectra in relation to very large-scale turbulent structures in experimental measurements in turbulent shear flows.

**Keywords.** Taylor's hypothesis, turbulence, channel flows, convection velocity

## 1. Introduction

With the rise of modern computational power, it is currently an exciting time for fluid mechanics. Large-scale computer simulations of fluid flow allow us to probe long-standing scientific questions in a fashion different from traditional experimental techniques. Using simulation, access to complete data sets describing the evolution of entire flow fields often sheds new light on old problems and guides further experimental investigation. The article by del Álamo & Jiménez (2009, this issue, vol. 640, pp. 5–26), which calls into question Taylor's famous hypothesis, serves as a case in point.

Turbulence is the process by which a broad range of structures and complexity arises out of a fluid system, a system which is, after all, governed by a relatively simple, yet crucially nonlinear, set of mathematical equations. Turbulence is ubiquitous, and often plays a key role in both natural and engineered fluid systems. Historically, simultaneous resolution of spatial and temporal components of turbulence has presented a challenge to laboratory experimentalists, because streamwise interference from multiple probes often prevents the reliable measurement of full spatio-temporal data. Alternative non-intrusive laser diagnostic techniques provide adequate spatial resolution, but are limited in temporal resolution. For this reason, Taylor (1938) proposed his famous hypothesis relating the spatial and temporal characteristics of turbulence. Taylor reasoned that if the turbulence intensity  $u'$  is small compared to the mean flow speed  $U$ , then the temporal response at a fixed point in space can be viewed as the result of an unchanging spatial pattern convecting uniformly past the point at velocity  $U$ .

For grid-generated compressible turbulence, Taylor's hypothesis performs well for the divergence-free part of the flow, but does not accurately represent the dilatational, i.e. inherently compressible part (Lee, Lele & Moin 1992). Also, it is well known that Taylor's hypothesis breaks down in the case of flows containing high

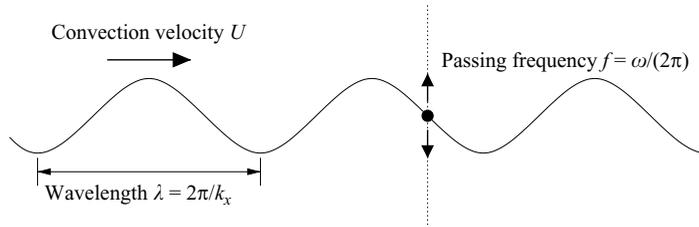


FIGURE 1. An elementary schematic of a single Fourier mode convected by uniform flow. The wave produces oscillations of frequency  $\omega$  as it passes by a fixed station (vertical dashed line).

shear (Lin 1953). Nevertheless, many experimentalists apply this hypothesis when complete spatio-temporal data is unavailable. For wall-bounded flows, the result has been ‘bimodal energy spectra’ (Hites 1997; Kunkel & Marusic 2006), which seem to contradict the theoretical  $k_x^{-1}$  spectrum,  $k_x$  being the streamwise wavenumber, predicted by Townsend (1976). Focusing on this case, del Álamo & Jiménez show from their simulation data that the long wavelength mode may be an artifact of Taylor’s hypothesis, and that the true spectrum matches theory more closely. Furthermore, they suggest a novel correction to Taylor’s hypothesis accounting for frequency-dependent convection velocities.

## 2. Overview

As shown in figure 1, the convection velocity of a single Fourier mode is determined from its frequency and wavelength as  $U = f\lambda = \omega/k_x$ . For turbulent flows, this definition is ambiguous because a spectrum of frequencies (shown as a shaded region in figure 2) is observed at each wavenumber  $k_x$ . Taylor’s hypothesis (more appropriately an approximation) removes this ambiguity by assuming that the convection velocity is independent of both frequency and wavenumber and is simply equal to the local mean velocity. One major contribution of del Álamo and Jiménez is to examine the consequences of this assumption in the case of turbulent channel flow, using full three-dimensional space–time data taken from simulations. In the highly sheared region near the wall, del Álamo and Jiménez found that modes with long wavelengths in fact propagate faster than the local mean velocity. This is consistent with the idea that these longer scales arise from very large-scale structures (with heights on the order of the width of the channel) whose convection velocity scales with the bulk mean velocity instead of the local mean. This is illustrated in the figure beside the title, where the red and yellow vortices propagate with the local mean profile at each vertical distance from the wall, whereas the larger purple and green velocity structures propagate at a speed that is close to the average of  $U(y)$  over their wall-normal extent.

Because these fast modes produce high-frequency signals, Taylor’s approximation aliases them to shorter wavelengths, creating artificial maxima and minima in the spatial energy spectrum. Figure 10(a) of del Álamo and Jiménez’s article shows that the true pre-multiplied energy spectrum appears flatter, reminiscent of Townsend’s  $k_x^{-1}$  theory, than the spectrum computed using Taylor’s approximation. Moreover, the spectrum computed via Taylor’s approximation matches closely the bimodal shape of spectra derived from experimental data, suggesting that the bimodal spectra found in

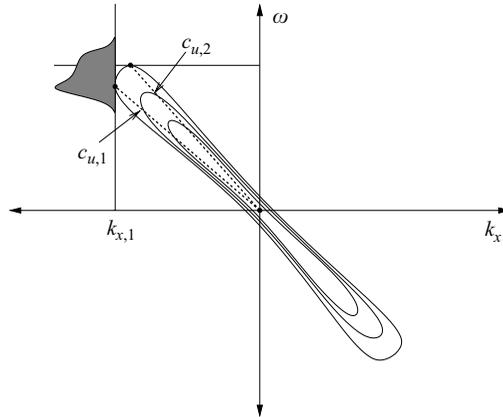


FIGURE 2. A sketch of the spatio-temporal spectrum of a turbulent flow. The convection velocities  $c_{u,1}$  and  $c_{u,2}$  shown are determined by the location of the maxima of spectra taken along constant  $k_x$  and  $\omega$ , respectively. Del Álamo and Jiménez's definition (not shown) corresponds to the centre of gravity of the shaded spectrum.

recent experiments may be explained by an artifact of Taylor's approximation rather than a physical effect. It is important to note, however, that the absence of a long-wavelength peak in the spatial energy spectrum is not necessarily inconsistent with the notion that energetic outer structures (such as packets of hairpin vortices) extend into the buffer layer. It does suggest, however, that their energy is more broadband than commonly supposed.

Several schemes have been proposed to define the convection velocity unambiguously, while retaining its dependence on both wavenumber and frequency. One possibility, proposed by Wills (1964), considers the maximum of the frequency spectrum observed along a constant wavenumber. This maximum defines a frequency which can be used to define a convection velocity  $c_{u,1}$  (figure 2). Alternatively, the maximum of the spectrum of wavenumbers at a constant  $\omega$  may be used to define a slightly different velocity  $c_{u,2}$  (in general,  $c_{u,1} \neq c_{u,2}$ ). Other definitions for convection velocity are possible as reviewed by Hussain & Clark (1981) and Goldschmidt, Young & Ott (1981). A common element of these previous definitions is that they rely on full  $k_x - \omega$  spectral information which, presents significant experimental difficulties and thus motivates experimentalists to apply Taylor's approximation in the absence of alternatives.

Del Álamo & Jiménez also suggest a physically motivated definition of convection velocity which depends on spectral information in only one direction (either space or time) and on a local derivative in the remaining direction. It seeks to minimize the variance of the total or derivative, finding the reference frame in which waves experience the least amount of change. For example, the single propagating wave of figure 1 is 'frozen' in a reference frame moving with the mean velocity  $U$ . Here, Taylor's hypothesis applies exactly and the variance of the total derivative would vanish. However, for a turbulent flow, the variance may be minimized, but not completely eliminated, thus providing a convenient measure of the validity of Taylor's hypothesis.

### 3. Future

Bimodal spectra derived from channel flow experiments have been the cause of much recent interest as they seem to provide additional concrete evidence of energetic very large-scale flow structures. Disturbingly, the long-wavelength spectral peak has been absent from numerical simulations, questioning their credibility. Using full spatio-temporal simulation data, however, del Álamo & Jiménez (in press) have shown that experimental bimodal spectra may well be artifacts of Taylor's approximation, and that when aliasing effects are taken into account, the resulting spectra agree better with those measured from simulation and predicted by theory. Clearly, experimental shear flow data based on Taylor's approximation will now need to be reconsidered. Also, while the existence of very large-scale structures is well known, flatter spectra imply that their energy shifts to longer wavelengths. This theoretical implication encourages future testing requiring higher Reynolds number simulations or long spatio-temporal experimental measurements.

While del Álamo and Jiménez pass a word of caution on direct application of Taylor's approximation to shear flows, their proposed correction should be valuable in a number of experimental and numerical settings. For example, this correction may be useful in the design of high-performance non-reflecting numerical boundary conditions. This is of paramount importance to aeroacoustic simulations where the sound radiated, while loud to human perception, actually comprises a very small percentage of the total aerodynamic energy of a flow. Numerical errors at an outlet boundary can overwhelm the physically generated sound and lead to gross mispredictions. This has led to the development of aeroacoustic boundary conditions estimating a convection velocity (Wang, Lele & Moin 1996) similar to del Álamo and Jiménez's correction.

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