BRUN, V. (1932). Gauss' fordelingslov. *Norsk Mat. Tidsskr.* XIV, 81.

[*c*, *d*, *e*, *ō*]

AUERBACH, H. (1933). Über die Fehlerwahrscheinlichkeit einer Summe von Dezimalzahlen. *Z. angew. Math. Mech.* XIII, 386.

[*c*, *e*, *ō*]

Two comments may be made on the above list. It will be noticed that I disagree with Mr Packer that Reitz's proof was any simpler than Laplace's. In fact it was Laplace's own derivation (for which, in modern notation, see the Appendix to my paper in the 1949 Swiss *Bulletin*) with only formal differences. Secondly, it may be mentioned that a 3-decimal table similar to Mr Packer's Table 3 is provided by Auerbach in the paper cited.

I hasten to assure you that the provision of this list, which contains four original proofs between Laplace and Rietz, is not intended as a criticism of Mr Packer's excellent note. The modest title of your *Journal* would, in any case, forbid the scoring of points on the part of your averagely priggish correspondent who signs himself

Yours faithfully,

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The variance-ratio distribution

Sirs,

The formula given by Bizley (*J.S.S.* X, 62) for the probability distribution of the variance-ratio distribution is certainly useful when the smaller of n_1 or n_2 is not too large to make the computation tedious. As a matter of historical record it should perhaps, however, be pointed out that the result, which is essentially obtained by integration by parts, has been known for a long time. As long ago as 1924 Karl Pearson (*Biometrika*, XVI, 202) showed that the ratio of the sum of the first $(n+1)$ terms of the binomial series $(1+u)^m$ to the whole sum was the Incomplete Beta Function

$$I_{(1+u)^{-1}}(m-n,\, n+1),$$

from which it can be inferred that the corresponding result for the binomial $(k+u)^m$ is $$I_{k(k+u)^{-1}}(m-n,\, n+1).$$

Now in the second of my two notes to the *Journal* on this subject (*J.S.S.* VII, 98), I showed that the integral of $f(u)\,du$ from u to infinity was

$$I_x(\tfrac{1}{2}n_2,\, \tfrac{1}{2}n_1),$$

where $x=n_2(n_2+n_1u)^{-1}$. In Bizley's notation x is $k(k+u)^{-1}$, while $\tfrac{1}{2}n_2=m-n$ and $\tfrac{1}{2}n_1=n+1$. This identifies the two results; what Bizley shows is that the required integral can be computed by working out the first $(n+1)$ terms of the binomial $(k+u)^m$, and dividing their sum by $(k+u)^m$, thus directly using a known result which has a habit of being re-discovered from time to time. The integral can, of course, be obtained directly from the *Tables of the Incomplete Beta Function* (1934; London: 'Biometrika' Office).

Yours faithfully,

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