LARGE SCALE CLUSTERING IN THE UNIVERSE

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1. INTRODUCTION

In 1924 Hubble presented the first generally convincing evidence, from the identification of variable stars, that some of the brightest "spiral nebulae" are galaxies of stars well outside the bounds of our own Galaxy. This led him to reconsider the idea that the faint "spiral nebulae," which were known to be much more abundant than bright ones, might be similar objects at greater distances. If the galaxies were uniformly distributed through space the number brighter than apparent magnitude m would vary as

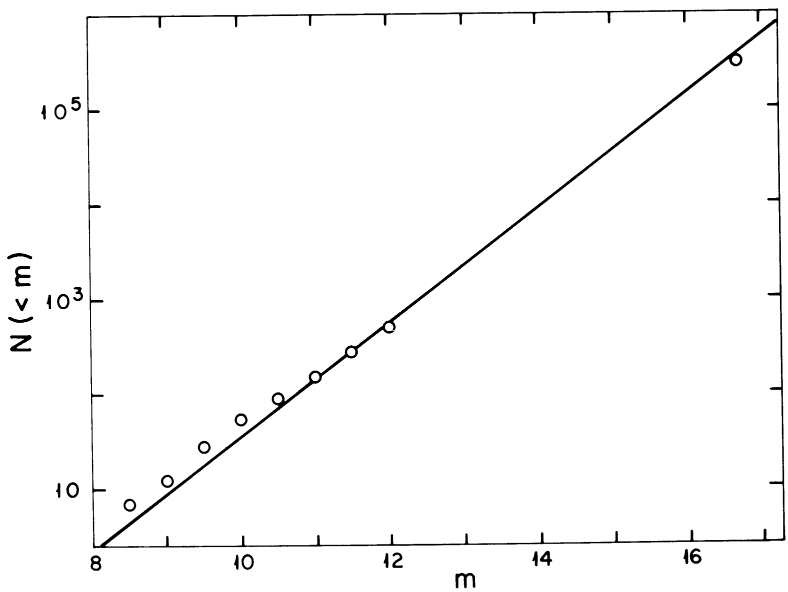

Fig. 1. Hubble's test of the distribution of galaxies.

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$$\log N(< m) = 0.6\ m + A.$$

Hubble (1926) tested this relation with the data shown in figure 1. The point at the extreme right hand side is based on Fath's (1914) counts of nebulae with diameters greater than $\sim 4''$. The straight line shows the slope expected under the homogeneity assumption. The agreement is very encouraging, though, as we now recognize, it must be in part fortuitous because the counts at $m \lesssim 12$ are influenced by the concentration of galaxies in and around the Virgo cluster, that is, the Local Supercluster, and also in the Local Group. Shapley (1938 and earlier references therein) persistently emphasized that the homogeneity assumption surely is not the whole truth for the galaxy distribution is strongly clustered on scales at least as large as that of the Local Supercluster, and he even ventured to ask whether there really is an ultimate scale on which the universe appears close to homogeneous. However, most people were quick to accept Hubble's conclusion that the galaxy distribution is fairly uniform. Hubble (1926) pointed out that this agrees with Einstein's (1917) homogeneous world model. Milne (1933) suggested that homogeneity might be a logical consequence of what we mean by the universe. Dingle (1933) was more cautious in arguing that, at least within the framework of general relativity theory, homogeneity is not a necessary property of the world model but rather a particularly simple mathematical model subject to empirical test. However, he seemed not to be inclined to question the observational situation.

As is discussed at this conference two recent developments indirectly confirm Hubble's intuition at an accuracy much better than anything previously available. First, the microwave background is isotropic about us, on angular scales $\gtrsim 10'$ to $\delta i/i \lesssim 10^{-3}$. This does not directly measure the isotropy of the matter distribution because the coupling of the microwave radiation to matter in the present universe is very weak. It does imply some strong indirect constraints: redshift, whether interpreted as cosmological (expansion) or gravitational, must be constant to tenth percent accuracy in different directions, as must the "initial values" at the horizon. The X-ray background appears to be isotropic to ~ 1 percent accuracy on scales larger than a few degrees. Since an appreciable fraction of this radiation does come from known sources - galaxies and clusters of galaxies - this implies a similar limit on fluctuations in the matter density integrated to the horizon, $ct \sim 3000\ h^{-1}$ Mpc ($H = 100\ h$ km s^{-1} Mpc^{-1}).

Despite the remarkable precision of these two tests there remains Shapley's point - the galaxy distribution is strongly clustered, and the nature of this clustering ought to be of considerable interest to cosmology. In the first place, of course, it tells us something about what the universe is like. More directly, galaxy clustering is interesting because the relevant dynamics seems to be simple enough that we might hope to deal with it quantitatively and in some detail. Perhaps if we can see how to measure the galaxy distribution in the

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right way it will prove to be the "Rosetta Stone" by which we learn the underlying significance of the clustering of matter!

The data on hand permit fairly detailed studies of the nature of the galaxy distribution (within "local samples") on scales $\lesssim$ 10 h^{-1} Mpc. Some aspects of this are summarized in the next section. The distribution on scales $\gtrsim$ 30 h^{-1} Mpc is much less well understood. Mandelbrot (1977) has given a particularly attractive discussion of the point that the usual description of the distribution of mass in the universe by a classical continuous and differentiable function $\rho(x, t)$ [or, more generally, $T_i{}^j(x^k)$] is in a sense a fiction because ρ fluctuates on whatever scale one chooses to measure it, from the limit of classical physics, perhaps 10^{-13} cm, to at least 30 h^{-1} Mpc $\sim 10^{26}$ cm. This is a range of some 39 decades. The standard Friedman-Lemaitre cosmological model can be a reasonable approximation only because (or if!) this progression of clustering ends within the next two decades, so the density is close to uniform on the scale ct $\sim$ 3000 h^{-1} Mpc. De Vaucouleurs (1971 and earlier references therein) has emphasized that this nominal two decade hiatus in the clustering is small compared to the 39 decades that came before, so it is well to check very carefully whether the clustering really does terminate. Some aspects of the observational situation are discussed in section 3.

2. CLUSTERING ON SCALES $\lesssim$ 10 h^{-1} Mpc

The best sample of the galaxy distribution on large scales is the Lick catalog (Shane and Wirtanen 1967), which lists counts of galaxies brighter than m = 19 in 10' by 10' cells. The effective depth of the catalog is about 250 h^{-1} Mpc. A more detailed but shallower sample is the Zwicky catalog (Zwicky et. al. 1961-68), with an effective depth $\sim$ 50 h^{-1} Mpc.

One very natural choice of statistics to use in analyzing the galaxy distribution is the n-point correlation functions used in the theory of non-ideal gases. (We know this is a natural choice because in the original analyses of Neyman, Scott and Shane 1953, Limber 1954, and Rubin 1954 all, apparently independently, chose statistics closely allied to the two-point correlation function.) Recent work along this line has been reported at length in the literature (Groth and Peebles 1977 and earlier references therein) so only a few of the main results will be summarized here.

The two-point correlation function, $\xi(r)$, is the generalization to a distribution of points of the usual auto-correlation function for a continuous density function,

$$\xi(r) = \langle\rho(\vec{x})\,\rho(\vec{x} + \vec{r})\rangle/\langle\rho\rangle^2 - 1.$$

The galaxy two-point correlation function at small scales is given to good accuracy by the simple power law model

$$\xi = (r_o/r)^\gamma, \quad r_o \gtrsim 5\ h^{-1}\ \mathrm{Mpc},$$
$$\gamma = 1.77 \pm 0.04, \quad 100\ h^{-1}\ \mathrm{kpc} \lesssim r \lesssim 10\ h^{-1}\ \mathrm{Mpc}. \tag{1}$$

We can be fairly sure the estimate of ξ has not been seriously affected by systematic errors in the catalogs because the results from the independent Lick and Zwicky samples are quite similar. The galaxy three-point correlation function is known to accuracy comparable to that of ξ. The galaxy four-point function is only just detected above the noise, but again the results from the two samples are consistent. All these statistics agree with the picture that the galaxies are arranged in a hierarchical clustering pattern, with the characteristic density within clusters of size r scaling as $r^{-\gamma}$. This also agrees very well with the conclusion of de Vaucouleurs (1971 and earlier references therein) that the most dense clusters of size r have density that scale as a power of r, with index very close to γ.

3. CLUSTERING ON SCALES $\gtrsim$ 10 h^{-1} Mpc

A first interesting question is whether the power law hierarchical clustering pattern observed at $r \lesssim 10\ h^{-1}$ Mpc might simply be extrapolated to arbitrarily large scales. A cosmology of this sort has been discussed by Wertz (1971). This extrapolation would imply that the map of angular positions of galaxies brighter than m should appear equally "rough," in the ensemble average, at all m. That is, suppose galaxies are counted in cells of fixed size and shape randomly placed in the sky. Then the expected rms fluctuation in the counts of galaxies brighter than m would be independent of m (Mandelbrot 1975). This is not what is observed: the ratio of rms fluctuations in the Zwicky and Lick samples is quite constant at a value of about 3, $10' \lesssim \theta \lesssim 2^\circ$. This value is consistent with what is expected if the Zwicky catalog is a "fair sample" of the clustering in the Lick catalog.

A related aspect is the size of r_o (eq. [1]). This is a characteristic clustering scale on which the density fluctuates by a factor of about 2. If clustering extended over the full depth of the sample r_o would be expected to be comparable to the sample depth. The fact that it is much smaller in the Lick survey, $\sim 250\ h^{-1}$ Mpc, suggests that r_o has not been affected by the sample size.

In the Lick sample there is in addition to the small-scale clustering a definite large-scale variation over angular distances of perhaps 40° (Shane 1976, Groth and Peebles 1977). It is difficult to know what part of this might be due to local effects such as variable obscuration in the Galaxy, what part might be a true large-scale component in the galaxy distribution. A rough upper limit to the latter at 10 r_o is (Peebles and Hauser 1974)

$$\xi(50\ h^{-1}\ \mathrm{Mpc}) \lesssim 0.025. \tag{2}$$

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That is, the large-scale component in any case seems to be a relatively small perturbation from homogeneity in the sample.

The clustering length r_o represents a mean over the distribution, and one certainly can find some spots in the Lick sample where the density remains higher than twice the overall mean over distances substantially larger than r_o. An example is given by the cross-correlation between the Lick counts and the positions of Abell clusters. One simply "stacks" the galaxy counts around all clusters in a chosen distance and richness class to get the mean number density as a function of angular distance from the cluster center. By using the galaxy luminosity function (which is itself constrained by the variation of this "stacked" density with cluster distance class) one can find from the angular cross-correlation the mean spatial density of galaxies as a function of linear distance from a cluster center. This mean space density is twice the overall mean in the Lick sample at the distance (Seldner and Peebles 1977)

$$r_a \approx 14\ h^{-1}\ \text{Mpc}. \tag{3}$$

A second example is the clustering in the positions of the Abell clusters. The auto-correlation function $\xi_{cc}(r)$ describing the spatial distribution of Abell clusters reaches unity at the clustering length (Hauser and Peebles 1973)

$$r_s \approx 30\ h^{-1}\ \text{Mpc}, \quad \xi_{cc}(r_s) = 1. \tag{4}$$

This can be compared to the depth of the Abell (1958) catalog, $640\ h^{-1}$ Mpc.

The three lengths r_o, r_a, and r_s measure an interesting progression in the scales of galaxy clustering in general, in the concentration around rich clusters, and in the clustering of the rich clusters themselves. Only in the second case can one trace the clustering effect to distances substantially larger than the characteristic clustering length given here. (This is because systematic errors in the catalogs cause systematic errors in the auto-correlation function estimates but not in the cross-correlation function estimates if the errors are not common to both catalogs.) However, it does appear that all three lengths are comfortably smaller than the samples sizes, which suggests that the data now available are adequate to measure the large scale clustering in the universe. Of course, one would very much like to test this against deeper samples.

A deep sample that is available is the angular distribution of radio sources, most of which are at distances $\sim ct = 3000\ h^{-1}$ Mpc. Seldner (1977) has found very weak but apparently significant clustering in the 4C radio catalog. This is illustrated in figure 2, which shows the angular power spectrum for the distribution of sources in the full catalog, $S \geq 2$ Jy. This statistic is a measure of the mean number of sources in excess of that expected for a random distribution and within

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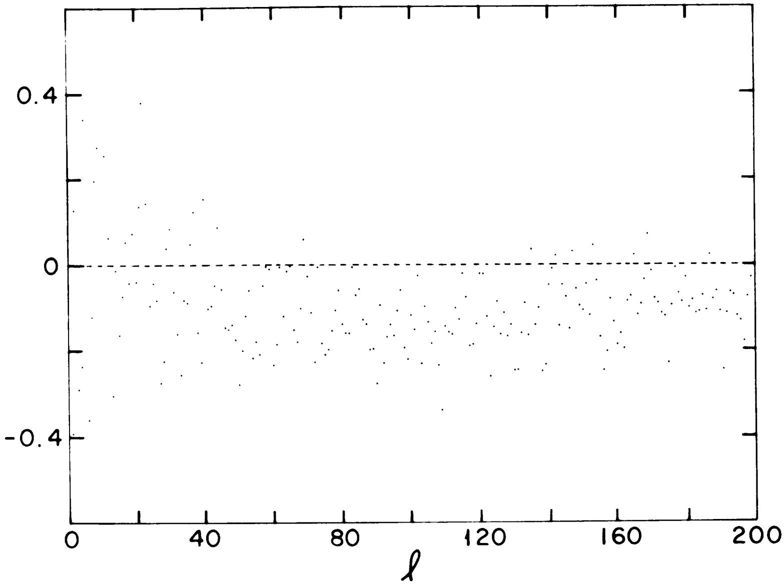

Fig. 2. Test of clustering in the 4C catalog.

angular distance $\sim 180^{\circ}/\ell$ of a randomly chosen source in the catalog. (This is the Legendre transform of the angular correlation function. The advantages of this power spectrum are that estimates at successive ℓ values are nearly statistically independent and the effects of clustering on different scales are displayed at different ℓ-values. For details see Peebles 1973.)

The power spectrum is negative, averaging about - 0.15, which says there are on the average about 0.15 <u>fewer</u> sources around a randomly chosen one than is expected for a uniform random distribution. This is known to be due to confusion: if two sources happen to be closer than $\sim 0\overset{\circ}{.}5$ in the sky they will be counted as one (Webster 1976). The spectrum reaches a minimum at $\ell \sim 100$. It rises at larger ℓ because it is starting to "resolve" the anti-correlation at $\theta \lesssim 0\overset{\circ}{.}5$ caused by confusion. Much more interesting is the fact that the spectrum at $\ell \sim 30$ ($\theta \sim 6^{\circ}$) is higher than at $\ell \sim 100$ ($\theta \sim 2^{\circ}$) by ~ 0.1. That is, the mean number of neighbors in excess of random increases by about 0.1 in going from 2° to 6° from a source, as if the sources were clustered on scales of perhaps 4°. The effect is larger if the sample is limited to a higher flux level.

Now this apparent clustering, if real, can be interpreted in two rather different ways. Most of the 4C sources are at distances $\sim 3000\ h^{-1}$ Mpc, and so one might well suppose that the clustering is among sources at about this distance. An angle of 4° subtends $\sim 200\ h^{-1}$ Mpc at this distance. To produce the observed degree of

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clustering the spatial clustering of the sources would have to be substantial because the effect is washed out by the many clusters seen in projection. A rough estimate of the wanted spatial two-point correlation function for the sources is

$$\xi_{ss}(200\ h^{-1}\ \mathrm{Mpc}) \sim 0.5. \qquad (5)$$

This goes well beyond the clustering scales mentioned above (cf. eq. [2]). The second and possibly more conservative interpretation is that we are seeing clustering in the small fraction of sources that are at distances much closer than 3000 h^{-1} Mpc. Some evidence that this is so is the fact that there is quite a substantial cross-correlation between the radio source positions and the Lick galaxy counts (at distances $\sim$ 250 h^{-1} Mpc). Thus a significant number of the 4C sources are fairly close, and, since these sources evidently tend to be near galaxies, and galaxies tend to cluster, these sources should tend to cluster. Seldner (1977) has been able to find a self-consistent model along these lines that does reproduce the observed clustering of sources around galaxies and among themselves for the range of flux levels $2 < S < 9$. The radio luminosity function that is wanted is close to standard estimates except that it is larger at the low power end, but not by a factor that seems unreasonable. In this interpretation the radio source clustering lengths are in line with equations (1), (3) and (4) rather than equation (5).

4. CONCLUSIONS

My impression is that we may understand at least the broad outlines of the large scale distribution of galaxies. The main question is, have we "fair samples" of the distribution? I think the two major and direct pieces of evidence that we do are (1) the n-point correlation functions derived from the Zwicky and Lick samples are related to each other as expected if both are fair samples, and (2) the characteristic clustering lengths (eqs. [1], [3] and [4]) are well within the sample sizes. It is possible to find contrary indications, as in equation (5), and clearly it will be a major task to decide whether such indications can be "explained away" as systematic errors in data or interpretation.

All the data discussed here were obtained "by hand," so to speak, and the more I have studied the data the more I have been impressed with the enormous effort and the scrupulous attention to detail that was devoted to each catalog. This means it will be no easy matter to improve the observational situation, even with automatic scanning devices, though surely that time eventually will come. There are two extensions of the data that would be very important and are technically feasible now. First is a very deep survey in limited selected areas, following Hubble (1936), for the purpose of extending the test of Hubble's count-magnitude relation (duly corrected for cosmology), extending the test of equation (1) to smaller r, and testing whether the apparent clustering in angular positions scales with depth as predicted from the clustering measures taken from the Lick and Zwicky samples. Some preliminary results on this subject have been reported by Phillips et. al. (1977). A more difficult project is to improve the estimates of the shape of the galaxy correlation function ξ at $r \sim 10\ h^{-1}$ Mpc. This could be done in a survey about twice the depth of the Lick catalog, in sample areas spread over some 10 to 20 degrees, with galaxy magnitude standards controlled and consistent in all samples to about $0^m.03$ (to assure that systematic errors in magnitudes do not introduce appreciable spurious clustering).

Finally, it might be mentioned that the situation will be very different when we have adequate samples of galaxy redshifts at $D \gtrsim 50\ h^{-1}$ Mpc. In the present analysis, where we have only very crude measures of individual galaxy distances, a major problem is that the angular distribution of a sample at great depth appears very close to random because we are seeing many clusters in projection. As a result a small systematic error in the angular distribution can be translated into a very large error in the estimate of the spatial clustering. Redshift data will allow us to avoid this problem and will present us with the great opportunity of studying the large scale kinematics as well as the large scale distribution of galaxies in the universe. Some aspects of this great project are discussed at this conference by M. Davis.

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DISCUSSION

Kiang: When I made the statement "galaxies are clustered on all scales" ten years ago (*Monthly Notices* 1967), I was looked at askance by most astronomers. I am glad that this idea is now getting generally accepted, thanks largely to the work of Dr Peebles and co-workers. But the impression should not be formed that this idea originated with Peebles, nor with his results on the covariance function. In fact, I arrived at this idea by first following Neyman and Scott's method of definite clusters but finding that the size of clusters obtained increased with the size of cells used in the analysis.

Longair: I will discuss in my lecture tomorrow the evidence on the isotropy of the distribution of radio sources. In brief, one may say that a large number of surveys have now been analysed using the method of power spectrum analysis by Webster and no evidence of anisotropy has been found. It is important that this result is found in large surveys such as the Molonglo surveys in which the effects of confusion are very small.

In interpreting the results of the cross-correlation analysis between Shane-Wirtanen counts and 4C radio sources, it should be remembered that the luminosity distribution of radio sources is very broad. Roughly, one would expect about 20% of the 4C sources to have redshifts less than $\sim$ 0.1-0.2. These sources will be correlated with the overall galaxy distribution and hence will certainly contribute to the effect found by Peebles.

Peebles: In Webster's analysis he only sets upper limits to the anisotropy. Our effect is at the upper limits he sets to the anisotropy. It should be noted that one can distinguish confusion and clustering using the power spectrum method.

Bolton: The relationship between radio sources and Shane-Wirtanen galaxies can be shown to be due to the identified galaxies alone if the radio sample is divided into three classes - identified galaxies, quasars and unidentified sources.

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Peebles: That is just the point I was trying to make: one can argue that the apparent clustering in the 4C sources is due to the relatively small fraction of relatively close radio galaxies. We find the effect you mentioned in the 3C catalogue. There does not seem to be enough data to repeat this test in the 4C catalogue.

THE ISOTROPY OF THE UNIVERSE ON SCALES EXCEEDING THE HORIZON

L. Grishchuk

I would like to describe in a few words work which was done by Zeldovich and myself. It gives some restrictions on the amplitude of possible very large-scale density fluctuations mentioned by Peebles. The main question investigated in this work is the following. What can be said, using known observational data and some general hypotheses, about structure of the Universe beyond the region accessible for observation at the present epoch? In fact we consider density fluctuations (as well as rotational perturbations and gravitational waves) with wavelengths larger than the horizon. We use the observational fact that the quadrupole-type anisotropy of the microwave background radiation is absent at the level of $\delta T/T < 10^{-4}$. It is interesting to know if it may happen that, at the present epoch, there exists a significant density of perturbations (say, with the dimensionless amplitude of the order of 10^{-1}) which we do not even suspect because the corresponding wavelength is very long and therefore direct observation of the entire perturbation is not possible. Such a direct observation will be possible only in the remote future when the horizon becomes equal to the corresponding wavelength. To answer the question we make a natural but very important assumption. Namely, we assume that the harmonic perturbations of different wavelengths are not correlated in any particular way. Otherwise, they might fit together in such a way that all perturbations (and, therefore, $\delta T/T$) would be especially small within the horizon while significant perturbations could take place just beyond the horizon. A situation of this kind would imply that an observer at the Earth occupies a unique position in the Universe. We assume, on the contrary, that all observers are equivalent. All of them, even causally unconnected observers, could detect similar restrictions on the anisotropy of the microwave background, $\delta T/T < 10^{-4}$. Nevertheless, the question still exists whether small perturbations unnoticeable by every observer within his horizon can represent different parts of a significant long wavelength limit. The main result of this investigation can be formulated in the following way. The observational data on $\delta T/T$ in combination with the natural hypothesis on the statistical independence of different harmonics leads to the conclusion that in the Universe there are no significant (i.e. with the amplitude exceeding $\delta T/T$) density fluctuations on any spatial scale larger than the horizon. (The paper will be published in *Astr. Zh. U.S.S.R.*, November-December, 1977.)