Macroeconomic Dynamics (2023), **27**, pp. 482–505
doi:10.1017/S1365100521000493

CAMBRIDGE UNIVERSITY PRESS

ARTICLE

Transitional dynamics of the saving rate and economic growth

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Abstract

We estimate the relationship between GDP per capita growth and the growth rate of the national saving rate using a panel of 130 countries over the period 1960–2017. We find that GDP per capita growth increases (decreases) the growth rate of the national saving rate in poor countries (rich countries), and a higher credit-to-GDP ratio decreases the national saving rate as well as the income elasticity of the national saving rate. We develop a model with a credit constraint to explain the growth-saving relationship by the saving behavior of entrepreneurs at both the intensive and extensive margins. We further present supporting evidence for our theoretical findings by utilizing cross-country time series data of the number of new businesses registered and the corporate saving rate.

Keywords: Economic growth, saving rate, credit constraints, entrepreneurs

JEL Classifications: D9; E2; O1

1. Introduction

There is wide evidence that a country's saving rate first rises then falls as the economy grows. In other words, the saving rate exhibits a hump-shaped over time (see Antràs, 2001). To explain this stylized fact, the literature has modified neoclassical models to include non-homothetic preferences, adjustment costs, and structural changes among others (e.g. Christiano, 1989; King and Rebelo, 1993; Laitner, 2000; Chen et al., 2006). More recently, Buera and Shin (2013) modified a neoclassical model to include heterogeneous agents and credit constraints to show that a country's saving rate follows a hump-shaped transitional dynamics after a reform that eliminates taxes and subsidies that distort the allocation of resources. This paper contributes to the literature as follows. First, we verify the hump-shaped saving rate of countries and the role of credit constraints by a large panel data analysis. Second, we build a theoretical model with a credit constraint to explain how the number of entrepreneurs and their saving behavior induce the hump-shaped relationship between the national saving rate and GDP per capita. Lastly, we provide supporting evidence for this mechanism by utilizing cross-country time series data of the number of new businesses registered and the corporate saving rate.

We start our analysis by estimating the relationship between per annum GDP per capita growth and the per annum growth rate of the national saving rate using panel data that cover 130 countries over the period 1960–2006.[1] The panel model estimates show that GDP per capita growth significantly increases the growth rate of the national saving rate in poor countries. However, the opposite is the case in rich countries. The estimated effects are quantitatively large. To illustrate their size, consider a low-income country with a GDP per capita (PPP-based) of USD 1000. For this country, our estimates suggest that a 1 percentage point increase in GDP per capita growth increases the growth rate of the national saving rate by about 4 percentage points. On the other hand, for a high-income country with GDP per capita of USD 50,000 a 1 percentage point increase in GDP per capita growth decreases the growth rate of the national saving rate by about 2 percentage points.

Furthermore, we find that the credit-to-GDP ratio (the GDP share of domestic credit to the private sector) significantly decreases the national saving rate: a 1 percentage point decrease in the growth rate of the credit-to-GDP ratio increases the growth rate of the national saving rate by around 0.2 percentage points. In addition, the effect of GDP per capita growth on the growth rate of the national saving rate decreases when the credit-to-GDP ratio is higher. So much so, that in countries with a low credit-to-GDP ratio GDP per capita growth increases the growth rate of the national saving rate, while in countries with a high credit-to-GDP ratio the opposite is the case.

To explain our empirical findings, we develop a model that introduces intertemporal saving decisions into the overlapping generations model by Matsuyama (2004).[2] The credit constraint in the Matsuyama model gives rise to a positive rent for entrepreneurial activities but not all agents can become entrepreneurs in the presence of indivisible investment. The ex ante identical young are endogenously divided into *entrepreneurs* and *investors*. In the Matsuyama model and its extensions (e.g. Kikuchi and Stachurski, 2009; Kikuchi and Vachadze, 2015; Kikuchi et al., 2018), however, agents consume only when old and saving is inelastic (the young save their entire wage), that is, saving is independent of the credit constraint and the wealth.[3]

Once we allow agents to consume in both periods in the Matsuyama model, both the credit constraint and the wealth affect saving of the young. This requires us to derive an equilibrium that is compatible with preferences of all agents, that is, both entrepreneurs and investors. The entrepreneurial rent now gives entrepreneurs incentives to save more than investors to overcome the credit constraint and fund their investment projects.[4] Hence, tighter credit constraints increase the saving rate as well as the income elasticity of the national saving rate. When the wealth increases, more agents can become entrepreneurs as they require less external finance. On the other hand, when the wealth increases, entrepreneurs save less. When the wealth is low, the extensive margin dominates the intensive margin; the opposite is true when the wealth is high. Because of this interplay of the intensive and extensive margins, GDP per capita growth increases the national saving rate in poor countries, while the opposite is true in rich countries.

To provide empirical support for the mechanism of the theoretical model, we use cross-country time series data of the number of new businesses registered and the corporate saving rate. The number of new businesses registered is a proxy for the extensive margin in the theoretical model. The corporate saving rate is a proxy for the intensive margin. Our panel model estimates show a significant positive effect of GDP per capita growth on the growth rate of new businesses registered, and that this effect is significantly larger for poorer countries. Moreover, GDP per capita growth has a significant negative effect on the growth rate of the corporate saving rate; and more so in richer countries.

These empirical results, which are consistent with the mechanism of the theoretical model, suggest that in poor countries the extensive margin is dominant—saving at the intensive margin plays only a minor role. It is the dominance of the extensive margin that gives rise to a significant positive effect of GDP per capita growth on the national saving rate in poor countries. On the other hand, in rich countries, the intensive margin dominates the extensive margin. That is why in rich countries there is a significant negative effect of GDP per capita growth on the growth rate of the national saving rate: in countries where GDP per capita is already high, entrepreneurs rely less on external funds so that when GDP per capita increases even more, there is a large decrease in the corporate saving rate and only a small increase in the number of new businesses registered.

Our analysis also speaks to the literature on the role of financial development. One view in this literature is that credit constraints inhibit capital accumulation by preventing a more efficient

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

allocation of credit to investment.[5] The other view is that the effects of credit constraints on savings rates are positive. In particular, it is well known from the life-cycle literature that credit constraints may increase saving rates (e.g. Bewley, 1986; Deaton, 1991; Aiyagari, 1994; Levine and Zame, 2002). Closely related to our paper with regard to the effects that credit constraints have on saving are Jappelli and Pagano (1994) and Ghatak et al. (2001). Jappelli and Pagano (1994) examines a three-period overlapping generations model, in which agents work, only when middle-aged, but consume in all three periods. When the credit constraint is binding, the consumption of the young is sub-optimal; however, a tighter credit constraint raises the saving of the middle-aged. In the presence of this trade-off, the authors show the existence of an optimal level of credit constraints. In a similar vein, Ghatak et al. (2001) analyzes a two-period overlapping generations model, with moral hazard in the labor market and transaction costs in the credit market. They show that higher transaction costs in the credit market induce the young to work harder. The increase in the work-effort by the poor young allows them to overcome the transaction costs and enjoy entrepreneurial rents when old.

The credit constraint in our model, as in Jappelli and Pagano (1994) and Ghatak et al. (2001), has a positive effect on the natinoal saving rate. However, our mechanism is different. In Jappelli and Pagano (1994), loans to consumers are facilitated between generations and consumers do not change their behavior, even in the presence of a binding credit constraint. In our model, loans are facilitated within one generation, between investors and entrepreneurs, and the young agents have dynamic incentives to save more to become entrepreneurs. The young supply extra effort to become self-financed entrepreneurs in the Ghatak et al. (2001) model, while in our model they become entrepreneurs through thrift alone.[6]

In terms of the mechanism that generates the hump-shaped saving rate, Buera and Shin (2013)'s model is closest to ours. In their model after a reform eliminates distortions in the resource allocation, the economy transits from one steady state to another.[7] Initially, the productive, high-saving entrepreneurs account for only a small fraction of the national income, but over time, they start to account for a larger fraction, and the national saving rate rises. Eventually, the saving rates of the entrepreneurs start to fall as they are less likely to be credit-constrained and face the diminishing marginal returns. The resulting hump-shaped transitional dynamics of the saving rate arises because the reform eliminates the distortions faced by ex ante heterogeneous agents. In contrast, in our model, there is no misallocation of resources, and the credit constraint alone causes ex ante homogenous agents to behave differently; it is optimal for agents, who choose to become entrepreneurs, to save more.

The rest of the paper is organized as follows. Section 2 presents the baseline regression results. Section 3 presents an interaction model to provide further evidence for our findings. Section Section 4 develops the theoretical model. Section 5 presents empirical support for the theoretical mechanism. Section 6 concludes. Appendix A–D contain all remaining proofs and some extensions.

2. The effects of GDP per capita and the Credit-to-GDP ratio on the saving rate

We begin by estimating the average effects that growth in GDP per capita and the credit-to-GDP ratio (the GDP share of domestic credit to the private sector) have on the growth rate of the national saving rate.[8] The lower is the credit-to-GDP ratio, the tighter is the credit constraint. The econometric model is

$$\triangle \ln (s_{it}) = \gamma \triangle \ln (y_{it}) + \theta \triangle \ln (\lambda_{it}) + a_i + b_t + u_{it}, \tag{1}$$

where $\triangle \ln (s_{it})$ is the year $t-1$ to t change in the log of the national saving rate; $\triangle \ln (y_{it})$ is the year $t-1$ to t change in the log of GDP per capita; $\triangle \ln (\lambda_{it})$ is the year $t-1$ to t change in the log of the credit-to-GDP ratio; a_i is a country fixed effect; b_t is a year fixed effect; and u_{it} is an error term that is clustered at the country level.

The above econometric model can be derived from our theoretical model, see Section 4.5. In Section 2.1, we discuss identification issues pertaining to the estimation of the econometric model. We present and interpret our empirical results in Section 2.2.

2.1 Identification issues

Potential endogeneity of GDP per capita and the credit-to-GDP ratio is an important empirical issue in the estimation of (1). Endogeneity biases could arise because a within-country change in the saving rate affects GDP per capita and the credit-to-GDP ratio or because of time-varying omitted variables that affect the savings rate beyond GDP per capita and the credit-to-GDP ratio. Moreover, it is well known that classical measurement error attenuates least squares estimates toward zero (thus leading to an understatement of the true causal effect that GDP per capita and the credit-to-GDP ratio have on the national saving rate). In order to correct for endogeneity and measurement error bias, we need plausible exogenous instruments for GDP per capita and the credit-to-GDP ratio. Instrument validity requires that the instruments should only affect the savings rate through their effects on the endogenous variables. Because the estimating equation includes country fixed effects, such instruments need to be time-varying.

We use year-to-year variations in the international oil price weighted with countries' average net export share of oil in GDP as an instrument for the change in GDP per capita.[9] It is important to note that because year-to-year variations in the international oil price are highly persistent (see Hamilton, 2009; Brückner et al., 2012*b*, for evidence on international oil prices' random walk behavior), the instrumental variable estimate of γ captures the effect that a persistent shock to countries' GDP per capita has on the growth rate of the saving rate. Because the oil price shock instrument is constructed based on countries' average net export shares (i.e. the net export shares are time-invariant), the time series variation comes exclusively from the variation in the international oil price. By weighting the variation in the international oil price with countries' average net export shares of oil in GDP, the instrument takes into account that how the international oil price affects GDP per capita growth differs across countries, depending on whether they are net importers or exporters of oil. We can reasonably assume that the majority of countries are price takers in the international oil market. In order to ensure that our estimates are not driven by potentially large oil exporting or importing countries, where the exogeneity assumption may be more questionable, we will also present estimates that are based on a sample, which excludes large oil exporters and importers.

We use a lagged credit-to-GDP ratio as an internal instrument for the current credit-to-GDP ratio. Using the lagged variable as an instrument should reduce concerns that our within-country estimate of θ is inconsistent. Moreover, the first difference specification eliminates omitted variables bias, arising from time-invariant cross-country differences in historical and geographical variables that may be affecting the saving rate and the credit-to-GDP ratio.

2.2 Empirical results

We obtained data for national saving rates and GDP per capita (constant price PPP-based) from the Penn World Table Heston et al. (2011), and the GDP share of domestic credit to the private sector from the World Development Indicators The World Bank (2012). The sample consists of 130 countries over the period 1960–2007. For a list of countries in the sample, see Table 5 in Appendix D.

Table 1 presents our baseline estimates of the average effect that growth in GDP per capita (y) and the credit-to-GDP ratio (λ) have on the growth rate of the national savings rate (s). In columns (1)–(4), we present instrumental variables estimates. For comparison, we show in column (5) estimates from the Pesaran and Smith (1995) mean group (MG) estimator and in column (6)

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

Table 1. Effects of growth in GDP per capita and the credit-to-GDP ratio on the growth rate of the national saving rate

	$\Delta \ln(s_{it})$					
	(1)	(2)	(3)	(4)	(5)	(6)
	IV	IV	IV	IV	MG	LS
$\Delta \ln(y_{it})$	2.50***	2.43***	2.47***	2.12***	1.82***	1.52***
	(0.81)	(0.82)	(0.83)	(0.71)	(0.20)	(0.15)
$\Delta \ln(\lambda_{it})$		−0.21**	−0.22**	−1.23*	−0.98**	−0.26***
		(0.09)	(0.10)	(0.68)	(0.42)	(0.10)
$\Delta \ln(s_{it-1})$			0.19***	0.20***		
			(0.07)	(0.07)		
Kleibergen–Paap F-stat	19.60	19.40	9.96	3.70	.	.
Cragg–Donald F-stat	283.67	274.86	136.37	14.04	.	.
Endogenous Regressors	$\Delta \ln(y_{it})$	$\Delta \ln(y_{it})$	$\Delta \ln(y_{it})$, $\Delta \ln(s_{it-1})$	$\Delta \ln(y_{it})$, $\Delta \ln(\lambda_{it})$, $\Delta \ln(s_{it-1})$	.	.
Instruments	OPS_{it}	OPS_{it}	OPS_{it}, $\ln(s_{it-2})$	OPS_{it}, $\ln(\lambda_{it-1})$, $\ln(s_{it-2})$	.	.
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3781	3781	3781	3781	3781	3781

Notes: The dependent variable, $\Delta \ln(s_{it})$, is the change in the log of the saving rate. $\Delta \ln(y_{it})$ is the change in the log of real GDP per capita; $\Delta \ln(\lambda_{it})$ is the change in the log of the credit-to-GDP ratio. The method of estimation in columns (1)–(4) is two-stage least squares; in columns (5) mean group estimation; column (6) least squares fixed effects. Huber robust standard errors (shown in parentheses) are clustered at the country level.

we show estimates from the least squares (LS) fixed effects estimator. All regressions control for country and year fixed effects (which are jointly significant at the 1% significance level).

The main result from the panel regressions is that GDP per capita growth, on average, has a significant positive effect on the growth rate of the national saving rate. The growth rate of the credit-to-GDP ratio has a significant negative effect on the growth rate of the national saving rate. Specifically, the IV estimates in column (1) show that unconditional on the credit-to-GDP ratio, the estimated coefficient on log GDP per capita is 2.5; its standard error is 0.8. Column (2) shows that the average effect of growth in GDP per capita on the growth rate of the national saving rate is not much different when we control for the growth rate of the credit-to-GDP ratio. In columns (3) and (4), we document that these results are robust to a dynamic panel specification and instrumenting the change in the credit-to-GDP ratio with its lag.

Columns (5) and (6) show that the MG and LS estimates of θ (γ) are also negative (positive) and significantly different from zero at the conventional significance levels. Quantitatively, the IV estimates are in absolute size somewhat larger than the MG and LS estimates. One possible reason for this is a classical measurement error that attenuates the LS and MG estimates but not the IV estimates.

Our first main empirical finding is thus that the response of the growth rate of the national saving rate to growth in GDP per capita (the credit-to-GDP ratio) is positive (negative). This is consistent with previous empirical literature that has examined the macroeconomic relationship between saving and GDP per capita (see Jappelli and Pagano, 1994; Loayza et al., 2000). The finding is also consistent with our theoretical predictions and suggests that the majority of countries have a level of GDP per capita and credit-to-GDP ratios, which are relatively low, so that growth in GDP per capita has a significant positive effect on the growth rate of the national saving rate.

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

3. Interaction of GDP per capita growth with GDP per capita and the Credit-to-GDP ratio

3.1 Estimation framework

In this section, we present estimates of a model where the income elasticity of the national saving rate varies as a function of countries' average GDP per capita and the credit-to-GDP ratio:

$$\triangle \ln (s_{it}) = \gamma' \triangle \ln (y_{it}) + \delta(\triangle \ln (y_{it}) * \lambda_i) \qquad (2)$$

$$+ \zeta(\triangle \ln (y_{it}) * y_i) + \theta' \triangle \ln (\lambda_{it}) + a_i' + b_t' + u_{it}'.$$

The above econometric model can be derived from our theoretical model (see Section 4.5).

We use countries' period average credit-to-GDP ratio, λ_i, to construct the first interaction term. This allows us to focus on how long-run cross-country differences in the credit-to-GDP ratio affect the income elasticity of the national saving rate. Note that we construct the interaction term as $\triangle \ln y_{it} * \lambda_i$. Likewise, we construct the second interaction term that captures the elasticity as $\triangle \ln (y_{it}) * y_i$ where y_i is a country's average GDP per capita. The particular construction of the interaction terms implies that the coefficient γ' captures the predicted elasticity when λ_i and y_i are zero.

3.2 Empirical results

Table 2 presents estimates from the above interaction model. We begin by reporting in column (1) estimates from a more parsimonious version of the interaction model that only has as interaction term $\triangle \ln (y_{it}) * \lambda_i$. In this model specification, the estimate of γ' captures the predicted income elasticity of the national saving rate when $\lambda_i = 0$. Our estimate of γ' is 6.2 and its standard error is 1.3. Thus, the estimate of γ' is positive and significantly different from zero at the 1% level. The estimate of δ is around -10.8 (s.e. 3.1), hence negative and significantly different from zero at the 1% level. The significant negative δ indicates that the income elasticity of the national saving rate significantly increases as countries' credit-to-GDP ratios are lower. So much so, that at sample minimum, $\lambda_i = 0.02$, the predicted elasticity is 5.9 with a standard error of 1.3; at sample maximum, $\lambda_i = 1.51$, the predicted elasticity is -10.1 with a standard error of 3.5. Column (2) shows that this result also holds when we control for the direct effect of $\triangle \ln (\lambda_{it})$ on $\triangle \ln (s_{it})$, which continues to be negative and significant at the 5% level.

We present in columns (3) and (4) estimates of an interaction model that has as interaction term only $\triangle \ln (y_{it}) * y_i$. In this model specification, the estimate of γ' captures the predicted elasticity when $y_i = 0$. We find that γ' is 4.8 and its standard error is 1.2. Thus, the estimate of γ' is positive and significantly different from zero at the 1% level. The estimate of ζ is -0.13 (s.e. 0.05), hence negative and significantly different from zero at the 5% level. The significant negative ζ indicates that the elasticity is significantly higher in poor countries. Column (4) shows that this result also holds when we control for the direct effect of $\triangle \ln (\lambda_{it})$ on $\triangle \ln (s_{it})$, which continues to be negative and significantly different from zero at the 5% level.

In columns (5) and (6), we present estimates from an econometric model that includes both interaction terms, $\triangle \ln (y_{it}) * y_i$ and $\triangle \ln (y_{it}) * \lambda_i$. In this model, the estimates of δ and ζ capture conditional interaction effects. That is, ζ captures how the elasticity varies across poor and rich countries when holding λ_i, the average credit-to-GDP ratio, constant. Likewise, δ captures how the average credit-to-GDP ratio affects the income elasticity of the national saving rate, when holding y_i, countries' average GDP per capita, constant. Hence, any effect that being rich or poor has on the elasticity within a country through the credit-to-GDP ratio, is shut down. Likewise, any effect that the credit-to-GDP ratio may have on the elasticity through y_i, the average GDP per capita level, is also shut down.

Table 2. Interactions between GDP per capita growth, GDP per capita, and the credit-to-GDP ratio

	$\Delta \ln(s_{it})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \ln(y_{it})$	6.16***	6.08***	4.82***	4.77***	6.81***	6.74***
	(1.33)	(1.33)	(1.22)	(1.22)	(1.27)	(1.25)
$\Delta \ln(y_{it}) * \lambda_i$	−10.82***	−10.87***			−9.23***	−9.25***
	(3.09)	(3.05)			(3.01)	(2.94)
$\Delta \ln(y_{it}) * y_i$			−0.13***	−0.13***	−0.06**	−0.06**
			(0.05)	(0.05)	(0.03)	(0.03)
$\Delta \ln(\lambda_{it})$		−0.29**		−0.20**		−0.27**
		(0.12)		(0.09)		(0.12)
Kleibergen-Paap	18.82	18.85	12.80	12.53	14.08	14.05
F-stat						
Endogenous	$\Delta \ln(y_{it})$,	$\Delta \ln(y_{it})$,	$\Delta \ln(y_{it})$,	$\Delta \ln(y_{it})$,	$\Delta \ln(y_{it})$,	$\Delta \ln(y_{it})$,
Regressors	$\Delta \ln(y_{it})^*\lambda_i$,	$\Delta \ln(y_{it})^*\lambda_i$	$\Delta \ln(y_{it})^*y_i$	$\Delta \ln(y_{it})^*y_i$	$\ln(y_{it})^*\lambda_i$,	$\ln(y_{it})^*\lambda_i$,
					$\Delta \ln(y_{it})^*y_i$	$\Delta \ln(y_{it})^*y_i$
Instruments	OPS_{it},	OPS_{it},	OPS_{it},	OPS_{it},	OPS_{it},	OPS_{it},
	$OPS_{it}{}^*\lambda_i$	$OPS_{it}{}^*\lambda_i$	$OPS_{it}{}^*y_i$	$OPS_{it}{}^*y_i$	$OPS_{it}{}^*\lambda_i$,	$OPS_{it}{}^*\lambda_i$,
					$OPS_{it}{}^*y_i$	$OPS_{it}{}^*y_i$
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3850	3850	3850	3850	3850	3850

Notes: The method of estimation is two-stage least squares. Huber robust standard errors (shown in parentheses) are clustered at the country level. The dependent variable is the change in the log of the saving rate. *Significantly different from zero at the 10% significance level, ** 5% significance level, *** 1% significance level.

With the above in mind, we can now interpret the estimates in columns (5) and (6). The estimate of γ' is 6.8, which captures the predicted income elasticity of the national saving rate when both $y_i = 0$ and $\lambda_i = 0$; its standard error is 1.3. Note that there is no country in the sample with $y_i = 0$ and $\lambda_i = 0$. It makes more sense to consider countries with very low λ_i and y_i, say, those at the bottom 5th percentile ($y_i = 0.6$ and $\lambda_i = 0.07$).[10] For computing the predicted elasticity, we need to consider that the estimates of δ and ζ are −9.2 and −0.06, respectively; their standard errors are 3.0 and 0.03, respectively. Hence, for a country at the bottom 5th percentile ($y_i = 0.6$ and $\lambda_i = 0.06$), a 1 percentage point increase in growth of GDP per capita is predicted to increase the growth rate of the national saving rate by around 6.1 percentage points (s.e. 1.1). This is a large effect. For higher values of y_i and λ_i, the effect is considerably smaller. For example, at the 50th percentiles ($y_i = 3.5$ and $\lambda_i = 0.27$), the predicted income elasticity of the national saving rate is 4.1, and at the 75th percentiles ($y_i = 3.5$ and $\lambda_i = 0.27$), it is 1.4. Moreover, the elasticity can be negative for sufficiently high values of y_i (and λ_i). This is shown in Figure 1, which plots for different values of λ_i the predicted elasticity over the sample range of y_i.

We provide further sensitivity analysis in Brueckner et al. (2021). Appendix E of Brueckner et al. (2021) shows that our results robust to excluding the top and bottom 1st percentile of the change in the national saving rate; using initial (1970) oil net export GDP shares to construct the oil price shock instrument; and splitting the sample into the post-1990 and pre-1990 period. Appendix F of Brueckner et al. (2021) shows that results are similar for the post- and pre-financial crisis period. Appendix G of Brueckner et al. (2021) examines parameter heterogeneity: we report in that section estimates when countries are clustered according to their GDP per capita, geographic location, population size, and sectoral composition.

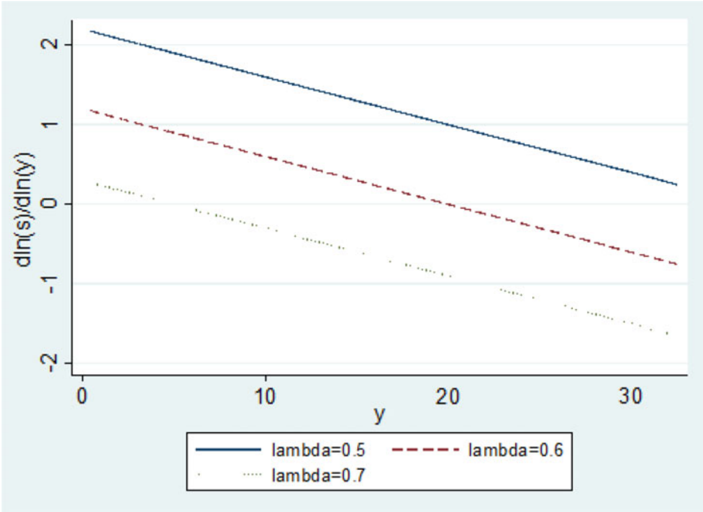

Figure 1. The predicted income elasticity of the national saving rate for different levels of average GDP per capita and credit-to-GDP ratio.
Note: The figure is based on the estimates in column (6) of Table 2.

To summarize: In poor countries, growth in GDP per capita has a significant positive effect on the growth of the national saving rate. In rich countries and countries where the credit-to-GDP ratios are high, the opposite is the case, that is, growth in GDP per capita has a significant negative effect on the growth rate of the national saving rate.

4. The theoretical model

The economy is inhabited by overlapping generations of agents living for two periods. Every agent supplies one unit of labor inelastically when young and consumes when both young and old. Successive generations have unit mass and are ex ante homogeneous by possessing one unit of labor endowment. Production combines the current stock of capital k_t supplied by old agents with one unit of labor supplied by young agents.[11] The resulting output per capita is $y_t = f(k_t)$ where $f{:}R_+ \to R_+$ is the production function in intensive form. We assume that $f(0) = 0$, $f{:}R_+ \to R_+$ is twice continuously differentiable on R_{++} and strictly increasing and strictly concave on R_+. Factor markets are competitive and rewards on physical capital and labor are determined by the marginal product rule, that is, $f'(k_t)$ is the rate of return on one unit of capital and $w_t = w(k_t) := f(k_t) - k_t f'(k_t)$ is the wage. The final commodity produced at time t may be either consumed or invested to produce capital which becomes available in period $t + 1$. Capital depreciates fully within a period, so that the next period capital stock is equal to the investment.

Following production and distribution of factor payments, the old consume and exit the model, while the young receive the wage—the wealth of the young—and make their *saving* and *investment* decisions. When making the saving decision at time t, young agents have two options to transfer the current saving into the second period. First, they may become *an investor* by lending the entire saving $s_t^l w_t$ in the competitive credit market for a rate of return r_{t+1}. Second, young agents may become *an entrepreneur* by starting an investment project, which requires one unit of the final commodity for investment and returns $R > 0$ units of capital in period $t + 1$. Later on, we restrict parameter values so that entrepreneurs must always borrow $(1 - s_t^b w_t)$ in order to start an investment project. That's why we use the superscript "b" (borrower) to refer to entrepreneurs and "ℓ" (lenders) to refer to investors. Produced R units of capital can be rented

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

to the final commodity producing firm in exchange for $Rf'(k_{t+1})$ units of the final commodity. Hence, the second period consumption of entrepreneurs is $c_{2t+1} = Rf'(k_{t+1}) - (1 - s_t^b w_t) r_{t+1} = (\phi_{t+1} - 1 + s_t^b w_t) r_{t+1}$ where $\phi_{t+1} := Rf'(k_{t+1})/r_{t+1}$ is the entrepreneurial rent.

We assume that entrepreneurs can pledge only up to a fraction $\lambda \in (0, 1)$ of the project revenue for debt repayment. Thus, they can borrow $1 - s_t^b w_t$ units of final commodity and start an investment project only if

$$(1 - s_t^b w_t) r_{t+1} \leq \lambda Rf'(k_{t+1}) \Leftrightarrow s_t^b \geq \tfrac{1-\lambda\phi_{t+1}}{w_t}. \tag{3}$$

We refer to this as the credit constraint. The parameter λ measures the severity of the credit constraint, with a higher (lower) value corresponding to a looser (tighter) credit constraint. This formulation is a parsimonious way of introducing a credit constraint in a dynamic macroeconomic model. One of the justifications of the credit constraint is the existence of a default cost, which is proportional to the project revenue. In such case, lenders avoid strategic default of borrowers by limiting their debt obligation.[12] It is clear from the above inequality that the saving rate of entrepreneurs must be above a threshold, which depends on the severity of the credit constraint (λ) and on the entrepreneurial rent (ϕ_{t+1}). The credit constraint binds when the wage is sufficiently low. We will see later that this creates an entrepreneurial rent and motivates entrepreneurs to save more than investors. The higher is the wage, the less do entrepreneurs require external funds, and thus, the lower are their incentive to save. The difference in saving between entrepreneurs and investors completely disappears as the wage exceeds a threshold.

4.1 Optimal behavior

Young agents maximize the following lifetime utility:

$$u(c_{1t}, c_{2t+1}) = \ln c_{1t} + \ln c_{2t+1}, \tag{4}$$

where we assume no time discount for simplicity. In Appendix C, we introduce a time discount and show that the main results hold. Investors choose a saving rate to maximize lifetime utility:

$$\max_{s_t^\ell \in [0,1]} \ln (1 - s_t^\ell) w_t + \ln s_t^\ell w_t r_{t+1},$$

which can be rewritten as $\ln U^\ell + \ln (w_t^2 r_{t+1})$ where

$$U^\ell := \max_{s_t \in [0,1]} \{(1 - s_t^\ell) s_t^\ell\}. \tag{5}$$

The optimal saving rate of investors is $s_t^\ell = 1/2$ and $U^\ell = 1/4$. Entrepreneurs choose a saving rate to maximize lifetime utility:

$$\max_{s_t^b \in [0,1]} \left\{ \ln (1 - s_t^b) w_t + \ln (Rf'(k_{t+1}) - (1 - s_t^b w_t) r_{t+1}) \middle| \, s_t^b \geq \frac{1 - \lambda\phi_{t+1}}{w_t} \right\},$$

which can be rewritten as $\ln U^b(w_t, \phi_{t+1}) + \ln (w_t^2 r_{t+1})$ where

$$U^b(w_t, \phi_{t+1}, \lambda) := \max_{s_t^b \in [0,1]} \left\{ \left(1 - s_t^b\right) \left(\frac{\phi_{t+1} - 1}{w_t} + s_t^b \right) \middle| \, s_t^b \geq \frac{1 - \lambda\phi_{t+1}}{w_t} \right\}. \tag{6}$$

The next proposition establishes the optimal saving of entrepreneurs.

Proposition 1. *For a given $(w_t, \phi_{t+1}, \lambda)$, the optimal saving rate of entrepreneurs is*

$$s_t^b = \max \left\{ \frac{1}{2} \left(1 - \frac{\phi_{t+1} - 1}{w_t} \right), \frac{1 - \lambda\phi_{t+1}}{w_t} \right\}, \tag{7}$$

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and

$$U^b(w_t,\phi_{t+1},\lambda)=\begin{cases}\frac{1}{4}\left(1+\frac{\phi_{t+1}-1}{w_t}\right)^2 & \text{if } w_t\geq 1-(2\lambda-1)\phi_{t+1}\\ \left(1-\frac{1-\lambda\phi_{t+1}}{w_t}\right)\frac{(1-\lambda)\phi_{t+1}}{w_t} & \text{if } 1-\lambda\phi_{t+1}\leq w_t<1-(2\lambda-1)\phi_{t+1}.\end{cases} \tag{8}$$

If $w_t < 1-\lambda\phi_{t+1}$, then young agents cannot become entrepreneurs because the credit constraint is violated even if they save their entire wage.

Proof of Proposition 1 can be found in Appendix B.

4.2 The entrepreneurial rent

When making the saving decision, young agents take the value of $(w_t, \phi_{t+1}, \lambda)$ as given and compare the value of U^ℓ and $U^b(w_t, \phi_{t+1}, \lambda)$. If $U^b(w_t, \phi_{t+1}, \lambda) < U^\ell$, then all young agents strictly prefer to become investors. If $U^b(w_t, \phi_{t+1}, \lambda) > U^\ell$, then all young agents strictly prefer to become entrepreneurs. Therefore, it must be that $U^b(w_t, \phi_{t+1}, \lambda) = U^\ell$ in equilibrium so that young agents are indifferent between becoming an entrepreneur and an investor. In equilibrium, some young agents become investors by lending $s^\ell w_t$ and the others become entrepreneurs by borrowing $1 - s_t^b w_t$ in the competitive credit market. The next proposition establishes the entrepreneurial rent, which must hold in equilibrium.

Proposition 2. *For a given (w_t, λ), young agents are indifferent between becoming an entrepreneur and an investor if and only if*

$$\phi_{t+1}=\phi\left(w_t,\lambda\right):=\begin{cases}\frac{1}{2\lambda}\left(1-w_t+\sqrt{1-2w_t+\frac{w_t^2}{1-\lambda}}\right) & \text{if } w_t<2(1-\lambda)\\ 1 & \text{if } w_t\geq 2(1-\lambda),\end{cases} \tag{9}$$

which is the solution to $U^b(w_t, \phi_{t+1}, \lambda) = U^\ell$.

Proof of Proposition 2 can be found in Appendix B. From (7) and (9), we can see that if $\phi_{t+1} = 1$, then the saving behavior of investors and entrepreneurs is identical and they consume the same amount of the final commodity when old. However, if $\phi_{t+1} > 1$ then entrepreneurs save more than investors (entrepreneurs consume more than investors when old). In either case, young agents will have to adjust the entrepreneurial rent toward the equilibrium level in order to maximize the lifetime utility and at the same time eliminate the excess demand/supply in the credit market. In other words, the equilibrium rent is the highest rent that entrepreneurs can earn and still be able to borrow all that they need, with no surplus or shortage in the credit market.

The entrepreneurial rent declines if the wage increases or the credit constraint is looser (λ increases) as shown in Lemma 1. If $0 < w_t < 2(1-\lambda)$, then $\phi_{t+1} = \phi(w_t, \lambda) > 1$ and entrepreneurs and investors achieve the same level of lifetime utility despite the fact that they have different saving rates. Figure 2(a) shows the entrepreneurial rent for different values of λ.

Substituting (9) into (7), we obtain the saving rate of entrepreneurs in equilibrium:

$$s_t^b=s^b(w_t,\lambda):=\begin{cases}\frac{1-\lambda\phi(w_t,\lambda)}{w_t} & \text{if } w_t<2(1-\lambda)\\ \frac{1}{2} & \text{if } w_t\geq 2(1-\lambda).\end{cases} \tag{10}$$

The saving rate of entrepreneurs $s^b(w_t, \lambda)$ declines as the wage increases or the credit constraint is looser (see Lemma 2). Figure 2(b) shows the saving rate for different values of λ. If $0 < w_t < 2(1-\lambda)$, then from (9) and (10) $\phi_{t+1} > 1$ and $s_t^b > s_t^\ell = 1/2$. Moreover, when the credit constraint is tighter, the entrepreneurial rent is higher and the saving rate of entrepreneurs is higher; entrepreneurs rely less on external funds. This captures the idea that "anybody can make

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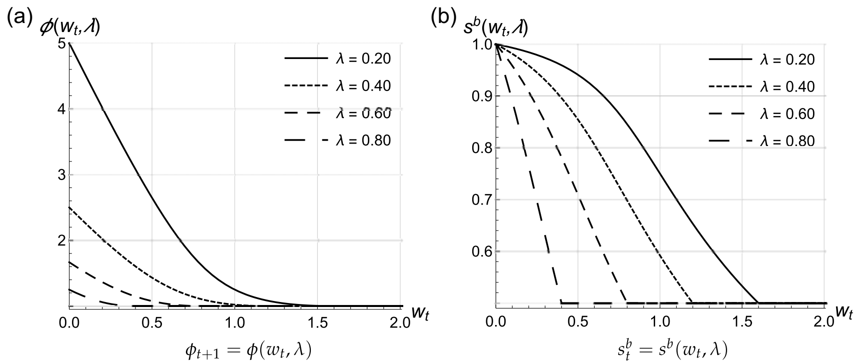

Figure 2. The entrepreneurial rent (ϕ_{t+1}) and the saving rate of entrepreneurs (s_t^b) for different values of parameter λ.

it through thrift." The young, who want to become entrepreneurs and face the credit constraint, save more in order to earn entrepreneurial rents in the future.[13] We can show that $s_t^b w_t$ increases as the wage increases. This implies that by restricting to $w_t \in (0, 2)$, we can guarantee $0 < s_t^b w_t < 1$ as $s_t^b = 1/2$ when $w_t = 2$. Hence, there is always a positive demand for credit; entrepreneurs always borrow in the credit market as it has been assumed (see Lemma 2).

4.3 The national saving rate

In equilibrium, there will be only as many entrepreneurs as there are resources available divided by the final commodity each entrepreneur requires for investment projects. Let s_t denote the national saving rate at time t. Since the resources available for the investment projects are the national saving $s_t w_t$ and each entrepreneur requires one unit of the final commodity for investment projects, the equilibrium size (i.e. the mass) of young agents who become entrepreneurs is $s_t w_t$, while the equilibrium size of young agents who become lenders is $1 - s_t w_t$. Since each entrepreneur borrows $1 - s_t^b w_t$ and each investor lends $s^\ell w_t = \frac{w_t}{2}$ units of the final commodity, the credit market clearing condition is

$$s_t w_t \left(1 - s_t^b w_t\right) = (1 - s_t w_t)\, \frac{w_t}{2}. \tag{11}$$

Substituting (10) into (11), we obtain the national saving rate in equilibrium:

$$s_t = s(w_t, \lambda) := \begin{cases} \frac{1}{w_t + 2\lambda\phi(w_t, \lambda)} & \text{if } w_t < 2(1-\lambda) \\ \frac{1}{2} & \text{if } w_t \geq 2(1-\lambda). \end{cases} \tag{12}$$

The national saving rate increases on $w_t \in (0, 1-\lambda)$, decreases on $w_t \in (1-\lambda, 2(1-\lambda))$, and is constant at $1/2$ for $w_t \in (2(1-\lambda), 2)$ leading to a hump-shaped saving rate with respect to the wage/wealth (see Lemma 3). To understand this relationship, we note that the wage affects the saving rate of entrepreneurs as well as the number of entrepreneurs (the saving rate of investors is constant). Let $\pi_t := s_t w_t \in (0, 1)$ denote the fraction of entrepreneurs. We can rewrite (11) as:

$$s_t = \pi_t \left(s_t^b - \frac{1}{2}\right) + \frac{1}{2}. \tag{13}$$

The fraction of entrepreneurs π_t increases but the saving rate of entrepreneurs s_t^b decreases as the wage increases.[14] Since the saving rate of entrepreneurs is higher than that of investors, the first effect causes the saving rate s_t to rise. However, the second effect causes it to fall. The first effect dominates when the wage is low but the second effect eventually dominates. Once the wage is high enough so that the credit constraint no longer binds, $s_t^b = 1/2$ and $s_t = 1/2$.

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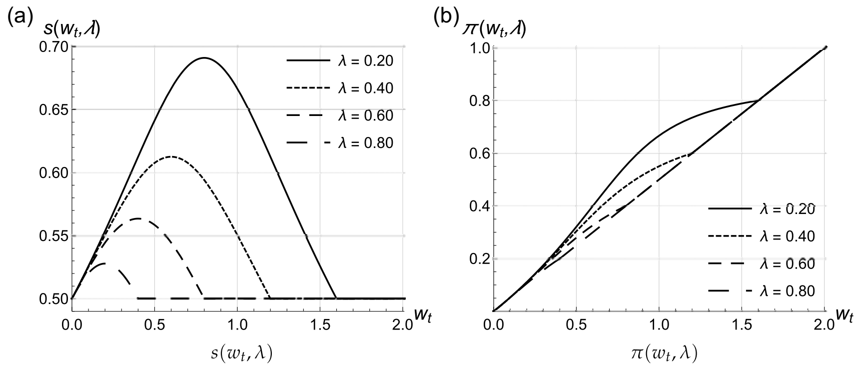

Figure 3. The national saving rate s_t and the fraction of entrepreneurs π_t for different values of λ.

Figure 3(a) and (b) show the saving rate and the fraction of entrepreneurs, respectively, for different values of parameter λ. We can see that $\pi(w_t, \lambda) = s(w_t, \lambda)w_t$ is strictly increasing in w_t and strictly decreasing in λ (see Lemma 3). The fraction of entrepreneurs increases when the wage increases as they need to rely less on external funds. This increases the national savings.

4.4 The transitional wage dynamics

We are now ready to describe the transitional dynamics of the wage. With full depreciation of capital within a period, $k_{t+1} = R\pi(w_t, \lambda)$ and the evolution of the wage is given by:

$$w_{t+1} = w(R\pi(w_t, \lambda)). \tag{14}$$

Two important properties of the wage function should be highlighted here. First, strict concavity of f implies strict monotonicity of w because $w'(k) = -kf''(k) > 0$. Second, $w(0) = 0$ because $f(0) = 0$. These properties on the one hand imply the existence of $R^+ > 0$ such that $w(R) < 2$ for $R \in (0, R^+)$ and on the other hand imply that if $R \in (0, R^+)$ and $w_t \in (0, 2)$ then $\pi(w_t, \lambda) \in (0, 1)$ and $w_{t+1} = w[R\pi(w_t, \lambda)] \in (0, w(R)) \subset (0, 2)$. Hence, (14) defines a dynamical system on the state space $[0, 2]$. Under (14), entrepreneurs always need to borrow in order to start an investment project. The following definition of equilibrium reconciles market clearing and individual optimality.

Definition 1. For a given parameter pair $(\lambda, R) \in (0, 1) \times (0, R^+)$ and for a given initial wage $w_0 = w(k_0) \in (0, 2)$, the transitional dynamics is a equilibrium sequence $\{w_t, s_t^b, s_t\}_{t=0}^{\infty}$ such that in every period

(1) The entrepreneurial rent adjusts so that young agents are indifferent between becoming an entrepreneur and becoming an investor; (10) holds.
(2) The fraction of entrepreneurs adjusts so that the credit market clears; (11) holds.
(3) The fraction of entrepreneurs determines the wage in the next period; (14) holds.

We can easily verify that $w = 0$ is the corner steady state because $\pi(0, \lambda) = 0$ and $w(0) = 0$. In general, multiple interior steady states may exist. When multiple steady states exist, the initial condition determines to which steady state the economy converges; however, the equilibrium sequence, $\{w_t\}_{t=0}^{\infty}$, of wages always forms a monotonic sequence for any $w_0 \in (0, 2)$. The capital stock and the output per capita also form a monotonic sequence because f and w are both strictly increasing functions and $k_t = w^{-1}(w_t)$ and $y_t = f[w^{-1}(w_t)]$.[15] Given the monotone relationship between w_t and y_t, we obtain

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Theorem 1. The national saving rate $s(w_t, \lambda)$

(1) first increases and then decreases with output per capita y_t.
(2) is higher when the credit constraint is tighter (λ takes a lower value).

The theorem is consistent with the main empirical results that GDP per capita growth increases the national saving rate in poor countries, while the opposite is true in rich countries, and that the growth of the credit-to-GDP ratio decreases the national saving rate. It is worthwhile to highlight here that the predictions in Theorem 1 hold even in an open economy, in which international credit and lending are allowed. In fact, the saving behavior of entrepreneurs depends on the wage and the severity of the credit constraint only and is independent of the interest rate. Therefore, we only need to confirm that the wage forms a monotonic sequence in a small open economy. In a small open economy, the capital accumulation is determined by:

$$k_{t+1} = \Phi(w_t) = (f')^{-1}\left(\frac{r^*\phi(w_t, \lambda)}{R}\right),$$

where r^* is the world interest rate. Note that Φ is a non-decreasing function in w_t since $(f')^{-1}$ is a strictly decreasing function while ϕ is nonincreasing in w_t. Hence, $w_{t+1} = W[\Phi(w_t)]$ implies that the equilibrium wage sequence, $\{w_t\}_{t=0}^{\infty}$, as in the closed economy, always forms a monotonic sequence for any $w_0 \in (0, 2)$.

The theorem predicts the equilibrium relationships between the national saving rate, output, and the credit constraint. The credit constraint has a positive effect on the national saving rate because it encourages entrepreneurial saving similarly as in Jappelli and Pagano (1994) where it reduces consumption. However, previous models such as Jappelli and Pagano (1994) do not generate our main result that the national saving rate first increases and then decreases as output per capita increases.

4.5 Motivation of the econometric model specification

We show how the econometric specification in (1) and (2) can be derived from our model.[16] Taking logarithms, we obtain $\ln s_t = \ln s((w \circ f^{-1})(y_t), \lambda_t)$. For small values of $\triangle \ln y_t = \ln y_t - \ln y_{t-1}$ and $\triangle \ln \lambda_t = \ln \lambda_t - \ln \lambda_{t-1}$, $\triangle \ln s_t = \ln s_t - \ln s_{t-1}$ can be approximated as:

$$\triangle \ln s_t \approx \gamma_{t-1} \triangle \ln y_t + \theta_{t-1} \triangle \ln \lambda_t, \tag{15}$$

where $\gamma_{t-1} := \frac{w_{t-1} s_1(w_{t-1}, \lambda_{t-1})}{s(w_{t-1}, \lambda_{t-1})} \frac{y_{t-1}(w \circ f^{-1})'(y_{t-1})}{(w \circ f^{-1})(y_{t-1})}$ and $\theta_{t-1} := \frac{w_{t-1} s_2(w_{t-1}, \lambda_{t-1})}{s(w_{t-1}, \lambda_{t-1})}$.[17] Proof of Lemma 3 in Appendix B derives both elasticities. The sign of the income elasticity of the national saving rate changes from positive to negative at $w = 1 - \lambda$. The elasticity of the saving rate with respect to the parameter λ is always negative. If the production function is Cobb–Douglas, then $\frac{y(w \circ f^{-1})'(y)}{(w \circ f^{-1})(y)} = 1$.

Suppose that the parameters do not vary significantly over time (we test this hypothesis in Section 2.2). Then, we can write (15) for a panel of $i = 1 \ldots N$ countries as:

$$\triangle \ln s_{it} \approx \gamma_i \triangle \ln y_{it} + \theta_i \triangle \ln \lambda_{it}. \tag{16}$$

The above equation shows that a within-country change in the log of the national saving rate should be related to a within-country change in the log of y and λ. In (1), γ and θ thus capture the average elasticity effect of a marginal within-country change in GDP per capita and in the credit-to-GDP ratio on a change in the national saving rate. In other words, γ and θ are sample mean elasticity effects. We check in Section 2.2 whether our restricted panel data model in (1) consistently estimates these average elasticity effects, using the Pesaran and Smith (1995) MG estimator.

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We note that for a relatively low value of y_i, $w_i = (w \circ f^{-1})(y_i) < 1 - \lambda$ while $w_i = (w \circ f^{-1})(y_i) > 1 - \lambda$ for a relatively high value of y_i. It follows that γ_i is positive (negative) for a low (high) value of y_i and any fixed value of λ_i because $\gamma_i = \frac{w_i s_1(w_i,\lambda_i)}{s(w_i,\lambda_i)} \frac{y_i (w \circ f^{-1})'(y_i)}{(w \circ f^{-1})(y_i)} > 0$ when $w_i \in (0, 1 - \lambda_i)$ and $\gamma_i = \frac{w_i s_1(w_i,\lambda_i)}{s(w_i,\lambda_i)} \frac{y_i (w \circ f^{-1})'(y_i)}{(w \circ f^{-1})(y_i)} < 0$ when $w_i \in (1 - \lambda_i, 2(1 - \lambda_i))$. The income elasticity of the national saving rate can therefore be either positive or negative. On the other hand, since $\theta_i = \frac{w_i s_2(w_i,\lambda_i)}{s(w_i,\lambda_i)} < 0$, it follows that θ_i is negative for any fixed values of λ_i, y_i, and $w_i = (w \circ f^{-1})(y_i)$. Hence, our model unambiguously predicts a negative elasticity effect of a higher credit-to-GDP ratio on the national saving rate.

Let $F(y, \lambda) \equiv \frac{(w \circ f^{-1})(y) s_1((w \circ f^{-1})(y),\lambda)}{s((w \circ f^{-1})(y),\lambda)} \frac{y (w \circ f^{-1})'(y)}{(w \circ f^{-1})(y)}$. Then the functional form of the interaction model can be derived from our model by applying the following first-order Taylor expansion:

$$\gamma_i \equiv F(y_i, \lambda_i) \approx F(\hat{y}, \hat{\lambda}) + F_1(\hat{y}, \hat{\lambda})(y_i - \hat{y}) + F_2(\hat{y}, \hat{\lambda})(\lambda_i - \hat{\lambda}),$$

where $\hat{y}$ is the unique solution to $(w \circ f^{-1})(y) = 1 - \lambda$ and $\hat{\lambda}$ is the cross-country average of λ_i. Substituting the above expression in (16) yields the following relationship between the change in the log of the national saving rate and the change in the log of GDP per capita and the change in the log of the credit-to-GDP ratio:

$$\triangle \ln s_{it} = \left(\gamma' + \delta \lambda_i + \zeta y_i\right) \triangle \ln y_{it} + \theta' \triangle \ln \lambda_{it}.$$

where $F(\hat{y}, \hat{\lambda}) = 0$, $\gamma' \equiv -\hat{y} F_1(\hat{y}, \hat{\lambda}) - \hat{\lambda} F_2(\hat{y}, \hat{\lambda})$, $\zeta \equiv F_1(\hat{y}, \hat{\lambda})$, and $\delta \equiv F_2(\hat{y}, \hat{\lambda})$. Note that $F_1(\hat{y}, \hat{\lambda}) < 0$ and $F_2(\hat{y}, \hat{\lambda}) < 0$. This implies that $\gamma' > 0$, $\delta < 0$, $\zeta < 0$ and $\theta' < 0$ as before. Hence, our model predicts that the income elasticity of the national saving rate is larger in countries with lower GDP per capita and a lower credit-to-GDP ratio.

5. Empirical support for the mechanism

The theoretical model in Section 4 explains the empirical findings that the national saving rate follows a hump-shaped transitional dynamics by the interplay of entrepreneurial savings at the intensive and extensive margins. The model predicts that when the wealth increases, more agents can become entrepreneurs but entrepreneurs save less, and when the wealth is low, the extensive margin dominates the intensive margin; the opposite is true when the wealth is high. This section examines the mechanism of the theoretical model by utilizing cross-country time series data of the number of new businesses registered as a proxy for the extensive margin and the corporate saving rate as a proxy for the intensive margin.

5.1 New businesses registered

In this section, we estimate the relationship between the growth rate of the number of new businesses registered and GDP per capita growth (constant price PPP-based). We also estimate the relationship between the growth rate of the number of new businesses registered and the growth rate of the credit-to-GDP ratio. We obtained data on the number of new businesses registered from the World Development Indicators by The World Bank (2020). Specifically, the variable we use is the number of new limited liability corporations registered in the calendar year (per 1000 people aged 15 to 64 years). These data are available annually from 2006 onward.

Column (1) of Table 3 shows least squares estimates of the average relationship between the growth rate of the number of new businesses registered and GDP per capita growth. The panel covers 105 countries over the period 2006–2017. The dependent variable is the year $t - 1$ to t change in the log of new businesses registered per capita and the right-hand-side variable is the year $t - 1$ to t change of the log of GDP per capita. Control variables are country and year fixed effects.

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Table 3. The relationship between GDP per capita growth, the growth rate of the credit-to-GDP ratio and the growth rate of new businesses registered

Δ ln (New Businesses Registered$_{it}$)			
	(1)	(2)	(3)
Δ ln (y_{it})	1.994***	2.417***	2.605***
	(0.319)	(0.391)	(0.394)
Δ ln (y_{it}) $*$ y_i		−0.024*	−0.035**
		(0.013)	(0.014)
Δ ln (λ_{it})			0.226**
			(0.104)
Δ ln (λ_{it}) $*$ y_i			−0.012**
			(0.005)
Country FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Observations	810	810	810

Notes: The method of estimation is least squares. Standard errors are shown in parentheses.. The dependent variable is the change in the log of the new business registered per 1000 people aged between 15 to 64 years. *Significantly different from zero at the 10% significance level, **5% significance level, ***1% significance level.

From column (1) of Table 3, one can see that on average there is a significant positive relationship between GDP per capita growth and the growth rate of new businesses registered. The estimated coefficient on the $t-1$ to t change in the log of GDP per capita, $\Delta \ln(y_{it})$, is around 1.9 and has a standard error of around 0.3. One can reject the hypothesis that the estimated coefficient is equal to 0 at the 1% significance level. Quantitatively, the estimate in column (1) of Table 3 can be interpreted as follows: a 1 percentage point increase in GDP per capita growth raises the growth rate of new businesses registered by around 2 percentage points.

Column (2) of Table 3 shows that the effect of GDP per capita growth on the growth rate of new business registered is larger for poorer countries. This can be seen from the significant negative coefficient on $\Delta \ln(y_{it}) * y_i$, which is the interaction between the $t-1$ to t change in the log of GDP per capita and countries' average GDP per capita (in the table, reported in 1000s of dollars) over the period 2006–2017. Quantitatively, the estimates in column (2) can be interpreted as follows. Consider a low-income country with average GDP per capita over the period 2006–2017 equal to USD 1000. The estimates in column (2) suggest that, for this country, a 1 percentage point increase in GDP per capita growth raises the growth rate of new businesses registered by around 2.4 percentage points. Consider now a middle-income country with average GDP per capita from 2006 to 2017 equal to USD 10,000. The estimates in column (2) suggest that, for this country, a 1 percentage point increase in GDP per capita growth raises the growth rate of new businesses registered by around 2.2 percentage points. For a high-income country with GDP per capita equal to USD 50,000, the effect is much smaller. For that country, the estimates in column (2) of Table 3 suggest that a 1 percentage point increase in GDP per capita growth raises the growth rate of new businesses registered by around 1.4 percentage points.

Column (3) of Table 3 shows that the relationship between the growth rate of new businesses registered and GDP per capita growth is robust to controlling for the growth rate of the credit-to-GDP ratio. From column (3) of Table 3, one can see that the growth rate of the credit-to-GDP ratio has a significant positive effect on the growth rate in new businesses registered, but less so, the higher is the GDP per capita in the economy.

Table 4. The relationship between GDP per capita growth, the growth rate of credit-to-GDP ratio, and the growth rate of the corporate saving rate

Δ ln (gross corporate saving rate$_{it}$)			
	(1)	(2)	(3)
$\Delta \ln(y_{it})$	−0.621***	−0.021	−0.029
	(0.166)	(0.195)	(0.079)
$\Delta \ln(y_{it}) * y_i$		−0.030***	−0.028***
		(0.005)	(0.004)
$\Delta \ln(y_{it}) * \lambda_i$			−0.025**
			(0.010)
$\Delta \ln(\lambda_{it})$			−0.015
			(0.009)
Country FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Observations	887	887	706

Note: The method of estimation is least squares. Standard errors are shown in parentheses. The dependent variable is the change in the log of gross corporate saving rate. *Significantly different from zero at the 10% significance level, ** 5% significance level, *** 1% significance level.

5.2 The corporate saving rate

In this section, we estimate the relationship between the growth rate of the corporate saving rate and GDP per capita growth. We also estimate the relationship between the growth rate of the corporate saving rate and the growth rate of the credit-to-GDP ratio. Our data on the gross saving rate of the corporate sector are from Chen et al. (2017). We compute the gross corporate saving rate as the gross saving of the corporate sector divided by the gross value added of the corporate sector. The data on GDP per capita (PPP-based) and the credit-to-GDP ratio (the GDP share of domestic credit to the private sector) are from the World Development Indicators by The World Bank (2020).

Column (1) of Table 4 shows least squares estimates of the average the relationship between the growth rate of the corporate saving rate and GDP per capita growth. The panel covers 59 countries over the period 1992–2013. The dependent variable is the year $t-1$ to t change in the log of the corporate saving rate and the right-hand-side variable is the year $t-1$ to t change of the log of GDP per capita. Control variables are country and year fixed effects.

From column (1) of Table 4, one can see that on average there is a significant negative relationship between GDP per capita growth and the growth rate of the corporate saving rate. The estimated coefficient on the $t-1$ to t change in the log of GDP per capita is around −0.6 and has a standard error of around 0.2. One can reject the hypothesis that the estimated coefficient is equal to zero at the 1% significance level. Quantitatively, the estimate in column (1) of Table 4 can be interpreted as follows: a 1 percentage point increase in GDP per capita growth decreases the growth rate of the corporate saving rate by around 0.6 percentage points, on average.

Column (2) of Table 4 shows that the negative effect of GDP per capita growth on the growth rate of the corporate saving rate is larger (in absolute value) for richer countries. This can be seen from the significant negative coefficient on $\Delta \ln(y_{it}) * y_i$, which is the interaction between the $t-1$ to t change in the log of GDP per capita and countries' average GDP per capita (in the table, reported in USD 1000s) over the period 1992–2013. Quantitatively, the estimates in column (2) can be interpreted as follows. Consider a low-income country with average GDP per capita over the period 1992–2013 equal to USD 1,000. The estimates in column (2) suggest that, for this country, a 1 percentage point increase in GDP per capita growth decreases the growth rate of the corporate saving rate by around 0.05 percentage points. Consider now a middle-income country with average GDP per capita over the period 1992–2013 equal to USD 10,000. The estimates in column (2) suggest that, for this country, a 1 percentage point increase in GDP per capita growth decreases the growth rate of the corporate saving rate by around 0.3 percentage points. For a high-income country with GDP per capita equal to USD 50,000 the effect is much larger. For that country, the estimates in column (2) of Table 4 suggest that a 1 percentage point increase in GDP per capita growth decreases the growth rate of the corporate saving rate by around 1.5 percentage points.

Column (3) of Table 4 shows that GDP per capita growth has a larger negative effect on the growth rate of the corporate saving rate, the larger is the credit-to-GDP ratio. This can be seen from the significant negative coefficient on $\Delta \ln(y_{it}) * \lambda_i$, which is the interaction between the $t-1$ to t change in the log of GDP per capita and countries' average credit-to-GDP ratio over the period 1992–2016. The estimated coefficient on this interaction term is -0.025 and has a standard error of 0.010. This coefficient can be interpreted as follows. For each 10 percentage points increase in the credit-to-GDP ratio, the effect of a 1 percentage point increase in the growth rate of GDP per capita reduces the growth rate of the corporate saving rate by an additional 0.25 percentage points.

6. Conclusion

Based on panel data covering 130 countries over the period 1960–2017, we find that the income elasticity of the national saving rate is significantly decreasing in GDP per capita and the credit-to-GDP ratio. There exists a threshold of GDP per capita above which the income elasticity of the national saving rate is negative. The elasticity is significantly positive in low-income countries. For the majority of high-income countries, it is significantly negative. In addition, the elasticity decreases when the credit-to-GDP ratio is higher. So much so, that in countries with a low credit-to-GDP ratio GDP per capital growth increases the saving rate, while in countries with a high credit-to-GDP ratio the opposite is the case.

To explain the empirical findings, we build a model in which entrepreneurs are credit-constrained and investment projects are indivisible. The credit constraint creates rents for entrepreneurs. The indivisible investment size does not permit all agents to obtain credit to finance entrepreneurial activities. This creates dynamic incentives for entrepreneurs to save more and rely less on external funds. The resulting saving behavior of entrepreneurs generates the relationship between GDP per capita growth, the national saving rate, and the credit constraint. We present supporting evidence for our theoretical findings by utilizing cross-country time series data of the number of new businesses registered and the corporate saving rate.

Acknowledgements. This is a substantially revised version of an earlier paper "Effects of Income Growth on Domestic Saving Rates: The Role of Poverty and Borrowing Constraints." We acknowledge financial support of Singapore Ministry of Education Academic Research Fund Tier 1 (R-122-000-112-112), the Lee Kuan Yew School of Public Policy, National University of Singapore, City University of New York PSC-CUNY Research Award (60124-00 48), Korea University (K2009811) and BrainKorea21 Plus (K1327408 and T192201). We thank Hiro Kasahara, Aditya Goenka, Liugang Sheng and Chong Kee Yip for their valuable comments.

Notes

1 We provide estimates of the growth-saving relationship over the post-global financial crisis period 2007–2017 in Appendix F of Brueckner et al. (2021).

2 In a standard two-period overlapping generation model, output in every period is produced by combining capital provided by the old and labor provided by the young. The young then receive the wage—the wealth of the young—and decide how much to save. This is in contrast to a standard growth model, in which the wage, saving, and output are determined simultaneously by a representative agent, who maximizes utility over an infinite horizon.

3 This means the young in the Matsuyama model are credit-rationed.

4 Kikuchi and Vachadze (2018) and Hillebrand et al. (2018) introduce intertemporal decision into the Matsuyama model too. However, Kikuchi and Vachadze (2018) only considers a symmetric equilibrium, in which everyone chooses the same level of saving and Hillebrand et al. (2018) considers a model, in which there are a fixed fraction of different types of agents. To the best of our knowledge, our model is the only extension of the Matsuyama model, in which ex ante identical agents make different saving decisions.

5 See Levine (1997) for a comprehensive survey of both the theoretical and empirical literature. The literature has identified different channels, through which credit constraints may adversely affect output per capita. Galor and Zeira (1993), for example, shows that credit constraints can create persistence in initial wealth inequalities by preventing children of poor families from obtaining human capital. Credit constraints can also reduce occupational mobility (e.g. Banerjee and Newman, 1993; Aghion, 1995; Piketty, 1997) and prohibit high ability workers from becoming entrepreneurs (e.g. Lloyd-Ellis and Bernhardt, 2000; Matsuyama, 2000)

6 Ghatak et al. (2001) refers to the dynamic incentives for the young to work hard and save in order to become self-financed entrepreneurs as the American Dream effect. The role of the credit constraint for encouraging saving to set up businesses is well documented. An excellent summary can be found in Ghatak et al. (2001). In a study using US data, Buera (2009) documents that people, who eventually become entrepreneurs, save more than people who expect to remain workers.

7 The reform-triggered transitional dynamics is motivated by the historical accounts of the so-called miracle economies such as China, Japan, Korea, Malaysia, Singapore, Taiwan, and Thailand.

8 Due to non-stationarity of time series of GDP per capita and the savings rate, we use first differences of the variables for our regression analysis.

9 See Brückner et al. (2012*a*,b). For an application of this IV strategy to US states, see Acemoglu et al. (2013).

10 Note that GDP per capita, y_i, is measured in thousands; see also Table 7 in Brueckner et al. (2021). Hence, $y_i = 0.6$ refers to a country with GDP per capita of USD 600.

11 "Labor" can be interpreted broadly to include any endowment held by young agents, while "capital" can be interpreted broadly to include human capital or any other reproducible good used in production.

12 See footnote 13 in Matsuyama (2004).

13 The main reasons, identified in the literature, why potential entrepreneurs save more than other investors are (1) to accumulate the minimal capital requirements needed to engage in entrepreneurship and to implement projects as in our paper; (2) to hedge against uninsurable entrepreneurial risks; or (3) to cover the cost of external financing as in Ghatak et al. (2001).

14 Lemma 3 demonstrates that $\pi(w, \lambda)$ is strictly increasing in w where $\pi(w, \lambda) = s(w, \lambda)w$.

15 For example, if the production function is Cobb–Douglas, $f(k_t) = Ak_t^{\alpha}$ where $A > 0$ is the total factor productivity and $\alpha \in (0, 1)$ is the capital share, $w_t = (1 - \alpha)y_t$. See Appendix A for a Cobb–Douglas example of the dynamics.

16 There is a fraction of the population that works (called the young in the OLG model) and a fraction of the population that is retired (called the old in the OLG model). For estimation of the econometric model, we need variation in the data for the variables of interest; that is, variation that comes from the cross section and variation that comes from the time series. Regarding the time series dimension, the panel data we use are at an annual frequency; this maximizes observations. In any given year, there is a fraction of the population that works and a fraction of the population that is retired.

17 Observe $\ln g(x_t) \approx \ln g(x_{t-1}) + \frac{g'(x_{t-1})}{g(x_{t-1})}(x_t - x_{t-1})$. Since $\Delta \ln x_t = \ln x_t - \ln x_{t-1} \approx \frac{x_t - x_{t-1}}{x_{t-1}}$, it follows that $\ln g(x_t) \approx \ln g(x_{t-1}) + \frac{x_{t-1} g'(x_{t-1})}{g(x_{t-1})} \Delta \ln x_t$.

References

Acemoglu, D., A. Finkelstein, and M. J. Notowidigdo (2013) Income and health spending: Evidence from oil price shocks. *Review of Economics and Statistics* 95, 1079–1095.

Aghion, P. (1995) A trickle-down theory of growth and development with debt overhang. *Review of Economic Studies* 64, 131–172.

Aiyagari, S. R. (1994) Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics* 109, 659–684.

Antràs, P. (2001) Transitional dynamics of the savings rate in the neoclassical growth model. *mimeo*.

Banerjee, A. V. and A. F. Newman (1993) Occupational choice and the process of Development. *Journal of Political Economy* 101, 274–298.

Bewley, T. (1986) Stationary monetary equilibrium with a continuum of independently fluctuating consumers. In W. Hildenbrand and W. Mas-Colell (eds.), *Contributions to Mathematical Economics in Honor of Gérard Debreu* pp. 79–102. Amsterdam: North-Holland.

Brückner, M., A. Chong, and M. Gradstein (2012a) Estimating the permanent income elasticity of government expenditures: Evidence on Wagner's law based on oil price shocks. *Journal of Public Economics* 96, 1025–1035.

Brückner, M., A. Ciccone and A. Tesei (2012b) Oil price shocks, income, and democracy. *Review of Economics and Statistics* 94, 389–399.

Brueckner, M., T. Kikuchi and G. Vachadze (2021) Transitional Dynamics of the Saving Rate and Economic Growth. arXiv:2012.15435v3

Buera, F. J. (2009) A dynamic model of entrepreneurship with borrowing constraints: Theory and evidence. *Annals of Finance* 5, 443–464.

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

Buera, J. F. and Y. Shin (2013) Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy* 121, 221–272.
Chen, K., A. İmrohoroğlu and S. İmrohoroğlu (2006) The Japanese saving rate. *American Economic Review* 96, 1850–1858.
Chen, P., L. Karabarbounis and B. Neiman (2017) The global rise of corporate saving. *Journal of Monetary Economics* 89, 1–19.
Christiano, J. L. (1989) Understanding Japan's saving rate: The reconstruction hypothesis. *Federal Reserve Bank of Minneapolis Quarterly Review* 13, 10–25.
Deaton, A. (1991) Saving and liquidity constraints. *Econometrica* 59, 1221–1248.
Galor, O. and J. Zeira (1993) Income distribution and macroeconomics. *Review of Economic Studies* 60, 35–52.
Ghatak, M., M. Morelli and T. Sjöström (2001) Occupational choice and dynamic incentives. *Review of Economic Studies* 68, 781–810.
Hamilton, D. J. (2009) Understanding crude oil prices. *The Energy Journal* 30, 179–206.
Heston, A., R. Summers and B. Aten (2011) *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.
Hillebrand, M., T. Kikuchi, and M. Sakuragawa (2018) Bubbles and crowding-in of capital via a savings glut. *Macroeconomic Dynamics* 22, 1238–1266.
Jappelli, T. and M. Pagano (1994) Saving, growth, and liquidity constraints. *The Quarterly Journal of Economics* 109, 83–109.
Kikuchi, T. and J. Stachurski (2009) Endogenous inequality and fluctuations in a two-country model. *Journal of Economic Theory* 144, 1560–1571.
Kikuchi, T., J. Stachurski and G. Vachadze (2018) Volatile capital flows and financial integration: The role of moral hazard. *Journal of Economic Theory* 176, 170–192.
Kikuchi, T. and G. Vachadze (2015) Financial liberalization: Poverty trap or chaos. *Journal of Mathematical Economics* 59, 1–9.
Kikuchi, T. and G. Vachadze (2018) Minimum investment requirement, financial market imperfection and self-fulfilling belief. *Journal of Evolutionary Economics* 28, 305–332.
King, G. R. and S. T. Rebelo (1993) Transitional dynamics and economic growth in the neoclassical model. *American Economic Review* 83, 908–931.
Laitner, J. (2000) Structural change and economic growth. *Review of Economic Studies* 67, 545–561.
Levine, R. (1997) Financial development and economic growth: Views and agenda. *Journal of Economic Literature* 35, 688–726.
Levine, K. D. and W. R. Zame (2002) Does market incompleteness matter? *Econometrica* 70, 1805–1839.
Lloyd-Ellis, H. and D. Bernhardt (2000) Enterprise, inequality and economic development. *Review of Economic Studies* 67, 147–168.
Loayza, N., K. Schmidt-Hebbel and L. Servén (2000) What drives private saving across the world? *Review of Economics and Statistics* 82, 165–181.
Matsuyama, K. (2000) Endogenous inequality. *Review of Economic Studies* 67, 743–759.
Matsuyama, K. (2004) Financial market globalization, symmetry-breaking, and endogenous inequality of nations. *Econometrica* 72, 853–884.
Pesaran, M. H. and R. Smith (1995) Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics* 68, 79–113.
Piketty, T. (1997) The dynamics of the wealth distribution and the interest rate with credit rationing. *Review of Economic Studies* 64, 173–189.
The World Bank (2012) *World Development Indicators 2012*, Washington, DC: Development Data Group, The World Bank.
The World Bank (2020) *World Development Indicators 2020*, Washington, DC: Development Data Group, The World Bank.

Appendix A. A Cobb–Douglas example

Suppose that the production function is Cobb–Douglas, that is, $f(k) = k^\alpha$ where $\alpha \in (0, 1)$. It follows that $w(k) = (1-\alpha)k^\alpha$, $R^+ = (\frac{2}{1-\alpha})^{\frac{1}{\alpha}}$ and $w'(0) = \infty$. This implies that the corner steady state is always locally unstable and there exists either a unqiue interior steady state or an odd number of interior steady states, which solve $\Pi(w, \lambda) = R$ where $\Pi(w, \lambda) = \frac{w^{-1}(w)}{s(w,\lambda)w}$. If

$$\frac{w\Pi_1(w, \lambda)}{\Pi(w, \lambda)} = \frac{1-\alpha}{\alpha} - \frac{ws_1(w, \lambda)}{s(w, \lambda)} > 0 \tag{17}$$

that is, if the elasticity of output is small relative to the elasticity of saving ($\alpha < 1/2$ is sufficient), $\Pi(w, \lambda)$ is monotonically increasing in w and thus there exists a unique interior steady state. Let $w^*(R, \lambda)$ denote the unique steady state. Suppose that $w^*(R, \lambda) > 1 - \lambda$. If $w_0 < 1 - \lambda$, then the saving rate s_t first increases and then decreases as w_t (or y_t) converges to the steady state in the long run.

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Appendix B. Remaining proofs

We eliminate time subscripts for notational convenience.

Proof of Proposition 1: Let

$$s_1^b = \tfrac{1}{2}\left(1 - \tfrac{\phi-1}{w}\right) \text{ and } s_2^b = \tfrac{1-\lambda\phi}{w}. \tag{18}$$

We can easily verify that $s = s_1^b$ solves the unconstrained optimization problem of entrepreneurs:

$$U^b = \max_{s\in[0,1]} \left\{(1-s)\left(\frac{\phi-1}{w} + s\right)\right\}. \tag{19}$$

If $w \geq 1-(2\lambda-1)\phi$, then $s_1^b \geq s_2^b$ and thus entrepreneurs can overcome the credit constraint if their saving rate is s_1^b. In such case, $U^b = \frac{1}{4}(1+\frac{\phi-1}{w})^2$.

If $w \in [1-\lambda\phi, 1-(2\lambda-1)\phi)$, then $s_1^b < s_2^b \leq 1$ and thus entrepreneurs can overcome the credit constraint if their saving rate is s_2^b. In such case, $U^b = (1 - \frac{1-\lambda\pi}{w})\frac{(1-\lambda)\phi}{w}$.

If $w < 1-\lambda\phi$, then $s_1^b < 1 < s_2^b$ and thus entrepreneurs cannot overcome the credit constraint even if they save their entire wage. □

Proof of Proposition 2: If $w \geq 1-(2\lambda-1)\phi$, then it follows from (8) that $U_t^b = U^\ell \Leftrightarrow \phi = 1$. Hence, $w \geq 1-(2\lambda-1)\phi \Leftrightarrow w \geq 2(1-\lambda)$. If $w \in [1-\lambda\phi, 1-(2\lambda-1)\phi)$, then it follows from (8) that $U_t^b = U^\ell \Leftrightarrow \phi = \frac{1}{2\lambda}\left(1 - w + \sqrt{1-2w+\frac{w^2}{1-\lambda}}\right)$. Hence, $w \in [1-\lambda\phi, 1-(2\lambda-1)\phi) \Leftrightarrow w \in [0, 2(1-\lambda))$. □

Lemma 1.

(a) For $\lambda \in (0,1)$, the entrepreneurial rent $\phi(w,\lambda)$ is a continuous and strictly decreasing function on $w \in (0, 2(1-\lambda))$ and satisfies the following boundary properties:

$$\lim_{w\downarrow 0} \phi(w,\lambda) = \tfrac{1}{\lambda} \text{ and } \lim_{w\uparrow 2(1-\lambda)} \phi(w,\lambda) = 1. \tag{20}$$

(b) For $w \in (0, 2(1-\lambda))$, $\phi(w,\lambda)$ is a strictly decreasing function, while $\lambda\phi(w,\lambda)$ is a strictly increasing function on $\lambda \in (0,1)$.

Proof of Lemma 1: If $\lambda \in (0,1)$ and $w < 2(1-\lambda)$, then the entrepreneurial rent is

$$\phi(w,\lambda) = \tfrac{1-w+\psi(w,\lambda)}{2\lambda} \text{ where } \psi(w,\lambda) := \sqrt{1-2w+\tfrac{w^2}{1-\lambda}}. \tag{21}$$

(a) Differentiating both sides of (21) with respect to w and re-arranging terms, we obtain

$$\phi_1(w,\lambda) = \frac{1}{\psi(w,\lambda)}\left(\frac{w}{2(1-\lambda)} - \phi(w,\lambda)\right) < 0 \tag{22}$$

because when $w \in (0, 2(1-\lambda))$, $\frac{w}{2(1-\lambda)} < 1 < \phi(w,\lambda)$ and $\psi(w,\lambda) \in (1, 1/\lambda)$. This implies monotonicity of $w \mapsto \phi(w,\lambda)$. Taking limits of both sides of (21), we obtain the boundary properties of ϕ, which along with $\phi(w,\lambda) \equiv 1$ for $w \geq 2(1-\lambda)$ imply continuity of ϕ.

(b) Differentiating both sides of (21) with respect to λ and re-arranging terms, we obtain

$$\frac{\lambda\phi_2(w,\lambda)}{\phi(w,\lambda)} = \frac{w^2}{4(1-\lambda)^2}\frac{1}{\psi(w,\lambda)\phi(w,\lambda)} - 1 \in (-1, 0) \tag{23}$$

because when $w \in (0, 2(1-\lambda))$, $\frac{w}{2(1-\lambda)} < 1 < \phi(w,\lambda)$ and $\psi(w,\lambda) \in (1, 1/\lambda)$. The monotonicity properties of $\lambda \mapsto \phi(w,\lambda)$ and $\lambda \mapsto \lambda\phi(w,\lambda)$ are implied by (23). □

Lemma 2.

(a) For $\lambda \in (0,1)$, the saving rate of entrepreneurs $s^b(w,\lambda)$ is a strictly decreasing function on $w \in (0, 2(1-\lambda))$ and satisfies the following boundary properties:

$$\lim_{w\downarrow 0} s^b(w,\lambda) = 1 \text{ and } \lim_{w\uparrow 2(1-\lambda)} s^b(w,\lambda) = \tfrac{1}{2}. \tag{24}$$

(b) For $w \in (0, 2(1-\lambda))$, $s^b(w,\lambda)$ is a strictly decreasing function on $\lambda \in (0,1)$.

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Proof of Lemma 2:

(a) In equilibrium $U^b = U^\ell \Leftrightarrow$

$$\left(1 - \frac{1}{w} + \frac{\lambda\phi(w,\lambda)}{w}\right)\frac{(1-\lambda)\phi(w,\lambda)}{w} = \frac{1}{4} \Leftrightarrow \frac{1-\lambda\phi(w,\lambda)}{w} = 1 - \frac{w}{4(1-\lambda)\phi(w,\lambda)}. \tag{25}$$

Monotonicity and boundary properties of $w \mapsto \phi(w,\lambda)$ with (24) imply monotonicity and boundary properties of $w \mapsto s^b(w,\lambda)$.

(b) Monotonicity of $\lambda \mapsto s^b(w,\lambda)$ follows from Lemma 1. □

Lemma 3.

(a) For $\lambda \in (0,1)$, the national saving rate:

$$s(w,\lambda) \equiv \begin{cases} \frac{1}{w+2\lambda\phi(w,\lambda)} & \text{if } w < 2(1-\lambda) \\ \frac{1}{2} & \text{if } w \geq 2(1-\lambda) \end{cases} \tag{26}$$

first increases and then decreases on $w \in (0, 2(1-\lambda))$ achieving its local maximum at $w = 1-\lambda$ and satisfying the boundary properties:

$$\lim_{w\downarrow 0} s(w,\lambda) = \lim_{w\uparrow 2(1-\lambda)} s(w,\lambda) = \tfrac{1}{2} \text{ and } \lim_{w\to 1-\lambda} s(w,\lambda) = \tfrac{1}{\lambda}. \tag{27}$$

(b) For $\lambda \in (0,1)$, the fraction of entrepreneurs $\pi(w,\lambda) = s(w,\lambda)w$ is an increasing function on $w > 0$ satisfying the boundary properties:

$$\lim_{w\downarrow 0} \pi(w,\lambda) = \tfrac{1}{2} \text{ and } \lim_{w\uparrow 2(1-\lambda)} \pi(w,\lambda) = \tfrac{\lambda}{2}. \tag{28}$$

(c) For $w \in (0, 2(1-\lambda))$, $s(w,\lambda)$ and $\pi(w,\lambda)$ are both strictly decreasing functions on $\lambda \in (0,1)$.

Proof of Lemma 3:

(a) It follows from (21) and (26) that

$$s(w,\lambda) = \frac{1}{w + 2\lambda\phi(w,\lambda)} = \frac{1}{1+\psi(w,\lambda)} \tag{29}$$

where ψ is defined in (21). Differentiating both sides of (29) and using the definition of ψ, we obtain

$$\frac{ws_1(w,\lambda)}{s(w,\lambda)} = \frac{[s(w,\lambda)]^2 w}{1-s(w,\lambda)}\left(1 - \frac{w}{1-\lambda}\right) \text{ and } \frac{\lambda s_2(w,\lambda)}{s(w,\lambda)} = -\frac{\lambda[s(w,\lambda)]^2}{2(1-s(w,\lambda))}\left(\frac{w}{1-\lambda}\right)^2 \tag{30}$$

where $s_1(w,\lambda) := \frac{\partial s(w,\lambda)}{\partial w}$ and $s_2(w,\lambda) := \frac{\partial s(w,\lambda)}{\partial \lambda}$. This implies that $s(w,\lambda)$ is strictly increasing on $w \in (0, 1-\lambda)$ and decreasing on $w \in (1-\lambda, 2(1-\lambda))$. This with the boundary properties of $s(w,\lambda)$ implies that the national saving rate is hump-shaped on $w \in (0, 2(1-\lambda))$ achieving its maximum at $w = 1-\lambda$.

(b) It follows from the definition of the national saving rate that

$$\pi(w,\lambda) = \frac{w}{w+2\lambda\phi(w,\lambda)} = \frac{1}{1+\frac{2\lambda\phi(w,\lambda)}{w}}. \tag{31}$$

Monotonicity of $w \mapsto \frac{\phi(w,\lambda)}{w}$ implies that $w \mapsto \pi(w,\lambda)$ is a strictly increasing function. In addition

$$\pi_1(w,\lambda) = s(w,\lambda)\left(1 - \frac{s(w,\lambda)w}{\psi(w,\lambda)}\left(\frac{w}{1-\lambda} - 1\right)\right). \tag{32}$$

This with the boundary properties of $s(w,\lambda)$ implies the boundary properties of $\pi(w,\lambda)$.

(c) Monotonicity of $\lambda \mapsto s(w,\lambda)$ and $\lambda \mapsto \pi(w,\lambda)$ follows from Lemma 1 and from definitions of s and π. □

Appendix C. Time discount and flexible investment size

This section shows that we can relax our assumptions of a zero time discount and a fixed investment size and still obtain essentially the same results. Suppose the agent's lifetime utility is $\ln c_{1t} + \beta \ln c_{2t+1}$ and that capital is produced by the following technology:

$$F(i_t) = \begin{cases} 0 & \text{if } i_t < I \\ Ri_t & \text{if } i_t \geq I \end{cases}$$

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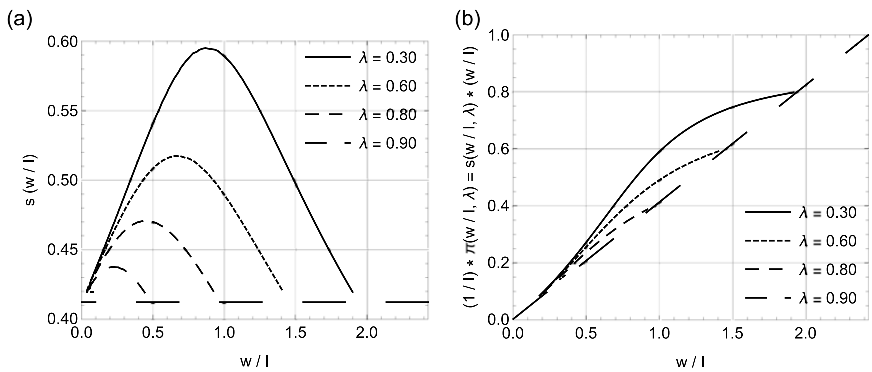

$s\left(\frac{w_t}{I}, \lambda\right)$ $\qquad$ $\frac{1}{I}\pi\left(\frac{w_t}{I}, \lambda\right) = s\left(\frac{w_t}{I}, \lambda\right)\frac{w_t}{I}$

Figure 4. The national saving rate and the fraction of entrepreneurs when $\beta = 0.70$.

where i_t is the investment of the final good, $F(i_t)$ is the produced amount of capital, and I is the minimum investment size. The lifetime utility of investors is $\ln U^\ell + \ln(w_t^{1+\beta} r_{t+1}^\beta)$ where $U^\ell = \max_{s\in[0,1]}\{(1-s)s^\beta\}$. This implies that $s^\ell = \frac{\beta}{1+\beta}$ and $U^\ell = \frac{\beta^\beta}{(1+\beta)^{1+\beta}}$. The lifetime utility of entrepreneurs is $\ln U^b(w_t/I, \phi_{t+1}) + \ln(w_t^{1+\beta} r_{t+1}^\beta)$ where

$$U^b(w_t/I, \phi_{t+1}, \lambda) = \max_{s\in[0,1]} \left\{ (1-s)\left(\frac{\phi_{t+1}-1}{w_t/I} + s\right)^\beta \middle| s \geq \frac{1-\lambda\phi_{t+1}}{w_t/I} \right\}.$$

This implies that the optimal saving rate of entrepreneurs is

$$s_t^b = \max\left\{ \frac{1}{1+\beta}\left(\beta - \frac{\phi_{t+1}-1}{w_t/I}\right), \frac{1-\lambda\phi_{t+1}}{w_t/I} \right\}$$

and

$$U^b(w/I, \phi, \lambda) = \begin{cases} \frac{\beta^\beta}{(1+\beta)^{1+\beta}}\left(1 + \frac{\phi-1}{w/I}\right)^{1+\beta} & \text{if } \frac{w}{I} \geq 1 - \frac{(1+\beta)\lambda - 1}{\beta}\phi \\ \left(1 - \frac{1-\lambda\phi}{w/I}\right)\left(\frac{(1-\lambda)\phi}{w/I}\right)^\beta & \text{if } \frac{w}{I} \in \left[1-\lambda\phi, 1 - \frac{(1+\beta)\lambda-1}{\beta}\phi\right). \end{cases}$$

If $\frac{w_t}{I} < 1 - \lambda\phi_{t+1}$, then young agents cannot become an entrepreneur because they cannot overcome the credit constraint even if they save the entire wage. The equilibrium entrepreneurial rent is $\phi_{t+1} = 1$ when $\frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}$ and $\phi_{t+1} = \phi(\frac{w_t}{I}, \lambda)$, which solves

$$\left(1 - \frac{1-\lambda\phi_{t+1}}{w_t/I}\right)\left(\frac{(1-\lambda)\phi_{t+1}}{w_t/I}\right)^\beta = \frac{\beta^\beta}{(1+\beta)^{1+\beta}}$$

when $1 - \lambda\phi_{t+1} \leq \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta}$. There is no closed-form solution of the above equation when $\beta \neq 1$. However, we can show that the properties of ϕ demonstrated in Lemma 1 hold for $\beta \neq 1$ as well. The credit market clears when

$$\frac{s_t w_t}{I}\left(I - s_t^b w_t\right) = \left(1 - \frac{s_t w_t}{I}\right)\frac{\beta w_t}{1+\beta}.$$

The saving rate of entrepreneurs is

$$s^b\left(\frac{w_t}{I}, \lambda\right) = \begin{cases} \frac{1-\lambda\phi(w_t/I,\lambda)}{w_t/I} & \text{if } \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta} \\ \frac{\beta}{1+\beta} & \text{if } \frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}. \end{cases}$$

The fraction of entrepreneurs is

$$s\left(\frac{w_t}{I}, \lambda\right) = \begin{cases} \frac{\beta}{\beta\frac{w_t}{I} + (1+\beta)\lambda\phi(w_t/I,\lambda)} & \text{if } \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta} \\ \frac{\beta}{1+\beta} & \text{if } \frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}. \end{cases}$$

Figure 4 shows the national saving rate and the fraction of entrepreneurs when $\beta = 0.70$. The figure indicates that the properties of the saving rate hold under a more general specification of the basic model.

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

Appendix D. Tables

Table 5. List of countries

Country	GDP p.c. (y) (in thousands)	Credit/GDP (λ)	Country	GDP p.c. (y) (in thousands)	Credit/GDP (λ)
Albania	3.171	0.092	Indonesia	3.142	0.319
Algeria	2.912	0.333	Iran	4.22	0.227
Angola	3.053	0.049	Ireland	13.204	0.604
Argentina	6.986	0.185	Israel	10.699	0.566
Armenia	5.062	0.079	Italy	13.643	0.644
Australia	14.364	0.501	Jamaica	4.439	0.238
Austria	14.83	0.738	Japan	14.311	1.517
Azerbaijan	4.316	0.064	Mauritius	9.942	0.428
Bahrain	14.837	0.442	Mexico	5.268	0.224
Bangladesh	1.268	0.175	Mongolia	1.87	0.154
Barbados	13.165	0.496	Morocco	2.468	0.261
Belarus	13.087	0.127	Mozambique	1.299	0.131
Belize	5.672	0.363	Nepal	0.897	0.129
Benin	0.746	0.156	Netherlands	14.625	0.863
Bolivia	1.915	0.258	New Zealand	11.354	0.552
Bosnia & Herz.	4.445	0.426	Nicaragua	1.601	0.25
Brazil	4.604	0.423	Niger	0.588	0.094
Bulgaria	6.262	0.368	Nigeria	0.849	0.11
Burkina Faso	0.663	0.108	Norway	16.985	0.484
Burundi	0.506	0.097	Oman	10.919	0.249
Cambodia	1.819	0.074	Pakistan	1.499	0.245
Cameroon	1.491	0.167	Panama	3.426	0.606
Canada	15.04	0.817	Papua New Guinea	1.459	0.185
Central Afr. Rep.	0.651	0.104	Paraguay	2.791	0.198
Chad	0.874	0.079	Peru	2.984	0.168
Chile	6.164	0.442	Philippines	2.084	0.271
China	2.624	0.874	Poland	8.417	0.276
Colombia	3.42	0.283	Portugal	8.506	0.749
Congo, Rep. of	1.421	0.145	Qatar	30.938	0.299
Costa Rica	5.111	0.234	Romania	6.685	0.149
Croatia	9.394	0.401	Russia	8.37	0.187
Cyprus	12.216	1.238	Rwanda	0.794	0.063
Czech Republic	15.303	0.487	Samoa	3.888	0.243
Denmark	14.335	0.642	Senegal	1.144	0.22
Djibouti	3.651	0.354	Sierra Leone	1.326	0.048
Dom. Republic	3.527	0.231	Singapore	14.802	0.744
Ecuador	3.029	0.217	Slovenia	16.958	0.382
Egypt	2.282	0.28	South Africa	5.125	0.902
El Salvador	2.88	0.303	Spain	11.61	0.79
Eq. Guinea	5.549	0.096	Tajikistan	2.2	0.173

https://doi.org/10.1017/S1365100521000493 Published online by Cambridge University Press

Table 5. Continued

Country	GDP p.c. (y) (in thousands)	Credit/GDP (λ)	Country	GDP p.c. (y) (in thousands)	Credit/GDP (λ)
Estonia	11.733	0.494	Tanzania	0.626	0.087
Ethiopia	0.746	0.151	Thailand	3.498	0.657
Fiji	2.983	0.286	Togo	0.684	0.185
Finland	13.084	0.565	Trinidad &Tobago	7.552	0.336
France	13.184	0.806	Tunisia	3.919	0.514
Gabon	4.584	0.147	Turkey	3.179	0.182
Gambia, The	0.823	0.134	Turkmenistan	6.589	0.017
Georgia	4.983	0.105	Uganda	0.58	0.066
Germany	17.353	0.913	Ukraine	6.208	0.191
Ghana	0.925	0.071	United Arab Emir.	32.473	0.29
Greece	10.546	0.354	United Kingdom	13.117	0.737
Guatemala	3.043	0.167	United States	18.805	1.219
Guinea-Bissau	0.641	0.088	Uruguay	5.732	0.323
Guyana	1.562	0.333	Venezuela	5.365	0.29
Haiti	1.431	0.138	Vietnam	2.432	0.43
Honduras	1.898	0.294	Yemen	0.928	0.055
Hungary	10.36	0.399	Zambia	1.006	0.119
Iceland	15.676	0.628	Zimbabwe	2.238	0.298
India	1.307	0.213			

Cite this article: Brueckner M, Kikuchi T and Vachadze G (2023). "Transitional dynamics of the saving rate and economic growth." *Macroeconomic Dynamics* **27**, 482–505. https://doi.org/10.1017/S1365100521000493