HOT-WATER DRILLING AND BORE-HOLE CLOSURE IN COLD ICE

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ABSTRACT. Drilling bore holes in deep, cold ice masses by hot-water methods and maintaining these holes with sufficient diameter to allow down-hole experimentation poses a major obstacle to the investigation of conditions beneath ice sheets and ice streams. Closure of the water-filled holes by refreezing is the dominant difficulty. In this paper, we describe calculations of heat transfer from the drilling system to the ice and the subsequent time-dependent motion of the phase boundary defining the bore-hole wall. Results are presented with the view of optimizing the bore-hole radius at depth for a fixed drill performance and a variable rate of drilling.

Calculation of melting/refreezing rates at the bore-hole wall requires the use of a one-dimensional, time-dependent numerical heat-flow model with a distorting mesh which follows the changing hole size. The delay of hole closure is discussed with a view to keeping holes open long enough to allow instruments to be lowered to the glacier bed, while realizing that drilling-system performance may be marginal because of logisitical and/or expenditure constraints. The relative merits of drilling a large hole, which is very time consuming with a small drill, and the use of water-soluble antifreezes, which have a history of creating plugs of ice slush, are discussed. A method of creating a stable hole filled with antifreeze in which ice slush does not occur is described.

The recent application of these theoretical ideas to the planning and implementation of successful hot-water drilling programs in Antarctica and Greenland is also presented.

1. INTRODUCTION

Hot-water drilling has become a standard technique for rapidly accessing the interior or basal regions of glaciers (e.g. Iken and others, 1977; Taylor, 1984). A vertical hole is melted with a hot fluid, either water or antifreeze, which is pumped into the ice under pressure. The fluid is injected down a hose that is lowered into the glacier. Cooled fluid and melt water circulate back up the bore hole to the surface and can be recovered if necessary. Most drilling has been in temperate ice; however, several recent projects have used hot-water drills to penetrate cold ice. In many cases (e.g. ice temperature $\geqslant -20^{\circ}$C), the drilling of cold ice is only marginally slower than drilling temperate ice. This is a consequence of the large latent heat of fusion for ice as compared to the sensible heat required to raise the ice to the melting temperature. The bulk of the heat delivered to the drilling nozzle goes to melt the ice in both cold and temperate regimes. A major difficulty when drilling in cold ice, as opposed to warm ice, is maintaining the hole against closure by refreezing, both during drilling and, more critically, after drilling so as to allow later access to the hole. Unless the fluid level in the bore hole is far below that required to balance the ice-overburden pressure, the closure due to ice deformation will be negligible in comparison with this closure by refreezing.

There has been limited experience with deep hot-water drilling in cold ice. Napoléoni and Clarke (1978) and Koci (1984) drilled to shallow depths, while Blatter (1987) drilled somewhat deeper into an Arctic glacier for temperature measurements. An essentially temperate-glacier drilling system was used to drill through the Ronne Ice Shelf (500 m) by Engelhardt and Determann (1987). Their objective was mainly to verify the ice thickness and there was no attempt to keep the hole open. In 1987, the Polar Ice Coring Office tested a large hot-water drill by drilling through Crary Ice Rise on Ross Ice Shelf (Koci and Bindschadler, 1989). The PICO operation avoided most of the problems discussed in this paper by drilling a large hole (nominal 0.35 m radius; personal communication from B. Koci) with an insulated hose. This large-diameter hole was required to remain open only long enough to insert a thermistor cable. The drilling system used was not easily portable and required large-scale logistic support.

In contrast, two ongoing drilling projects, one on Jakobshavns Isbræ, Greenland (Iken and others, 1988; Echelmeyer and others, 1989), and one on Ice Stream *B*, Antarctica (Engelhardt and others, in press), require bore holes in deep, cold ice which must be maintained unfrozen and accessible for a period of hours or days, allowing for access for down-hole instruments. These two projects are attempting to drill to considerable depths (1 km at Ice Stream B and up to 1.7 km at Jakobshavns Isbræ) with light-weight drills. The need for a light-weight system is especially true of the Greenland project, where surface ice conditions limit logistic support to helicopters and where the drilling system must be moved manually and piecewise between neighboring holes. The drilling has been highly successful at reaching great depths, but the difficulty of keeping the bore holes open against refreezing has made it obvious that accurate modeling of heat transfer and melt/freezing rates in the bore hole is necessary. Much of this thermal modeling of the drilling process can be adequately achieved using heat-exchanger types of calculations derived from the engineering literature. This is true in particular of the estimation of heat losses in the flow of the drilling fluid down the high-pressure hose, and the decay of heat in the return flow of fluid up the bore hole, as described by Iken and others (1977) and Iken (1988).

However, two problems, the melting and freezing of the bore-hole wall and the behavior of antifreeze in a bore hole, require more detailed calculations. These problems require knowledge of the temperature in the ice surrounding the bore hole and the motion of the ice/fluid phase boundary. The presence of the moving boundary limits the use of an analytical approach. Solutions may be obtained using a finite-element (or finite-difference) model of radial heat flow in the ice surrounding the bore hole.

In the following, numerical models are developed which allow the position of the moving boundary at the melting/refreezing bore-hole wall to be followed. The problem

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constitutes a classical "Stefan" problem. Related problems have previously been discussed by Harrison (1972) and Jarvis and Clarke (1974). Harrison (1972) described the closure of an unheated bore hole in a temperate glacier, where the rate of closure can be used to infer the ambient ice temperature. Jarvis and Clarke (1974) described a numerical solution of the unheated bore-hole closure problem. However, they did not describe the evolution of the bore hole through time. Preliminary results of modeling bore-hole closure along the lines presented herein have been described by Iken and others (1989) with application to the drilling program on Jakobshavns Isbræ.

We begin with a description of the drilling system in terms of heat-exchanger type equations. Much of this has previously been described by Iken and others (1977), Iken (1988), and Iken and others (1989). We present the discussion here because it forms the background necessary for bore-hole closure models since heat is added to the ice along the drill hose for the duration of drilling. Also, our development differs somewhat from that of Iken and her collaborators. Following this, we give a detailed discussion of the thermal behavior of the bore-hole walls as they refreeze in either water or antifreeze. We present an alternative to drilling with large, high heat-through-put drills which are not easily portable. Although virtually all the problems discussed in this paper could be solved by quickly drilling a large hole (0.5 m diameter or larger), the emphasis here is on solving the refreezing problems using a much more efficient and mobile drill capable of drilling a hole on the order of 0.1 to 0.2 m in diameter.

II. DRILLING AS A THERMAL SYSTEM

A hot-water drill consists of two heat exchangers coupled by a long, thermally leaky connection. The heat exchanger at the glacier surface adds some hundreds of kilowatts of heat and a few kilowatts of pressurization to the drilling fluid. The fluid then flows down the drill hose to the drilling nozzle. At the nozzle, the heat is transferred from the drilling fluid to the ice. Any inefficiency at the nozzle releases waste heat which, along with the melted ice, returns up the drilled hole towards the surface. While the fluid is flowing back up the bore hole, it is acting as both a heat-sink for the hose and a heat source to the walls of the bore hole. Although all the parts of the system thermally affect each other, in practice it is possible to separate the system into four zones: the surface heaters, the high-pressure hose, the nozzle, and the return flow. The zones are coupled together by the heat advected to the next zone in the flow path, and small leakage terms across non-adjacent zones. Each thermal zone is defined by dominant heat sources and sinks, and each thermal zone acts as a boundary condition for the adjacent zones. Separating the flow system greatly simplifies calculations and makes it possible to identify the specific heat losses in the system.

The solution techniques presented here should be useful for a wide range of drilling systems. Two cases are used for numerical examples, and are given in Table I. Case A is based on the drilling system used at the Upstream B camp ("Up-B") on Ice Stream B (Antarctica) to drill to depths of 1050 m in the 1988–89 field season (Engelhardt and others, in press). Case B is a system that has essentially twice the capacity and power output of case A. It was used to drill to a depth of 1630 m on Jakobshavns Isbræ in 1988 and 1989 (Echelmeyer and others, 1989; Iken and others, 1989).

The actual drilling at Up-B and Jakobshavns Isbræ always took longer than the nominal time indicated in Table I to drill a hole. The drills used several lengths of hose, and the drilling style involved redrilling (or "reaming") sections of the bore hole or pulling the drill back up several tens of meters as each hose length was added. And, of course, the glacier gremlins are always hard at work deep in the ice causing time-delaying problems. As a result, modeling of any particular bore hole is quite complicated, and these cases are only to be considered as representative of the capabilities of an idealized drilling system.

Surface zone

At the surface, the drilling fluid is pumped out of the hole or reservoir, through heaters and into the high-pressure hose going down the bore hole. The sensible-heat input to the bore hole is easily determined from the water temperature and flux at the pump outlet (the temperature drop is minimal, ~5 K, along the coiled hose to the top of the bore hole). When drilling through a firn layer, the potential energy in the water from its height above the fluid surface in the hole is negligible (less than 0.2 °C equivalent water temperature), but the energy in the water from the pressure pump may be important. Typically, pressures are in the 100–200 bar range, which translates to about 2° or 3 °C equivalent of water temperature. Although usually only of minor importance, this energy is not subject to the exponential decay that is typical of the sensible heat in the hose as it travels into the glacier. In small drilling systems with considerable heat loss, the flow energy can become a significant source of thermal energy at the drilling nozzle.

TABLE I

	Case A	*Case B*
Water flux	$0.5 \times 10^{-3}\ m^3\ s^{-1}$ ($30\ l\ min^{-1}$)	$1.33 \times 10^{-3}\ m^3\ s^{-1}$ ($80\ l\ min^{-1}$)
Water temperature (at surface, maximum)	90 °C	85 °C
Pressure (nominal)	180 bar	90 bar
Hose diameter, inner (nominal)	12.7 mm	19.1 mm
Hose diameter, outer (nominal)	22.6 mm	29.1 mm
Effective thermal resistance of hose, Z (Equations (2)–(3))	$0.29\ m\ K\ W^{-1}$	$0.27\ m\ K\ W^{-1}$
Bore-hole diameter (nominal)	0.1 m	0.2 m
Ice temperature (minimum)	−24 °C	−24 °C
Temperature profile	Increasing with depth	Decreasing with depth
Hose decay length, λ	605 m	1500 m
Bore-hole depth	1000 m	1600 m
Time to drill (nominal)	24 h	20 h

Heat losses in the hose

The drilling fluid takes some number of minutes to travel down the hose to the drill tip. During that time, the hose conducts heat from the drilling fluid to the surrounding fluid in the bore hole. A parcel of fluid traveling down the hose loses heat only through the walls of the hose, since the temperature gradients along the direction of flow in the hose (the y-direction) are negligible (over six orders of magnitude less than the radial gradients).

Further simplification occurs, because the pump and heaters supply a nearly constant volume of warm water to the drilling hose, and thus the temperature at a particular depth within the hose is constant with time. Also, although there is some structure to the radial temperature profile within the hose, the highly turbulent flow will lead to mixing. Following Bird and others (1965), the difference between the inner-wall temperature and that of the center line can be calculated to be on the order of 0.01 K, which is negligible.

Based on these simplifications, the temperature in the drilling fluid, T_d, at a depth y in the hose, can be determined from Iken and others (1977, Equation (3)).

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$$T_d(y) - T_{wall} = T_{in} e^{-y/\lambda} \quad (1)$$

where T_{in} is the entrance temperature at $y = 0$, T_{wall} is the bore-hole wall temperature ($\approx 0\,^\circ$C if drilling with water), and λ is a characteristic decay length (assumed to be independent of depth) given by

$$\lambda = \rho_d c_d Q_d Z \quad (2)$$

where ρ_d and c_d are, respectively, the density and specific heat capacity per unit mass of the drilling fluid, Q_d is the volume flux of drilling fluid through the hose, and Z is the effective thermal resistance of the hose ("effective impedance") plus that of the surrounding fluid layer within the bore hole (Iken, 1988; equal to 1/(effective thermal conductivity)). (The drilling fluid is generally water, but we admit the possibility of drilling with antifreeze at this stage; hence the subscript d instead of w.) The thermal resistance of the hose is given by

$$Z_{hose} = \frac{1}{2\pi K} \ln(r_2/r_1) \quad (3a)$$

(from Carslaw and Jaeger, 1959, chapter 10) where K is the thermal conductivity of the hose material and r_2, r_1 are the outer and inner radii of the hose under pressure as shown in Figure 1. K is often difficult to determine for a many-layered hose. The thermal resistance of the fluid layer surrounding the hose will have a maximum value given by

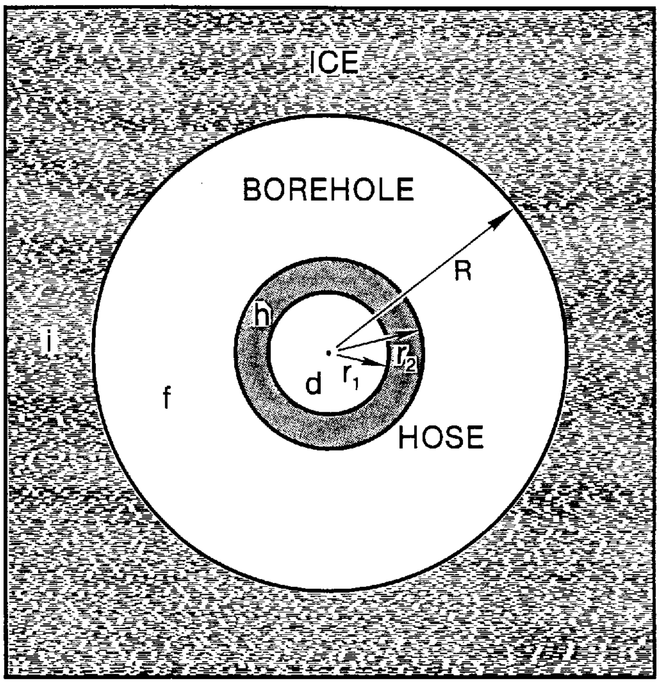

Fig. 1. Geometry of bore hole (radius R) in ice (i) and drilling hose (h) of inner radius r, and outer radius r_2. d denotes drilling fluid, and f the return flow.

$$Z_d = \frac{1}{2\pi K_d} \ln(R/r_2) \quad (3b)$$

where R is the radius of the bore hole and K_d is the thermal conductivity of the drilling fluid. This strictly conductive limit is on the order of 0.5 m K W^{-1}. However, turbulent flow in the returning fluid will diminish the effective resistance of the layer by adding advective heat transfer, and, in practice Z (= $Z_{hose} + Z_d$) from Equations (3a, b) is only an aprroximate upper board. We have found it is best to determine an effective value of Z from temperature measurements within the hose while drilling (a difficult procedure) or from maximum-speed drilling tests as explained by Iken and others (1989). Z has been found to be approximately 0.29 m K W^{-1} for case A and 0.27 m K W^{-1} for case B following the latter method.

The decay length λ given in Equation (2) is a linear function of fluid discharge. Because of the exponential decay of temperature along the hose (Equation (1)), it is important to maximize the discharge within the pressure capability of the drilling hose. Generally, $\lambda \approx$ 500–1500 m for small drilling systems, as shown in Table I.

The temperature of the drilling fluid as a function of depth is shown in Figure 2a and b for the two systems described in Table I. Also shown in these figures are approximate ice tmperatures at depth as measured in bore holes and extrapolated to the bed. Note that the temperature profiles in Ice Stream B, Antarctica, and in Jakobshavns Isbræ show markedly different behavior at depth. This is important in determining the evolution of bore holes in these respective ice streams. (These profiles are used in the following calculations. If, however, the actual temperatures are not known prior to drilling, an "educated" guess of $T_{ice}(y)$ will suffice. It is best to err on the cold side when predicting bore-hole closure rates.)

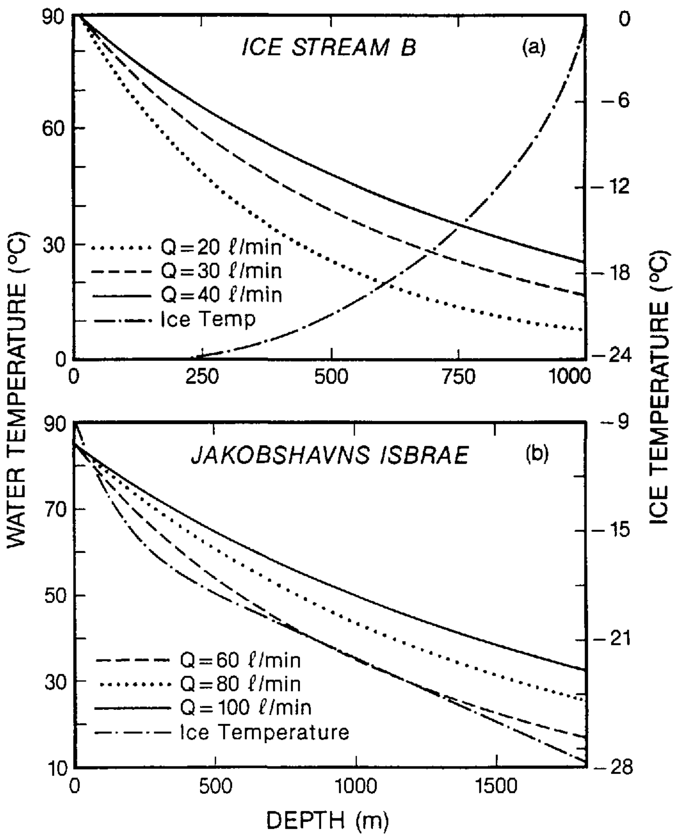

Fig. 2. Hot-water temperature along drilling hose and ice temperature at depth (estimated) in (a) Ice Stream B, and (b) Jakobshavns Isbræ.

The exponential decay of fluid temperature along the hose has several implications. The loss from the hose, which also decays exponentially with depth, is available as a source of heat to prevent refreezing of the hole and to warm the ice surrounding the bore hole during drilling. This warming is minimal at depth, and a hole in cold ice may actually close in on the drill hose at depth *while drilling* if the heat loss is insufficient. This could lead to severe problems!

The rate of drilling will decrease exponentially with depth. The rate of drill advance, $v(y)$, required to produce a hole of radius R_0 in ice of ambient temperature $T_0(y)$ (in °C) may be obtained from Iken and others (1989, equation (4)) (and setting $T_{wall} = 0$)

$$v(y) = \frac{\rho_d c_d Q_d T_d(y)}{\pi R_0^2 \mathscr{L}_v \left[1 + \dfrac{\rho_i c_i}{\mathscr{L}_v} \left| T_0(y) \right| \right]} \quad (4)$$

where $\mathscr{L}_v$ is the latent heat of fusion per unit volume and ρ_i, c_i are the density and specific heat per unit mass of ice. (R_0 is the radius of the bore hole at some height above

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the tip (typically 20–30 m), where all heat available from the drilling fluid at the tip has been transferred to the ice. This radius is not attained at the drill tip because the thermal efficiency there is not unity. The radius of the hole at depth y will change with time while drilling. R_0 is taken to be the nominal initial value.) Equation (4) may be solved for $R(y)$ as a function of depth and drilling speed.

The time to drill to a depth L is obtained from Equations (4) and (1):

$$t(L) = \frac{\pi R_0^2 \mathscr{L}_v}{\rho_d c_d Q_d T_{in}} \int_0^L \left[1 + \frac{\rho_i c_i}{\mathscr{L}_v} \,|\, T_0(y) \,|\right] e^{y/\lambda} dy . \qquad (5)$$

In most cases, the heat required for melting will dominate that required to warm the ice to the melting point and the term in brackets in the integrand may be approximated by using a constant, yet representative, temperature at depth, $\overline{T}_0$, and the integral can be evaluated explicitly. (The rate of drilling and the time to drill given by Equations (4) and (5) differ from those given by Iken and others (1977) and Iken (1988) in that these works have derived the maximum rate of drilling and minimum time to reach a depth L at a radius just large enough to let the drill pass. The radius in these rapidly drilled holes will be less than R_0 for a distance above the drill tip which, in cold ice, is long compared to the scenario described here. As such, our formulation provides a conservative estimate, which is important for drilling in cold ice if down-hole experimentation is expected.)

The rate of drilling and time to drill to a given depth are shown in Figures 3 and 4 for the two cases A and B (with water). An increase in the required bore-hole depth causes a substantial increase in fuel requirements and strain on the drilling equipment and the drillers themselves. Also shown in Figures 3 and 4 are the effects of changing the system parameters such as the number or size of pumps (varying Q_d) and the output of the heaters (thus changing T_{in}). It is seen that a system must be run at maximum performance. The loss of a single heater or a decrease in pump efficiency can lead to a substantial slow-down in the drilling and, possibly, a termination of the drilling if fuel is limited.

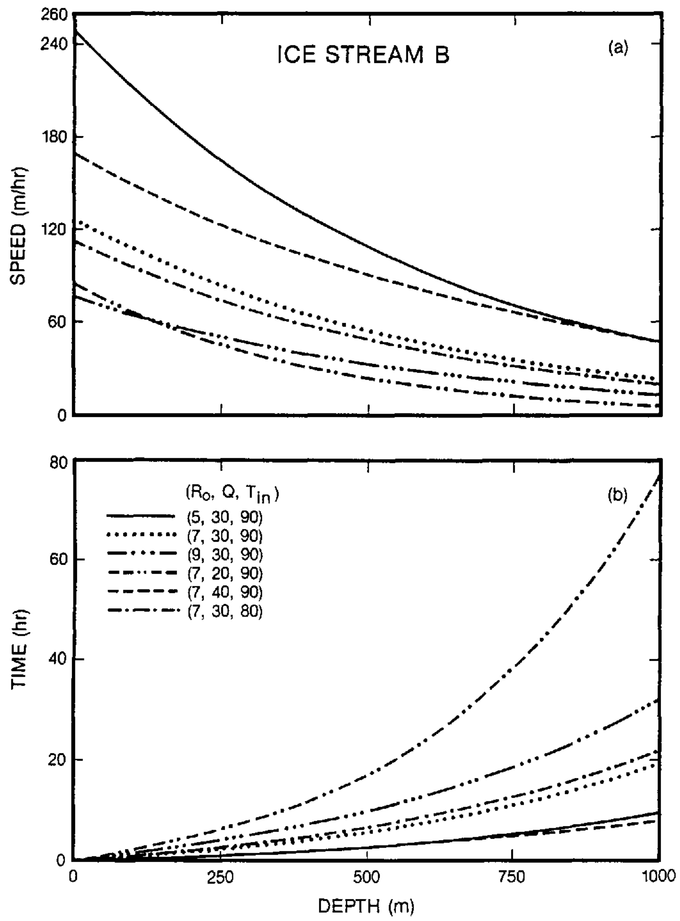

Fig. 3. Drilling speed (a) and time required to drill a bore hole to depth (b) for case A drill in Ice Stream B. Curves are labeled according to initial radius, R_0 (in cm), hot-water discharge, Q, in $l\ min^{-1}$; and inlet temperature, T_{in} ($^\circ C$).

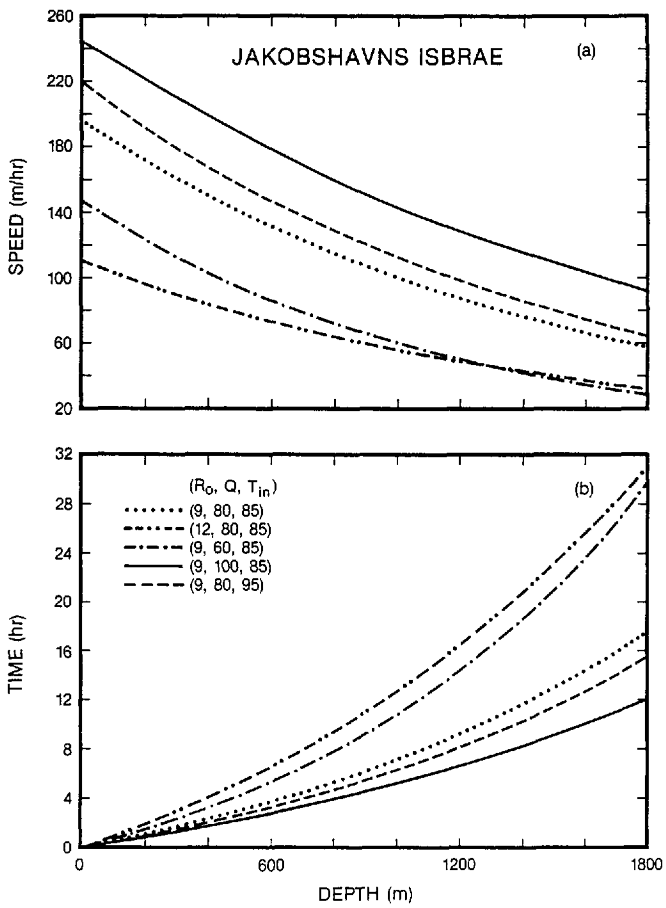

Fig. 4. Drilling speed (a) and time required to drill a bore hole to depth (b) for case B drill in Jakobshavns Isbræ labeled as in Figure 3.

As the depth of the hole is increased, the ability of a driller to sense the optimum drilling speed (either physically or with a load cell) which ensures a vertical hole of relatively uniform cross-section is reduced since vibration in the hose or its weight are not easily observed. Rate curves, such as those shown in Figures 3a and 4a, are invaluable for guiding such operations. Also, a proper choice of R_0 must be made, as discussed below.

Pressure losses in the hose and tip

There is a correction to the above development from the energy dissipated by the flow of fluid in the hose which depends on the pressure drop in the hose and at the drill tip. The pressure drop along the hose can be determined by standard, turbulent-flow theory (Reynolds number Re ~ 10^5) in which the drop depends on the square of the discharge, inversely to the fifth power of the inner hose radius and linearly with a hose-dependent friction factor (Bird and others, 1965). We have found that the friction factor for the type of smooth hydraulic hose used in both field projects described here (i.e. Synflex) is well-described by the $(\mathrm{Re})^{-\frac{1}{4}}$ — dependence of the Blasius equation (Bird and others, 1965).

For both cases A and B, the energy dissipation due to the pressure drop in the hose (about 70 bar at 1500 m in case B) and at the drill tip (about 70 bar in case B) is small in comparison with the thermal dissipation. However, if the drilling hose is of small diameter, then the frictional dissipation may be an important source of heat.

The tip-mixing region

Once the drilling fluid leaves the drill nozzle, it enters the highly turbulent mixing zone directly ahead of the tip.

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Several experimental studies have investigated this transfer (Iken and others, 1977; Taylor, 1984; Iken, 1988, Iken and others, 1989). By measuring bore-hole diameter at depth, Iken and others (1988) found a 90% efficiency of heat transfer over the first 10 m above the drill tip using system B and drilling at a rate approximately equal to that given by Equation (4). B. Koci (personal communication) has reported that, while drilling in Crary Ice Rise with a large drilling system, the outer water temperature at a position 5 m above the tip was one-half that at the tip (80° → 40°C). If this temperature were to decrease exponentially with distance above the tip, then the predicted temperature drop over the 10 m of the hose yields a comparable thermal efficiency of 75% for this larger system. It is reasonable that smaller drilling systems will have larger efficiencies, since the thermal transfer lengths are reduced. If efficiency is small, we might expect the quality of the hole (e.g. constancy of radius and roundness) to be reduced.

The remainder of the heat traveling upward from the nozzle region with the fluid will continue to melt ice, enlarging the hole to the radius R_0. The heat will decay exponentially with a Deissler-type heat-transfer coefficient (Bird and others, 1965). Based on the above data, the decay length is on the order of 5–10 m for common drilling systems. Thus, at a distance of a few tens of meters, we can expect all of the heat delivered to the tip to be utilized in melting ice. This is the condition which leads to relation (4) for drilling speed and the associated bore-hole radius, R_0. In other words, at a distance of 20 m or so above the tip, $r = R_0$ initially at the bore-hole wall. Also, when drilling at the speed given by Equation (4), the tip will be hanging free at all times and the hole will have maximum verticality.

The return flow

As the cooled drilling fluid and melted ice travel back up along the bore hole to the surface, it is heated by the "leakage" from the hose, as described above.

With the effective value of Z, the heat available from hose leakage per cent length, $Q_h(y)$, at a depth y is given by

$$Q_h(y) = \frac{T_d(y) - T_{wall}}{Z} \quad (6)$$

where $T_d(y)$ is the temperature of the drilling fluid and T_{wall} equals the pressure-melting temperature if the drilling fluid is water (see Equation (1)). This heat is available to heat the surrounding ice. However, some of this heat is wasted, as is indicated by a surface temperature of 1.8°C in the water returning from the drill tip at a depth of 1000 m on Jakobshavns Isbræ while drilling. This indicates an overall efficiency of heat transfer for case B of 98%.

THE PHASE CHANGE AT THE BORE-HOLE WALLS

Refreezing will occur as heat flows from the hole to the distant cold ice. The source of the heat in the bore hole during drilling is the heat leakage from the hose (as determined from Equations (1) and (6)). After drilling, the heat source is the latent heat of fusion for water freezing to the bore-hole walls. For drilling in cold ice, the fundamental question is: how long will the hole remain unfrozen given this heat flow? To solve it, we have developed a numerical-solution scheme as it has proven intractable analytically.

The problem may be formulated in cylindrical coordinates with y directed downward from the surface along the axis of the hole. Symmetry allows no azimuthal dependence. Because temperature gradients along the length of the hole are generally small relative to those away from the hole in radial direction, we may simplify the problem to one of heat flow in the radial direction only. The flow of heat in the ice is then governed by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\rho_i c_i}{K_i}\frac{\partial T}{\partial t} \quad (7)$$

where it is understood that T is the ice temperature at a depth y. The coefficients K_i and c_i are well defined in the ice, at least until the melting point is approached (Harrison, 1972). The imposed boundary conditions include the wall temperature, T_{wall}, and the ambient ice temperature at large distances from the hole, T_0. In this section, we assume the fluid is water, and thus $T_{wall} \approx 0°C$. The heat flux into the wall from the hot water within the hose is specified. The difference between this heat flux and that away from the wall down the temperature gradient in the ice determines the motion of the phase boundary through a Stefan boundary condition. If the phase boundary is located at $r = R$ (see Fig. 1), then

$$\mathscr{L}_v \frac{dR}{dt} = \frac{Q_h}{2\pi R} + K_i \frac{\partial T}{\partial r}\bigg|_R . \quad (8)$$

Equations (7) and (8), plus the boundary conditions and an initial condition, constitute a well-posed problem. For a non-constant heat input, we may take $Q_h = Q_h(t)$.

In order to generalize the solutions to bore holes drilled under various conditions, we place the problem defined by Equations (7) and (8) into non-dimensional form. For a hole drilled to initial radius R_0 (Equation (4)), let

$$T^* = \frac{T}{|T_0|} \quad (9a)$$

$$r^* = \frac{r}{R_0} \quad (9b)$$

$$R^* = \frac{R}{R_0} \quad (9c)$$

$$t^* = \frac{t}{t_0} \quad (9d)$$

$$Q^* = \frac{Q_h}{2\pi K_i |T_0|} \quad (9e)$$

and define

$$\kappa^* = \frac{\mathscr{L}_v}{\rho_i c_i |T_0|} \quad (10)$$

and

$$t_0 = \frac{R_0^2 \mathscr{L}_v}{K_i |T_0|} = \frac{\rho_i c_i}{K_i} \kappa^* R_0^2 \quad (11)$$

with T_0 given in °C. Values of t_0 and κ^* for representative depths in Ice Stream B and Jakobshavns Isbræ are given in Table II. The strong decrease in t_0 with depth on Jakobshavns Isbræ will be seen to make drilling in such a situation much more problematical than in a situation such as Ice Stream B, Antarctica.

TABLE II

Drilling system	*Depth*	κ^*	Q^*	t_0		
	m			h		
				R_0 = 0.06	0.09	0.12 m
Ice Stream B	Surface	6.6	0.98	6.0	13.4	23.8
(Case A)	250	6.8	0.66	6.1	13.7	24.3
	500	7.6	0.49	6.8	15.3	27.2
	750	10.3	0.46	9.2	20.8	36.9
	1000	31.9	4.52	26.0	64.3	114.3
Jakobshavns	Surface	17.7	2.60	15.8	35.6	63.4
Isbræ	400	9.1	1.04	8.1	18.3	32.6
(Case B)	800	7.9	0.75	7.1	15.9	28.3
	1200	6.8	0.46	6.1	13.7	24.3
	1600	6.1	0.31	5.5	12.3	21.9

The complete formulation of the moving phase-boundary problem with heating at the bore-hole wall ($r = R$) derived from the heat loss along the hose is then given by:

$$\kappa^* \frac{1}{r^*}\frac{\partial}{\partial r^*}\left[r^*\frac{\partial T^*}{\partial r^*}\right] = \frac{\partial T^*}{\partial t^*} \quad (12)$$

$$\frac{dR^*}{dt^*} = \frac{\partial T^*}{\partial r^*} + \frac{Q^*}{R^*} \quad (13)$$

$$\lim_{r^*\to\infty}\left[T^*(r^*,t^*)\right] = -1 \quad (14a)$$

$$T^*(R^*,t^*) = 0 \quad (14b)$$

$$T^*(r^*,0) = -1 \text{ for } r^* > 1 \quad (14c)$$

$$R^*(0) = 1 \quad (14d)$$

$$T(r^*,t^*) = 0,\ r^* < 1,\ \forall t^* \quad (14e)$$

for $T^* = T^*(r^*,t^*)$, $R^* = R^*(t^*)$, and Q^* given by Equations (1), (6), and (9).

The problem defined by Equations (12)–(14) can be solved using finite-difference expansions of the derivatives, taking care to retain equal accuracy in the first-order and second-order spatial derivatives in Equation (12). A Crank–Nichelson implicit scheme, second-order accurate in time, is helpful in calculating the time evolution of the boundary with reasonably large time steps. This was the approach originally used in Iken and others (1988).

A faster and more elegant method of solution involves finite-element methods and the substitution

$$\omega = \int \frac{dr^*}{r^*} = \ln r^* \quad (15)$$

(e.g. Jarvis and Clarke, 1974). Equations (12) and (13) become

$$\kappa^* \exp(-2\omega)\frac{\partial^2 T^*}{\partial \omega^2} = \frac{\partial T^*}{\partial t^*} \quad (12')$$

$$\frac{dR^*}{dt^*} = \exp(-\omega)\frac{\partial T^*}{\partial \omega} + \frac{Q^*}{R^*} \quad (13')$$

$\omega \to \infty$ as $r^* \to \infty$ and $\omega \to 0$ as $r^* \to 1$. This substitution allows simplified coding in the finite-element scheme and increased resolution near the phase boundary, where temperature gradients are large. The solution region is limited to $r > R$, while for $r < R$ the temperature remains constant at the melting point.

Evenly sized elements are used in the logarithmic space. The mesh is moved at each time step to maintain the $\omega = 0$ node at the fluid/ice boundary. Interpolations and integrations are performed with linear functions.

Numerical solutions are well behaved for useful time steps, except as the hole actually freezes closed. At closure, the logarithmic transform introduces a singularity, and Equation (12′) is not valid. However, closure may be approached arbitrarily closely. The temperature profile at closure can be used as input to the analytical solution outlined in the section on "temperatures after hole freezing" given below.

Solutions to Equations (12)–(14) with no heat input from the hose ($Q^* = 0$) for different values of ambient temperature yield the one-parameter family of curves shown in Figure 5. Because of the non-dimensionalization (Equation (9)), these curves are applicable to the closure of any bore hole drilled instantaneously to a radius R_0. The ambient ice temperature enters through the dimensionless parameter κ^*. R_0 determines the closure time, which is on the order of t_0 ($\propto R_0^2$) as defined in Equation (9). These curves apply to the bottom of a drill hole for which the

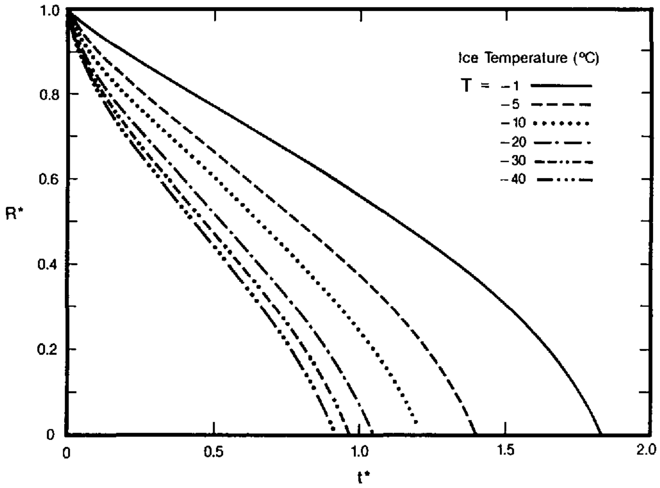

Fig. 5. Universal curves of bore-hole closure with no heating. Hole is assumed to be drilled instantaneously.

drill is quickly removed when $R = R_0$. Figure 5 also shows the danger of drilling a small-radius bore hole with a well-insulated hose (for which $Q_h \to 0$)). It is seen that closure is very rapid indeed if the ice temperature is low (e.g. 4–23 h with an ice temperature of −25°C and initial radius 50–120 mm, respectively). Only if the bore hole is very large ($R_0 \geqslant 0.5$ m) or, as is shown below, if the time of heating is significant, does the hole remain open for a useful period of time (especially considering most instruments are 50 mm or greater in diameter!). Otherwise, the ambient ice temperature must be near the melting point if a small hole is to remain open for a useful period of time.

When drilling under normal conditions, heat is applied to the bore-hole wall at depth $y < L$ from the time the drill tip passes this depth until the bottom of the hole ($y = L$) is reached. The heat loss is constant at y (if surface input is constant) until the drill is retrieved above y. The time of heating, T_Q, is then equal to the difference in time required to drill the bottom and to y, which, in non-dimensional form, is

$$t_Q^* = t_Q/t_0 = [t(L) - t(y)]/t_0 \quad (16)$$

where the drilling times are given by Equation (5). (Any time spent retrieving the hose under heat must also be included.) In some cases, the hole is then reamed using hot water. In this case, Q_h is a function of time at y, and t_Q and Q_h must be appropriately adjusted. However, the results are the same whether the heat is added by the hose loss during drilling or applied via repeated reaming operations.

Figure 6 shows a representative solution for a heating time approximately equal to the unheated closure time

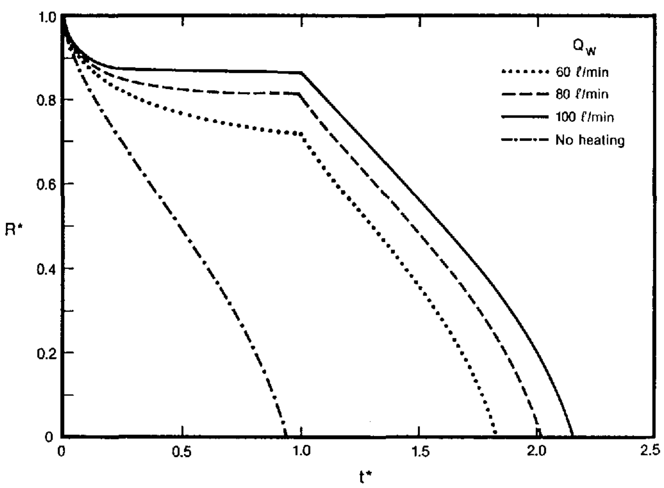

Fig. 6. Representative curve of bore-hole closure. Time of heating $t_Q^* = 1$ with drill system B at 1000 m in Jakobshavns Isbræ. Closure following drilling with different values of hot-water discharge is shown.

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t_Q^* = 1). The hole initially closes (as it will in all cases since the infinite temperature gradient imposed as part of the initial temperature condition drives closure until the imposed heat supply, Q_h, matches the temperature gradient-induced heat flow away from the hole $(-K_i \partial T/\partial r|R_0)$). The time period for which the hole initially closes is related to the ambient ice temperature and the magnitude of the heat loss from the hose.

Once the two heat fluxes balance (into and away from the wall), further heating will cause the hole to enlarge slowly. Depending on the ice temperature, time of drilling, and heat loss from the hose, the hole radius can increase to some radius which may be greater than the initial radius. Such is the case in the upper part of many deep holes, as will be seen below. A zone of warm ice grows around the hole until the heat is turned off, as shown in Figure 7. The dashed curves represent warming during drilling, and the solid curves show cooling after the heat is turned off. An interesting feature of the solution is the complexity of the time evolution of the temperature field. The newly formed ice near the refreezing hole is cooling while the more distant ice is still warming.

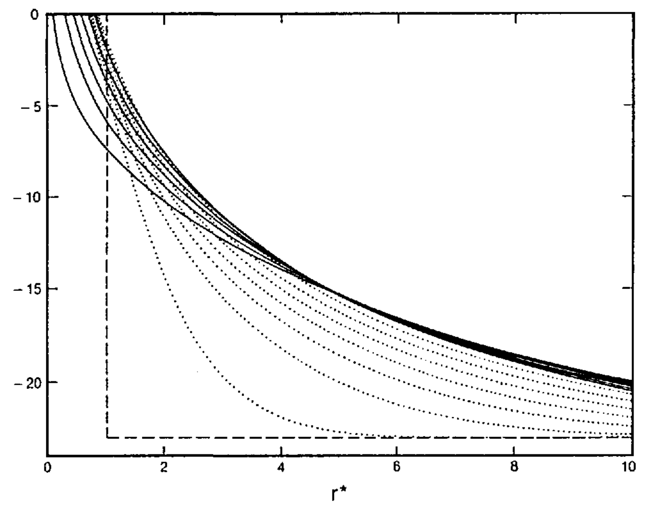

Fig. 7. Temperature in ice around bore hole drilled to 200 m (T_0 = −24°C) with drill A and heat for t_Q equal to 5.8 (1 d for R_0 = 5 cm). Dashed curves show development of temperature field while drilling is in progress below 200 m depth; solid shows evolution of the field after drill is removed above this depth. Time interval Δt* for dashed curves is 0.82; for solid it is 0.27.*

The benefits of heating the hole for longer periods of time are shown in Figure 8. Not only is the hole returned to a radius equal to or greater than R_0 but the time from heat removal to closure is increased. Of course, there are practical limits on the time of heating. Drilling slowly may enable the hole to enlarge and store sufficient preventative heat at shallow depths, but heat loss from the hose (i.e. system performance) may not be able to counteract hole closure while drilling at greater depths, especially if ice temperatures are low. Similarly, re-reaming the hole several times may put substantial heat into the ice, but bore-hole

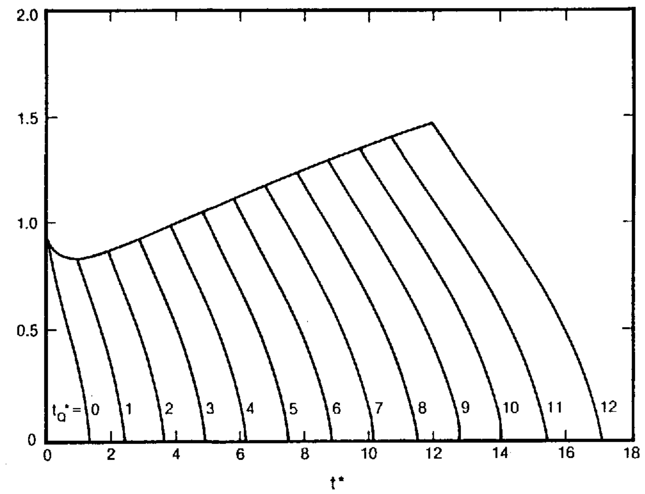

Fig. 8. Closure of bore hole at 200 m in Ice Stream B heated by system A for different lengths of time (t_Q). Hole initially closes, then enlarges until heating is stopped.*

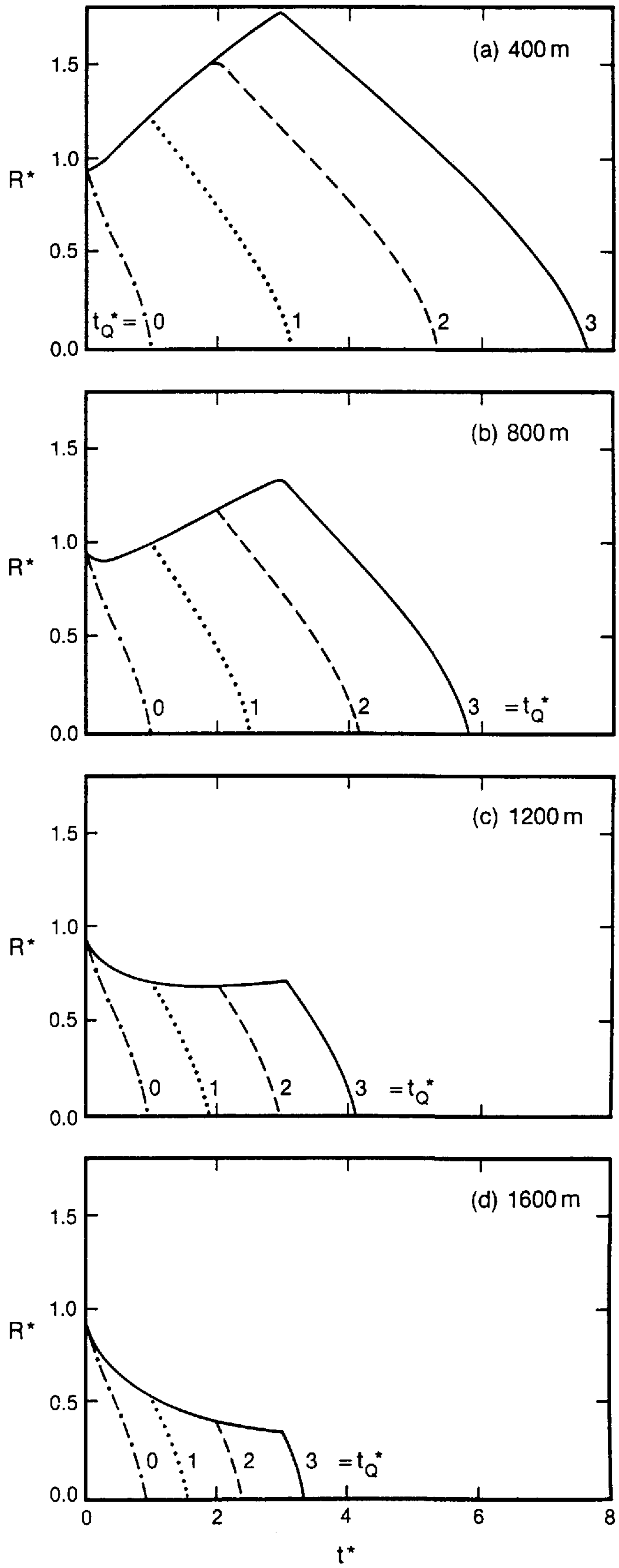

Fig. 9. Closure of bore holes at different depths in Jakobshavns Isbræ (drilled with system B) for different heating times t_Q. Vertical and time scales are the same from figure to figure.*

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closure between reaming operations (while the reamer is being pulled up) may cause loss of the hole.

Application to Jakobshavns Isbræ bore hole

Using the above models, the drill described in case B, and the temperature profile for Jakobshavns Isbræ shown in Figure 2b, we may develop a drilling strategy and model the time evolution of a bore hole drilled to 1600 m. The effects of heating for $t_Q^* = 0,1,2,3$ at representative depths are shown in Figure 9. Note that at shallow depths (y < 1000 m) the case B drilling system will cause hole enlargement for all heating times (and, therefore, in all bore holes drilled deeper than this). At greater depths (y > ~1200 m), the hole radius is reduced from its initial value at all times if $t_Q^* < 5$. At depths on the order of 1400–1600 m, the reduction in radius is monotonic and rapid down to about half the original radius. This is true for all practical heating times. This implies that drilling system B is marginal for holes about 1600 m deep for which the ambient ice temperature steadily decreases with depth. It is also important to note that drilling a small-diameter hole at these depths could be disastrous since the rapid drop to $R^* \approx 0.5$ could lead to a hole radius less than that of the drill tip or hose. If the ice temperature is warmer at depth than that shown in Figure 2b, the closure will proceed at a slower rate.

Increasing the entrance temperature to the practical limit of 95°C does not significantly change these considerations. The only feasible method to drill below 1600 m in such cold ice is to increase the hot-water discharge, Q_w. This increases the characteristic decay length λ in Equations (1) and (2) and thus increases the heat loss along the hose as given by Equation (6). Because the pressure rating of the hose and other components of the system is limited, a substantial increase in Q_w (say to 180 l min^{-1} or 3 × 10^{-3} m^3 s^{-1}) requires an increase in hose diameter, say to 25.4 mm inside diameter.

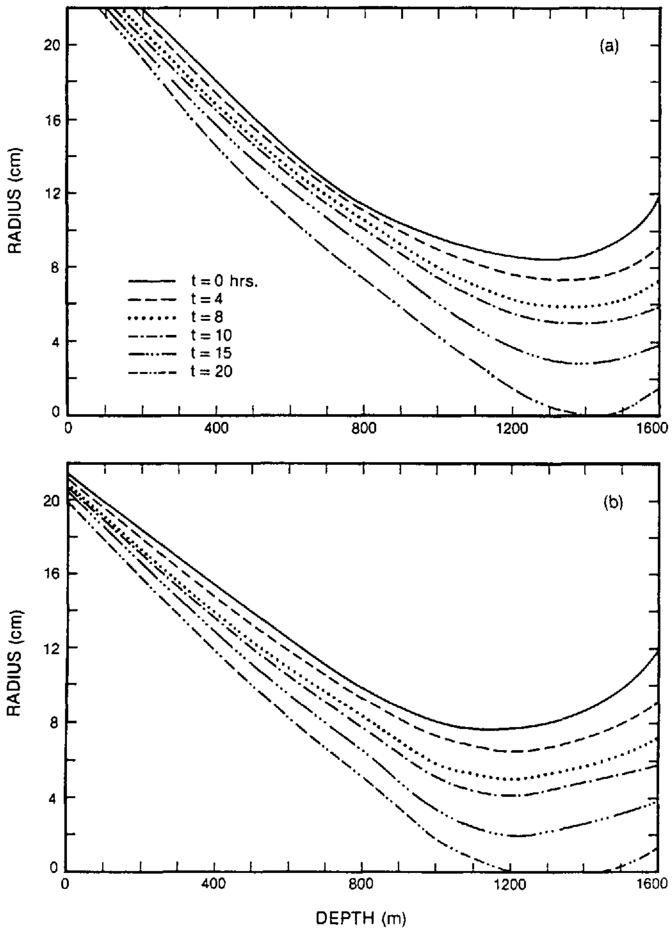

Fig. 10. (a) Time evolution of bore-hole radius at depth after drilling to 1600 m. R_0 is constant at 0.12 m. t equal to zero signifies the time when the drill reached 1600 m and power was shut off. (b) Similar to (a) except R_0 was 0.09 m above 1000 m depth, 0.10 m from 1000 to 1200 m, and 0.12 m below 1200 m.

Suppose that, based on examination of illustrations such as Figure 9, we choose to drill a hole of constant initial radius R_0. The hole will be drilled at a speed given by Equation (4) (e.g. see Fig. 4). The drill is removed upon reaching the bottom (leaving this drill down for any reasonable length of time does not serve to combat hole closure at great depths, as seen in Figure 9c and d). What then is the time evolution of the bore hole? Using Equations (15) and (14) to calculate t_Q^* and Q^* at a given depth, we may model the closure of the hole there. Combining several such models allows the bore-hole wall to be followed through time along its length. Figure 10a shows the result of such a study in dimensional space for R_0 = 120 mm. In this figure, t = 0 h corresponds to the time when the drill reaches the bottom (L = 1600 m; total of 26 h to drill). As shown in this figure, the radius at large depths is significantly smaller than R_0, even at the time the drill is removed. The radius decreases steadily and rapidly at those depths, making the hole almost unusable near the bottom after only 6 h. Enlargement occurs at depths of less than 800 m. A critical region of depth between 1300 and 1400 m exists where heat losses from the hose are insufficient to keep the hole open at a reasonable radius. Drilling below this depth must be accomplished quickly if the entire hole is to be useful for experimentation.

Hole enlargement to such large radii at shallow depths would seem to be inefficient and to be avoided. However, such avoidance is generally not the prerogative of the driller, since the maximum speed of drilling can only be increased slightly above that given by Equation (4). This is because the maximum drilling speed, as determined by Iken and others (1988), is reached. Above this speed, inefficient heat transfer leads to a non-vertical and poor-quality bore hole. In some cases, using a larger system with a better-insulated hose (such as that used by PICO (personal communication from B. Koci)) to drill a hole of larger radius may prove more efficient. The increased bulk and weight of insulated hose must be compared to the decrease in overall efficiency in drilling the bore hole at improper radius.

In order to minimize drilling time, the initial radius of the hole may be decreased from 120 to 90 mm at depths less than 1000 m and to 100 mm from 1000 to 1200 m. The decrease in drilling time is substantial (26 to 19 h). The critical depth is shifted to about 1200 m because of the trade-off in smaller R_0 and longer $t_Q(y)$.

The model results shown in Figures 9 and 10 proved invaluable for drilling to the bed of Jakobshavns Isbræ (1627 m) in 1989 (Echelmeyer and others, 1989). Drilling to depths much in excess of 1700 m in cold ice would require an increased hot-water discharge, as explained above.

Optimal drilling

While the numerical approach just described for determining drilling speed and initial radius as a function of depth is useful in planning a field operaton, it does not provide continuous and complete specifications of $v(y)$ and $R_0(y)$. The actual solution for the best $v(y)$ would require a non-linear optimization of the rate of drilling, time of heating, and bore-hole closure rate. This optimal solution would produce a $v(y)$ which leads to simultaneous hole closure for all depths at some time after drilling. In this case, no "extra" heat would be applied nor any time wasted in drilling too large a hole at any depth. While this optimization cannot be obtained analytically, a few simpler cases show the pertinent points of such a solution.

As a first model, assume that the ice is isothermal. Also, assume there is *no* heat loss along the hose and that all heat available is used to melt ice near the tip. This is the case of a well-insulated drill hose and slow drill speed. As such, it might be more applicable to the system designed by PICO and used on Crary Ice Rise, Antarctica. Then, the speed of drilling will be related to the radius by Equation (4) with the term in brackets set equal to one (or nearly so). (Assume the drilling fluid is water.)

As the non-dimensionalization (Equation (11)) shows, the time of complete hole closure when no heat is applied after drilling (Q_h = 0) is approximately equal to the time t_0 (i.e. $t^* \approx 1$ at complete closure). This is also shown in Figure 5. Thus, the total time of refreezing, t_f, is proportional to R_0^2:

$$t_f(y) \approx t_0[R_0(y)] = \frac{\mathcal{L}_v}{K_i|T_0|}R_0^2(y). \qquad (17)$$

Suppose we wish the entire bore hole, of length L, to freeze shut simultaneously at a time τ since the initiation of drilling, where $\tau > t(L)$. Then

$$t_f(y) = \tau - \int_0^y \frac{d\zeta}{v(\zeta)} \qquad (18)$$

which, using Equations (4′) and (17) leads to

$$R_0^2(y) = \frac{K_i|T_0|}{\mathcal{L}_v}\tau - \frac{1}{\Lambda}\int_0^y R_0^2(\zeta)d\zeta \qquad (19)$$

where $\Lambda = (\rho cQT)_w/\pi K_i|T_0|)$, which has units of length. This integral equation for R_0 may easily be solved by differentiating each side of Equation (19) with respect to y, yielding a differential equation for R_0. Solving this equation subject to the boundary condition that $t_f(0) = \tau$ gives

$$R_0(y) = \left[\frac{K_i|T_0|}{\mathcal{L}_v}\tau\right]^{\frac{1}{2}} e^{-y/2\Lambda}. \qquad (20)$$

From Equation (4)

$$v(y) = \frac{\Lambda}{\tau}e^{y/\Lambda}. \qquad (21)$$

Thus, for an isothermal glacier with no heat loss along the hose, the speed of drilling should increase exponentially with depth. This leads to an exponential decrease in the initial radius with depth. The scale length Λ is directly proportional to the power output of the drilling system, $(\rho cQT)_w$. For system A, $\Lambda \sim 1100$ m, while for case B (and for the drill used on Crary Ice Rise (personal communication from B. Koci)), $\Lambda \sim 3000$ m. Basically, this large value of Λ means that the initial radius and, to a lesser extent, the speed of drilling, should be nearly constant down to a depth of 1500 m or so for the larger drilling systems. If the bore hole should remain open for 20 h, then the speed of drilling down to 1500 m would be in the range 150–250 m h^{-1} with a radius of 0.11–0.09 m with such a system. The drilling time would be about 8 h. These simple calculations suggest that drilling with an insulated hose may be appropriate if the power output of the system and logistical/budgetary constraints do not dictate otherwise.

If there is a finite and depth-dependent heat loss along the hose, then the exponential increase in speed indicated by Equation (21) will no longer be valid. A non-linear integral equation results for the optimization problem if both the exponential heat loss along the hose (Equation (1)) and an increase in closure time which is approximately proportional to the square root of the total heat applied to the bore hole (after drilling to R_0) are taken into account. The latter assumption is based on the model results depicted in Figure 8.

If instead the simplification is made that the heat loss along the hose is given by Equation (1), but we assume that none of the lost heat is used to prevent hole closure (i.e. no dependence of closure rate on the time of heating), then

$$R_0(y) \approx \left[\frac{K_i|T_0|}{\mathcal{L}_v}\tau\right]^{\frac{1}{2}} \exp\left\{-\left[\frac{\lambda}{2\Lambda}e^{y/\lambda}\right]\right\} \qquad (22)$$

and

$$v(y) \approx \frac{\Lambda}{\tau}\exp\{-\frac{1}{\lambda\Lambda}[\lambda^2 e^{y/\lambda} + \Lambda y]\} \qquad (23)$$

where λ is given by Equation (2) and Λ is as stated above for an isothermal ice mass. For system B, this gives an extreme (nearly exponential) decrease in drilling speed with depth (and also a similar decrease in radius). Indeed, for a bore hole with a closure time of 20 h ($v_{surface}$ = 150 m h^{-1}) the speed at 500, 1000, and 1500 m would need to be 53, 28, and 14 m h^{-1}, respectively. Such slow speeds would surely make a great deal of heat available to the bore-hole wall, which violates the assumptions used in this simple model.

As the numerical results for the Jakobshavns Isbræ bore hole (Figs 9 and 11) show, the actual optimal variation of radius and drilling speed with depth lies somewhere in between the results given by Equations (20), (21), and (22), (23). An increase in bore-hole radius and a decrease in speed are indicated for drilling system B in ice where the temperature decreases with depth.

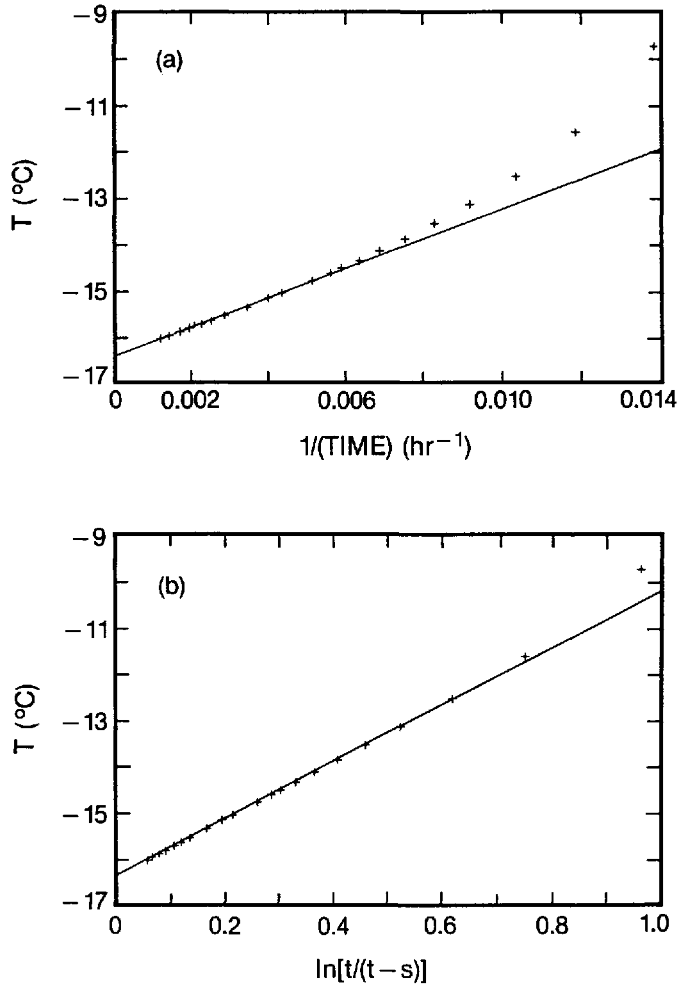

Fig. 11. (a) Temperature of thermistor at depth in Jakobshavns Isbræ versus 1/t, where t is hours since drill was removed. Line shows extrapolation to steady-state ice temperature using Equation (24). (b) Same thermistor record versus ln[t/(t − s)], where s is the time since complete hole closure at this depth (16.25 h). Line represents extrapolation from Equation (25).

IV. TEMPERATURE PROFILES AFTER COMPLETE RE-FREEZING

Ater the hole has completely refrozen (in the case of no antifreeze), the Stefan boundary condition is removed, and the temperature field around the bore hole may be obtained analytically for times significantly longer than the time of heating. This solution is useful in analyzing temperature measurements in refrozen bore holes. The asymptotic behavior at times that are long compared with the time the hole was open (including the drilling time) has been given by Carslaw and Jaeger (1959, chapter 10). Along the axis of the refrozen hole,

$$T(t) - T_0 = \frac{q}{4\pi K_i}\frac{1}{t} \tag{24}$$

where q is the total amount of heat per unit length put into the hole while drilling and refreezing, and t is the time since drilling occurred at the depth in question.

If the duration, s, of the time the hole was open is not small compared with the elapsed time, t, since the drill reached a particular depth, a better approximation is

$$T(t) - T_0 = \frac{1}{4\pi K_i}\frac{q}{s}\ln\left[\frac{t}{t-s}\right] \tag{25}$$

(Lachenbruch and Brewer, 1959, equation (3)). The $1/t$ and "nearly" $1/t$ dependencies shown by Equations (24) and (25), respectively, are useful in extrapolating temperature-sensor measurements made over relatively long but limited time to find ambient ice temperatures. Figure 11 gives an example of such an extrapolation using both Equations (24) and (25). Equation (25) provides a better extrapolation over shorter measurement periods. Jarvis and Clarke (1974) and Humphrey (paper submitted) solved the diffusion equation, Equation (9), using a numerical scheme similar to that described here (with $Q_h = 0$) in order to extrapolate temperature measurements over very limited time periods when even Equation (25) is invalid.

V. THE ANTIFREEZE PARADOX

It is commonly believed that a water-soluble antifreeze may be injected into hot-water bore holes in order to combat refreezing in cold ice. However, in practice, ice slush forms in the holes despite the antifreeze. This has been the experience at both Ice Stream B and Jakobshavns Isbræ, and in the bore hole at J9 (Zotikov, 1986). The cause of much of the slush formation is undoubtedly zones of mismatched antifreeze concentration and wall temperature. However, slush formation is a predictable outcome of injecting antifreeze into almost any water-drilled hole in cold ice (some exceptions are described below). Our experience has shown that the vexing problem of slush formation occurs even in holes that are filled with antifreeze at a nominal concentration considerably higher than needed to prevent freezing.

Slush-ice formation with antifreezes

The long-term behavior of an antifreeze-filled hole is a complex problem that depends on the solubility of the antifreeze in ice as well as consideration of surface energies for the antifreeze/ice interface. Here, we discuss only the short-term behavior that depends on the thermal evolution in the first tens of hours. Ethanol is used as the antifreeze for an illustrative, non-toxic example.

After completion, a bore hole drilled with hot water lies in a cylinder of ice that contains a large amount of heat with respect to the cold bulk ice. Typically, if the hole takes a day to drill, then the energy in the ice surrounding the bore hole is several times the energy that was required to melt the bore hole itself. Antifreeze added to such a hole contacts a bore-hole wall which is much warmer than the bulk ice temperature. Ice immediately around the hole is near 0°C, and is quickly dissolved by the antifreeze. The required heat initially comes from the fluid in the hole, causing the temperature of the fluid to drop, until the freezing point of the antifreeze mixture in the hole is reached. The fluid temperature is then lower than the surrounding ice temperature, at least out to some radius. Heat flows down this temperature gradient allowing further melting and also raising the fluid temperature as the antifreeze is diluted.

This process continues, with dilution of the antifreeze and a slow rise in its temperature. The cylinder of warm ice around the hole is losing heat to the hole and also to the distant ice. If enough melt occurs, the fluid temperature can rise above the distant ice. When the temperature of the wall ice drops below the fluid temperature, the diluted antifreeze is at its freezing point. Further cooling causes dissolved water to freeze out into slush ice. The amount of slush formation depends on the rate of wall dissolution. The faster the wall melts, the more stored heat goes into melting before the reservoir of heat is lost to the bulk ice. This leads to more water in solution and, thus, more slush following the final cooling.

The rate of dissolution depends on both heat conduction and fluid diffusion near the wall. At the wall, a flow of melt water diffuses away from the ice and into the bulk fluid, creating a zone of low antifreeze concentration next to the wall. There is a counter flow of antifreeze diffusing towards the wall. The wall is a heat sink to the ice, since it is melting (and either a source or sink of heat to the fluid, depending on the relative dilution of the fluid; during most of the dilution the wall is a source of heat to the fluid). If the fluid were unstirred and convective motion suppressed, the rate of wall melting would be effectively controlled by the low molecular diffusion rate of water away from and antifreeze towards the wall. However, in the bore hole, we expect effective fluid mixing initially during the hour or so that it takes to inject the alcohol, and convective mixing is likely even after the external disturbances are removed from the hole. Mixing the fluid greatly increases the diffusion of water and antifreeze, and molecular diffusion is confined to a narrow laminar sub-layer near the wall boundary. If this is the case, the rate of wall melting is controlled by the rate of heat delivery. In the calculations below, we assume that the thermal flow dominates the problem and that it determines the rate of wall melting. This sets an upper bound on the formation of slush.

A calculation of slush formation

To illustrate this process, we analyze the freezing of a bore hole at Up-B, Antarctica, in the winter of 1988–89. After successfully drilling a nominal 0.1 m diameter hole to the bed of the glacier, 3100 l of alcohol were pumped into the hole to create high-alcohol concentration in the upper, colder part of the hole. The final concentration was expected to be between 35 and 45% alcohol by volume, which should have been sufficient to prevent the freezing of the hole in −24°C ice. One sampler was lowered to the bed and retrieved, but when a second was lowered 3 h later, it stuck in slush at the water surface. The hole was effectively frozen and had to be redrilled.

The time evolution of the hole can be modeled approximately using the finite-element heat-flow model described in the previous section. However, in this particular problem, the temperature at the wall interface is not fixed at 0°C, but depends on the concentration of antifreeze in the fluid near the wall. If the concentration is large, then the ice wall will melt until the local temperature drops to the point where the wall is in equilibrium with

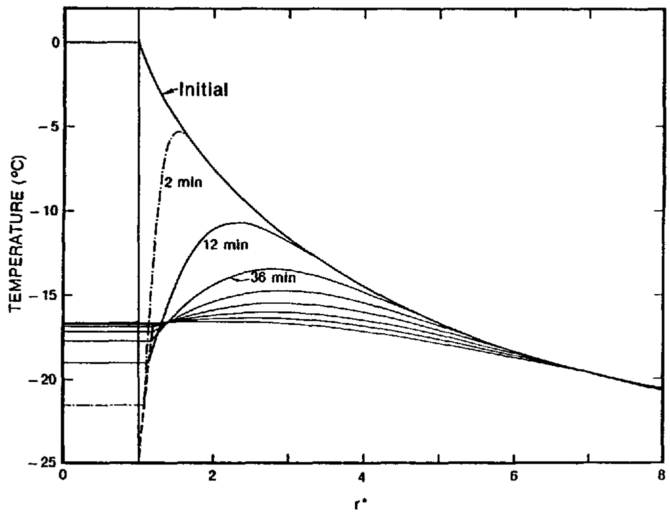

Fig. 12. Model of time evolution for the temperature in a 200 m bore hole in Ice Stream B after antifreeze was added to 50% concentration. Heavy solid line is initial temperature profile about bore hole before antifreeze addition. Dash–dot curve represents a time 2 min after injection; the next solid line is 12 min later. Each following curve is taken 24 min apart. Initial radius is 0.05 m.

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the fluid. Similarly, if the antifreeze concentration is low, fluid will tend to freeze. Slight gradients in antifreeze, with higher concentrations near the wall, will cause this freezing to occur as nucleation of slush throughout the fluid.

Heat flow in ice is described by Equation (7). The initial conditions of temperature are given from a calculation on the temperature around a water-filled hole after drilling (as in Figure 7). We assume that the hole is reasonably well mixed and the problem is dominated by thermal gradients, not diffusion/concentration gradients, and that the temperature of the ice/fluid boundary is at the freezing point of the average antifreeze mixture in the hole. Temperature changes within the bore-hole fluid are used as a heat source or sink but the actual heat flow in the fluid is ignored. The concentration of antifreeze in the hole depends on the amount of water melted from the wall.

Figure 12 shows the results of a thermal calculation in which it was assumed that the hole took 1 d to drill. The calculated initial temperature profile in the ice is shown by the upper solid line. Antifreeze, with a temperature of 0°C, was instantly injected to achieve an initial 50% concentration. Rapid melting occurs at the wall as antifreeze (with a freezing point of about −40°C) comes in contact with the ice that is at 0°C. The majority of the heat for the initial melting comes from the heat capacity of the bore-hole fluid. This melting dilutes the antifreeze concentration so that soon after injection the fluid is at −23°C and the temperature profile in the wall is that given by the dash–dot curve. The important point is that the fluid temperature and the wall temperature are well below the ice temperature near the hole. For the next several hours, melting at the wall further dilutes the fluid. Eventually, as the temperature of the fluid slowly rises and the heat around the hole diffuses away to the distant cold ice, the fluid and the near-hole ice reach the same temperature. At this point, the melting stops (3 h after adding the antifreeze). After this, the heat flows from the hole, which now contains fluid that has a melting point of about −17°C.

This flow of heat away from the hole results in slush formation, although the amount formed will depend upon how well the hole is mixed. Slush generation will be a maximum for no mixing, because, while antifreeze will be concentrated at the bore-hole wall by freezing, molecular diffusion is too slow to maintain the necessary concentration within the hole to prevent slush formation as cooling occurs. The time of initial slush formation is the same in the numerical solution and in the observed bore hole. The creation of slush in the hole within a few hours of introducing antifreeze should be contrasted with the 8 h required for the hole to freeze closed if left filled with water only.

Note that a higher initial antifreeze concentration does not alleviate this "slushing" problem. Typically, enough heat is stored in the annular zone of ice around the hole to dilute even pure alcohol to a point where its freezing temperature is above the bulk ice temperature. Since it is heat stored in the ice that causes the antifreeze to follow this melting/freezing cycle, the problem could be alleviated by drilling with antifreeze. This would reduce the heat that is released to the ice after the drill tip has passed a region. Unfortunately, drilling with antifreeze in current drills is technically difficult and expensive.

Solution to the antifreeze problem

The heat driving the dilution comes from antifreeze in the warm ice around the bore hole. If the temperature gradients can be maintained outward, then dilution will not occur. However, outward gradients cause refreezing/slush formation. A minimum of dilution or refreezing can be achieved if the antifreeze in the hole is always at equilibrium with the temperature of the wall. This eliminates the heat source/sink of the phase change at the wall. Since the heat in the ice near the hole drains to the distant cold ice, the antifreeze concentration must slowly be increased with time to match the cooling ice.

Figure 13 shows the cooling of the wall and the required alcohol concentration that would be needed to fill a length of bore hole drilled with system A in ice at −24°C. In practice, the ice temperature is not constant with depth, nor is the stored heat after drilling. Thus, multiple versions of Figure 13 are required to cover the bore-hole conditions. With this information, a careful program of alcohol injection could be devised. From Figure 13, it is seen that in the first 12 h requires over 90% of the total alcohol must be added. After a length of time, comparable to the drilling time of the hole, the temperature will change only slowly and an instrument could be placed in the hole for a period of time before the next re-injection.

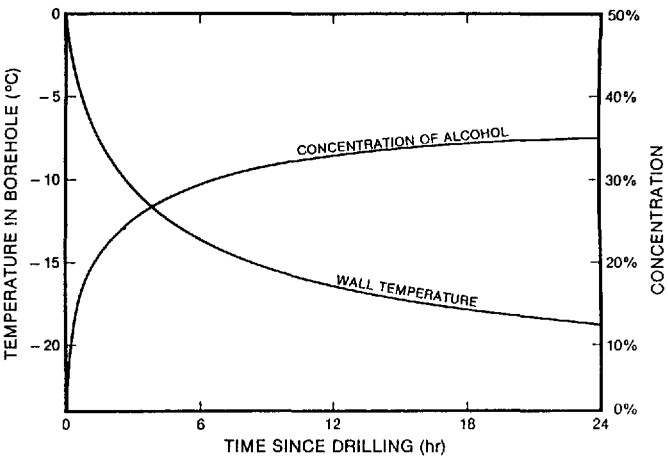

Fig. 13. Bore-hole temperature and optimal antifreeze concentration as a function of time since drilling. T_0 = −24°C and 1 d of heating at this depth (200 m).

VI. CONCLUSIONS

Hot-water drilling provides a rapid and relatively inexpensive method for drilling holes in warm and cold ice. The major problem facing a driller in attempting to drill a bore hole in cold ice to great depths is that the hole will refreeze rapidly over time. If the heat loss from the hose is insufficient, as it might be at depths approaching the limit for a particular drill system, this closure will be an ongoing problem even while drilling. In any case, once the hose is removed from the bore hole, the hole will start to close. The time of closure at any depth depends on the heat put into the bore hole (e.g. the heat-loss characteristics of the hose and the time of drilling and reaming), the square of the initial radius, and the ambient ice temperature. Heat is stored both in the ice itself and in the melted ice of the bore hole. A bore hole of large radius (≥0.5 m) could be drilled using a large drilling system (e.g. personal communication from B. Koci and R. Bindschadler), which would minimize the effects of this closure. However, drilling at many scientifically interesting locations requires ease of portability, minimal logistical support, and low cost. Iken (1988) and Iken and others (1989) have shown how such a small drilling system can be designed to drill these bore holes.

In the present paper we have described the rate of closure and its relation to the thermal properties of the drill, the initial radius, and the ice temperature. In order to maximize the usefulness of a deep bore hole, the initial radius and, thus, the speed of drilling must be appropriately chosen at depth. This choice depends upon the ice-temperature profile with depth and heat-loss characteristics of the drill hose.

It is important to plan any deep hot-water drilling program in cold ice based on these ideas of bore-hole closure. Even when the temperature profile shows an adverse decrease with depth (as in Fig. 4), useful bore holes may be successfully drilled to great depths as shown by recent drilling programs in Greenland (Echelmeyer and others, 1989; Iken and others, 1989) and Antarctica (Engelhardt and others, in press).

If a bore hole must remain open for more than a short time (say, more than 1 d), it must either be very large (requiring a relatively unportable drilling system) or it must be filled with antifreeze. If antifreeze is used, it must be carefully injected following a strict, calculated, time-dependent schedule. Otherwise, slush formation will occur and the hole will quickly become unusable. The actual antifreeze injection schedule requires accurate knowledge of the temperature profile at depth. In some cases, the amount of antifreeze required may increase logistical support requirements to the point that a much larger drill could be used more effectively.

The rapid closure rates of non-antifreeze-filled hot-water bore holes in cold ice suggests that many measurements should be made using *in-situ* sensors (such as for tilt, temperature, and basal conductivity). Such devices have recently been emplaced in the deep bore holes drilled in Jakobshavns Isbræ.

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