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PARACOMPACTNESS IN LOCALLY LINDELÖF SPACES

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This paper contains a set of results concerning paracompactness of locally nice spaces which can be proved by (variations on) the technique of "stationary sets and chaining" combined with other techniques available at the present stage of knowledge in the field. The material covered by the paper is arranged in three sections, each containing, in essence, one main result.

The main result of Section 1 says that a locally Lindelöf, submeta-Lindelöf ($= \delta\theta$-refinable) space is paracompact if and only if it is strongly collectionwise Hausdorff. Two consequences of this theorem, respectively, answer a question raised by Tall [**7**], and strengthen a result of Watson [**9**]. In the last two sections, connected spaces are dealt with. The main result of the second section can be best understood from one of its consequences which says that under $2^{\omega_1} > 2^{\omega}$, connected, locally Lindelöf, normal Moore spaces are metrizable. In the third section we prove that under $2^{\omega_1} > 2^{\omega}$, all connected, normal, locally compact, submetaLindelöf spaces are paracompact. In connection to both of these results, there is a number of related examples and theorems known (in the literature). These are briefly discussed in the remarks. The conclusion is that in our theorems, all the hypotheses are necessary.

Our terminology and notation will follow the standards of set-theoretic topology. All spaces are assumed to be regular T_1 topological spaces. In particular $[A]^{\leqq\kappa}$ will denote the set of all subsets of A of cardinality $\leqq \kappa$. Given a collection $\mathscr{A} = \{A_i : i \in I\}$ of sets, $\{A'_i : i \in I\}$ will be called an *expansion* of $\mathscr{A}$ if

$$A'_i \cap (\cup\mathscr{A}) = A_i \quad \text{for every } i \in I.$$

A space is said to be *strongly collectionwise Hausdorff* (or T_2) if every closed discrete collection of points has an open discrete expansion. In an analogy of H. Yunnila's term "submetacompact" we introduce the corresponding term "submetaLindelöf" in place of the old term "$\delta\theta$-refineable". We shall say that a space X is *submetaLindelöf* if for every open cover of X, there is a sequence $\{\mathscr{G}_n : n \in \omega\}$ of open refinements such that each $\mathscr{G}_n$ covers X, and for every point $x \in X$, there is an $n = n(x) \in \omega$ such that

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$$(\mathscr{G}_n)_x = \{G \in \mathscr{G}_n : x \in G\}$$

is countable. If the requirement "each $\mathscr{G}_n$ is a cover" is weakened to "$\cup_{n\in\omega} \mathscr{G}_n$ is a cover", then we shall speak about weakly submetaLindelöf spaces.

1. Paracompactness of locally Lindelöf spaces.

LEMMA 1.1. *Let $\kappa > \omega$ be a regular cardinal, and $S \subset \kappa$ be a stationary subset. Suppose that $Y = \{y_\alpha : \alpha \in S\}$ is a set of distinct points of a submetaLindelöf space X such that each point of X has a neighbourhood which meets Y in a set of cardinality $< \kappa$. Then there is a stationary (in κ) set $S' \subset S$ such that*

$$Y' = \{y_\alpha : \alpha \in S'\}$$

is a closed discrete subset of X.

Moreover, if $\kappa = \lambda^+$ for some $\lambda \geqq \omega$, then Y is the union of $\leqq\lambda$ closed discrete subsets of X.

Proof. Since X is submetaLindelöf, we can find a sequence $\{\mathscr{G}_n : n \in \omega\}$ of open covers of X such that

(i) for every $G \in \mathscr{G} = \cup_{n\in\omega} \mathscr{G}_n$, $|G \cap Y| < \kappa$;

(ii) for every $x \in X$, there is an $n(x) \in \omega$ with $(\mathscr{G}_{n(x)})_x$ countable.

Define Y_n by

$$Y_n = \{y_\alpha \in Y : n(y_\alpha) = n\} \quad (n \in \omega).$$

Since $Y = \cup_{n\in\omega} Y_n$, there is an $n_0 \in \omega$ such that

$$S_{n_0} = \{\alpha \in S : y_\alpha \in Y_{n_0}\}$$

is stationary. Now, define the equivalence relation $\sim$ on Y_{n_0} by putting $x \sim y$ if and only if there is a finite sequence $G_0, \dots, G_i \in \mathscr{G}_{n_0}$ such that $x \in G_0$, $y \in G_i$ and

$$G_j \cap G_{j+1} \cap Y_{n_0} \neq \emptyset \quad (j = 0, \dots, i-1).$$

Let $\mathscr{E}$ be the set of equivalence classes. By regularity of κ, $|E| < \kappa$ for every $E \in \mathscr{E}$. Now, for every $E \in \mathscr{E}$ put

$$\alpha(E) = \min\{\alpha \in S_{n_0} : y_\alpha \in E\}.$$

By the Pressing Down Lemma,

$$S' = \{\alpha(E) : E \in \mathscr{E}\}$$

is a stationary subset of κ. On the other hand, by the definition of $\sim$, every member of $\mathscr{G}_{n_0}$ meets

$$Y' = \{y_\alpha : \alpha \in S'\}$$

in at most a singleton. Therefore Y' is a closed discrete subset of X.

Now, let $\kappa = \lambda^+$ $(\lambda \geqq \omega)$. Then $|E| \leqq \lambda$ for each $E \in \mathcal{E}$, so

$$Y_{n_0} = \cup_{\xi \in \lambda} Y_\xi$$

in such a way that $|E \cap Y_\xi| \leqq 1$ for every $E \in \mathcal{E}$, and $\xi \in \lambda$. Again, for each $\xi \in \lambda$, $\mathcal{G}_{n_0}$ witnesses that Y_ξ is closed discrete in X.

LEMMA 1.2. *Let $\kappa \geqq \omega$ be a cardinal, and X be a submetaLindelöf space such that*

(1.2) *every closed discrete set A of κ^+ many distinct points has a subset A' of size κ^+ such that A' has a locally countable open expansion in X.*

Then the closure of every κ-Lindelöf subspace of X is κ-Lindelöf.

Proof. Let Z be a κ-Lindelöf subspace of X and suppose indirectly that $\bar{Z}$ is not κ-Lindelöf.

Then first we show that $\bar{Z}$ contains a closed discrete subset A of size κ^+. To see this, let $\mathcal{G}$ be an open (in X) cover of X with no subcover of cardinality $\leqq \kappa$. Since X is submetaLindelöf, we may assume that $\mathcal{G} = \cup_{n \in \omega} \mathcal{G}_n$ in such a way that each $\mathcal{G}_n$ covers $\bar{Z}$, and for every $x \in \bar{Z}$, there is an $n(x) \in \omega$ with

$$|(\mathcal{G}_{n(x)})_x| \leqq \omega.$$

Let

$$Z_n = \{x \in \bar{Z} : n(x) = n\}.$$

By Zorn's Lemma, for every $n \in \omega$, there is a maximal subset A_n of Z_n such that no member of $\mathcal{G}_n$ contains two points of A_n. By maximality,

$$\mathcal{G}' = \cup_{n \in \omega}(\cup\{(\mathcal{G}_n)_x : x \in A_n\})$$

covers $\bar{Z}_n$. Since $\mathcal{G}$ has no subcover of cardinality $\leqq \kappa$, there is an $n \in \omega$ with $|A_n| \geqq \kappa^+$.

Now, let us take a closed discrete subset A of $\bar{Z}$ of size κ^+. By (1.2) we may assume that A has a locally countable open expansion $\mathcal{O}$ in X. Clearly, $|\mathcal{O}| = \kappa^+$, and since $A \subset \bar{Z}$, each member of $\mathcal{O}$ meets Z. This contradicts our assumption that Z is κ-Lindelöf.

THEOREM 1.3. *A locally Lindelöf, submetaLindelöf space X is paracompact if and only if it is strongly collectionwise Hausdorff.*

Proof. Only the "if" part needs proof.

We shall prove the "if" part by induction on the Lindelöf degree $L(X)$ of X. If $L(X) = \omega$, then there is nothing to prove, so let $L(X) = \kappa > \omega$ and assume that for every space with Lindelöf degree $< \kappa$ we have already proved the theorem. Then there are two cases.

Case 1. κ is regular. Then let $\mathcal{G} = \{G_\alpha : \alpha \in \kappa\}$ be a cover of X by open sets with Lindelöf closures.

To make use of our induction hypothesis, it is enough to show that

$$S = \{\alpha \in \kappa : \overline{\cup_{\beta\in\alpha} G_\beta} - \cup_{\beta\in\alpha} G_\beta \neq \emptyset\}$$

is a non-stationary subset of κ. (Indeed, if S is non-stationary, then X is the free union of subspaces with Lindelöf degree $< \kappa$.)

Suppose indirectly that S is stationary, and pick, for every $\alpha \in S$, a point

$$x_\alpha \in \overline{\cup_{\beta\in\alpha} G_\beta} - \cup_{\beta\in\alpha} G_\beta.$$

Let $\nu(\alpha)$ be the least element of κ with $x_\alpha \in G_{\nu(\alpha)}$. Clearly, $\nu(\alpha) \geqq \alpha$. Let C be a c.u.b. subset of κ such that for every $\alpha \in C$,

$$\nu''(S \cap \alpha) \subset \alpha.$$

Then the points of $A = \{x_\alpha : \alpha \in S \cap C\}$ are all distinct, and $S \cap C$ is stationary in κ. By Lemma 1.1, there is a stationary $S' \subset S \cap C$ such that

$$A' = \{x_\alpha : \alpha \in S'\}$$

is a closed discrete set. (Remember that $|A \cap G_\beta| < \kappa$ for every $\beta \in \kappa$.) Since X is strongly collectionwise T_2, there is an open discrete expansion

$$\mathcal{O} = \{O_\alpha : \alpha \in S'\}$$

of A' in X. Since

$$x_\alpha \in \overline{\cup_{\beta\in\alpha} G_\beta} \quad \text{for } \alpha \in S',$$

for every $\alpha \in S'$, there is an $f(\alpha) \in \alpha$ such that

$$G_{f(\alpha)} \cap O_\alpha \neq \emptyset.$$

By the Pressing Down Lemma, there is an ordinal $\beta \in \kappa$ with $|f^{\leftarrow}(\beta)| = \kappa$, i.e., κ many members of $\mathcal{O}$ intersect G_β. This contradicts our assumption that $\overline{G}_\beta$ is Lindelöf.

Case 2. κ is singular. Then let $\mathrm{cf}(\kappa) = \tau < \kappa$, and let $\{\kappa_\xi : \xi \in \tau\}$ be an increasing sequence of regular cardinals $< \kappa$ converging to κ. Again, let us consider a cover $\mathcal{G} = \{G_\alpha : \alpha \in \kappa\}$ of X by open sets with Lindelöf closures. By Lemma 1.2, the subspaces

$$Z_\xi = \overline{\cup\{G_\alpha : \alpha \in \kappa_\xi\}}$$

have Lindelöf degrees $< \kappa$ $(\xi \in \tau)$, and are paracompact by our induction hypothesis. Therefore, for each $\xi \in \tau$, there is a discrete in X family $\mathcal{A}_\xi$ of open sets with Lindelöf closures (in X) such that

$$\cup\mathcal{A}_\xi = \cup\{G_\alpha : \alpha \in \kappa_\xi\}.$$

Let us consider the open cover $\mathcal{A} = \cup_{\xi\in\tau} \mathcal{A}_\xi$ of X. Since every member of $\mathcal{A}$ meets $\leqq \tau$ other members of $\mathcal{A}$, by a standard chaining argument, X

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is the free union of clopen subspaces with Lindelöf degrees $\leqq \tau$. Applying the inductive hypothesis once more, we are done.

Remark. Note that the proof of Theorem 1.3 works if, instead of "strongly collectionwise Hausdorff", we only assume that "every closed discrete collection of points has a σ-locally countable expansion." In the present formulation of the theorem, however, we sacrificed maximal strength for the sake of brevity.

COROLLARY 1.4. *Every normal, locally Lindelöf, screenable space is paracompact.*

This corollary answers Question E in [**7**].

In [**9**], S. Watson proved that in $V = L$, every normal, locally compact space is (strongly) collectionwise Hausdorff. Combining his result with Theorem 1.3 gives

COROLLARY 1.5 ($V = L$). *Every normal, locally compact, submeta-Lindelöf space is paracompact.*

Making use of the technique of proof of Theorem 1.3 the author also proved the following results.

THEOREM 1.6. (a) *Every locally c.c.c., submetaLindelöf, collectionwise Hausdorff space is the free union of Lindelöf subspaces.*

(b) *Every locally c.c.c., weakly submetaLindelöf, hereditarily collectionwise Hausdorff space is the free union of Lindelöf subspaces.*

A further application of the technique is given by

THEOREM 1.7. *A normal, locally ω_1-compact, collectionwise Hausdorff space is collectionwise normal with respect to closed Lindelöf subsets.*

The proofs of all these results are (simplified) versions of the proof of Theorem 1.3, and are therefore omitted.

2. Paracompactness in locally Lindelöf, connected spaces. In the rest of the paper, essential use is made of a result of Taylor [**8**]. We shall state and use here a somewhat more general form of his theorem which, however, can be proved in the same way as Lemma 2.1 in [**8**].

LEMMA 2.1 (Taylor, in essence). *Assume $2^{\omega_1} > 2^{\omega}$. Then in a normal space X, the following principle holds*:

(2.1) *If C is a* c.u.b. *subset of ω_1, and $\{F_\alpha : \alpha \in C\}$ is a σ-closed discrete family in X such that $\chi(F_\alpha, X) \leqq 2^{\omega}$ holds for every $\alpha \in C$, then there is a stationary subset $S \subset \omega_1$ such that $\{F_\alpha : \alpha \in S\}$ has an open discrete expansion.*

LEMMA 2.2. *Suppose X is a connected, ω_1-Lindelöf, locally Lindelöf, submetaLindelöf space such that*

(2.2) *if C is a* c.u.b. *subset of ω_1 and $Y = \{y_\alpha : \alpha \in C\}$ is σ-closed discrete set of distinct points in X, then there is a stationary subset $S \subset \omega_1$ such that $Y' = \{y_\alpha : \alpha \in S\}$ has an open locally countable expansion.*

Then X is Lindelöf.

Proof. Suppose indirectly that there is a cover $\mathcal{G} = \{G_\alpha : \alpha \in \omega_1\}$ of X by open sets with Lindelöf closures such that

$$G_\alpha - \cup_{\beta \in \alpha} G_\beta \neq \emptyset \quad \text{for every } \alpha \in \omega_1.$$

Since X is connected, for every $0 \neq \alpha \in \omega_1$, there is point

$$y_\alpha \in \overline{\cup_{\beta \in \alpha} G_\beta} - \cup_{\beta \in \alpha} G_\beta.$$

As in the proof of Theorem 1.3 we may assume that the points y_α are distinct on a c.u.b. $C \subset \omega_1$. Further, since each G_β contains y_α for only countably many $\alpha \in C$, by Lemma 1.1 we conclude that $Y = \{y_\alpha : \alpha \in C\}$ is a σ-closed discrete subset of X. By (2.2) there is a stationary subset $S \subset \omega_1$ such that $Y' = \{y_\alpha : \alpha \in S\}$ has an open locally countable expansion $\{O_\alpha : \alpha \in S\}$. As in the proof of Theorem 1.3 again, the Pressing Down Lemma implies that there are ω_1 many sets O_α meeting the same G_β for some $\beta \in \omega_1$. This contradicts our assumption that $\overline{G_\beta}$ is Lindelöf.

THEOREM 2.3 ($2^{\omega_1} > 2^\omega$). *Let X be a connected, normal, locally Lindelöf, submetaLindelöf space with $t(X) \leqq \omega$ and $\chi(X) \leqq 2^\omega$. Then X is paracompact (and thus, Lindelöf).*

Proof. By Lemma 2.1, X satisfies Condition (2.2) of Lemma 2.2. Therefore, to apply Lemma 2.2, it is enough to show that X is ω_1-Lindelöf.

To see this, take an arbitrary cover $\mathcal{G}$ of X by open sets with Lindelöf closures. Since, by Lemma 1.2, the closure of every Lindelöf subspace of X is Lindelöf, we can inductively define a sequence

$$\{\mathcal{G}_\beta : \beta \in \omega_1\} \subset [\mathcal{G}]^{\leqq \omega}$$

in such a way that $\overline{\cup \mathcal{G}_\beta} \subset \cup \mathcal{G}_{\beta+1}$ holds for every $\beta \in \omega_1$. Let

$$\mathcal{G}' = \cup_{\beta \in \omega_1} \mathcal{G}_\beta.$$

Since we are in a space of countable tightness,

$$\cup \mathcal{G}' = \cup_{\beta \in \omega_1} (\overline{\cup \mathcal{G}_\beta})$$

is a clopen subset of X. Since X is connected, it follows that $\mathcal{G}' \subset \mathcal{G}$ is a cover of X with $|\mathcal{G}'| \leqq \omega_1$.

Remark. One may ask whether the tightness and character restrictions on X in Theorem 2.3 are really necessary. The present author can prove with methods similar to those used in this section that CH + $2^{\omega_2} > 2^{\omega_1}$ together imply that $t(X) \leqq \omega$ can be omitted from Theorem 2.3. The case with the character restriction seems to be more complicated, because in the absence of $\chi(X) \leqq 2^{\omega}$ it is not immediate how to obtain the separation principle (2.2). In case of locally compact spaces, however, we shall show that both restrictions can be omitted (see Section 3).

COROLLARY 2.4 ($2^{\omega_1} > 2^{\omega}$). *Every connected, locally Lindelöf (or locally* c.c.c.) *normal Moore space is metrizable.*

Remark. Some set-theoretic hypothesis in Corollary 2.4 (and thus, in Theorem 2.3) is needed, since under MA + $\neg$CH, the "bubble space derived from a Q-set" (see **[6]**, e.g.) is an example of a (locally) connected, locally Lindelöf, nonmetrizable Moore space. "Connected" is also necessary, since Devlin and Shelah **[2]** gave an example, consistent with CH, of a locally countable, nonmetrizable normal Moore space. (Note, however, that under $V = L$, the conclusion of Corollary 2.4 remains true even if "connected" is omitted. This follows from Fleissner's theorem in **[3]**, and a result of Worrel **[10]**, independently obtained by Alster and Pol **[1]**.) Thus Corollary 2.4 seems to be the "strongest possible" metrization theorem for normal Moore spaces which is implied by $2^{\omega_1} > 2^{\omega}$.

We can prove a result corresponding to Theorem 2.3 in the class of locally c.c.c. spaces:

THEOREM 2.5. *Let X be a connected, locally c.c.c., submetaLindelöf space such that*

(2.5) *if C is a* c.u.b. *subset of ω_1 and $Y = \{y_\alpha : \alpha \in C\}$ is a σ-closed discrete collection of distinct points of X, then there is a stationary subset S of ω_1 such that $Y' = \{y_\alpha : \alpha \in S\}$ has an open disjoint expansion.*

Then X is Lindelöf.

Assuming Condition (2.5) hereditarily, we can weaken "submetaLindelöf" to "weakly submetaLindelöf."

3. Paracompactness in locally compact, connected spaces.

LEMMA 3.1 **[9]**. *Let X be a normal, locally compact space, and $\mathscr{C} = \{C_\beta : \beta \in \omega_1\}$ be a closed discrete family of compact subsets in X. Then there is a closed discrete expansion $\{C_\beta' : \beta \in \omega_1\}$ of $\mathscr{C}$ such that*

$$\chi(C_\beta', X) \leqq \omega_1$$

holds for $\beta \in \omega_1$.

LEMMA 3.2. *Let X be a connected, locally compact, submetaLindelöf space such that every Lindelöf subset has Lindelöf closure. Then X is ω_1-Lindelöf.*

Proof. Since X is submetaLindelöf, there is a sequence $\{\mathscr{G}_n : n \in \omega\}$ of covers of X by open sets with compact closures such that for every $x \in X$ there is an $n(x) \in \omega$ with

$$|(\mathscr{G}_{n(x)})_x| \leqq \omega.$$

Let $\mathscr{G} = \cup_{n \in \omega} \mathscr{G}_n$, and define

$$X_n = \{x \in X : n(x) = n\} \quad (n \in \omega).$$

Since the closure of every Lindelöf subspace in X is Lindelöf we can inductively define an increasing sequence $\{\mathscr{G}_\beta : \beta \in \omega_1\} \subset [\mathscr{G}]^{\leqq \omega}$ in such a way that $\overline{\cup \mathscr{G}_\beta} \subset \cup \mathscr{G}_{\beta+1}$ holds for every $\beta \in \omega_1$. We are going to prove that $\mathscr{G}' = \cup_{\beta \in \omega_1} \mathscr{G}_\beta$ covers X. By connectedness of X, it is enough to prove that

$$\cup \mathscr{G}' = \cup_{\beta \in \omega_1} \overline{\cup \mathscr{G}_\beta}$$

is a closed subset of X. Suppose indirectly that there is a point $x \in \overline{\cup \mathscr{G}'} - \cup \mathscr{G}'$. Take a compact neighbourhood C of x, and let $P = C \cap \cup \mathscr{G}'$. We shall show that for every $n \in \omega$, $P_n = P \cap X_n$ can be covered by some $\mathscr{G}_\beta$. This will lead us to a contradiction since then there is $\gamma \in \omega_1$ with

$$P \subset \bar{P} \subset \overline{\cup \mathscr{G}_\gamma} \subset \cup \mathscr{G}' \not\ni x,$$

and this implies that $C - \bar{P}$ is a neighbourhood of x avoiding $\cup \mathscr{G}'$.

So let $n \in \omega$, and take a maximal subset A_n of P_n such that each $G \in \mathscr{G}_n$ meets A_n in at most a singleton. Then A_n is a closed discrete subset of X, and thus of

$$P = C \cap (\cup \mathscr{G}') = \cup_{\beta \in \omega_1} (\overline{\cup \mathscr{G}_\beta} \cap C).$$

Since each $\overline{\cup \mathscr{G}_\beta} \cap C$ is compact, P is countably compact. Hence A_n is finite. By maximality,

$$\cup \{(\mathscr{G}_n)_x : x \in A_n\} \subset \mathscr{G}'$$

is a countable cover of P_n, and so it is included in some $\mathscr{G}_\beta$.

THEOREM 3.3 ($2^{\omega_1} > 2^\omega$). *Every connected, normal, locally compact, submetaLindelöf space X is paracompact.*

Proof. By Lemmas 3.1 and 2.1, X satisfies (2.2) of Lemma 2.2. Therefore, by Lemma 1.1, every Lindelöf subspace of X has Lindelöf closure. Hence by Lemma 3.2, X is ω_1-Lindelöf. By Lemma 2.2 we conclude that X is Lindelöf.

Remarks. $2^{\omega_1} > 2^\omega$ in the hypothesis of Theorem 3.3 cannot be omitted, since Gary Gruenhage has a construction which modifies, under MA + $\neg$CH, the Cantor tree space to obtain an example of a connected, locally compact, nonmetrizable normal Moore space. Since the example of [2] is also locally compact, "connected" cannot be omitted under $2^{\omega_1} > 2^{\omega}$, either. However, as is shown by Corollary 1.5, "connected" can be omitted under $V = L$. (Note that for submetacompact spaces, this is a result of Watson [9].) Finally, it is a result of Gruenhage [4] that every locally connected, normal, locally compact, submetacompact space is paracompact (in ZFC).

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