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Processes in the decomposition of networks of queues

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There are two possible approaches to the problem of a network of service facilities, where an individual customer passes through the network, queueing for service at some or all of these facilities. The system may be kept intact and an attempt made to solve for the resultant vector valued random processes. Even if sufficient assumptions are made to ensure that these processes are Markov few networks have comprehensive solutions as yet. The second approach is to consider methods by which the network may be decomposed into simpler structures, often consisting of individual servers, which can be studied separately.

A natural place to split up the network is at those points where a stream of customers is broken into several output streams by some kind of switching rule. Conditions are established for a class of decomposition switches which ensure that if the input stream to the switch is a Markov renewal process then so are each of the output streams.

The behaviour of one of these output streams at a subsequent service facility is then considered, first with exponentially distributed service times. The $SM/M/1$ queue has been investigated by Çinlar [2]. Here an analysis based on the busy period process considerably extends and simplifies Çinlar's results, avoiding the use of Laplace transforms where possible. Some attempt is made to relax the condition of the service time distribution. Since a Poisson process approximation for the arrival stream

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may be acceptable, the dual of this queue, where the sequence of service times forms a Markov renewal process, is considered. This generalisation allows for different classes of service requirements, perhaps reflecting the origin of customers from various parts of the network. In addition some results based on an identity due to Arjas [1], are presented for a *SM*/*SM*/1 queue, in which both the arrival times and service requirements depend on the sequence of customer types.

Since the output from a queue in a network will form part or all of the input to a subsequent queue, the departure processes from queues of the types that have been mentioned are examined. A few of these can be represented as Markov renewal processes and for the rest, in particular the *SM*/*SM*/1 queue, Arjas' identity is again used to find the distributions of the interdeparture intervals and the number of departures. The last of these has been published in [3].

References

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