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Distorted Dipole Magnetic Fields

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1. Background

Mestel et al. (1985; MRΩ^2) introduced an axisymmetric pulsar magnetosphere model in which electrons leave the star with non-negligible speeds and flow with moderate acceleration, and with poloidal motion that is closely tied to poloidal magnetic field lines, before reaching S_L, a limiting surface near which rapid acceleration occurs. As well as these Class I flows, there exist Class II flows which do not encounter a region of rapid acceleration (Burman 1984, 1985b). The formalism introduced by MRΩ^2 to describe the moderately accelerated flows can be interpreted in terms of a plasma drift across the magnetic field, following injection along it (Burman 1985a).

The MRΩ^2 formalism fully incorporates the toroidal magnetic field generated by the poloidal flow. The general formalism leaves the poloidal magnetic field unspecified, but, in the detailed development of MRΩ^2, and in my papers, that field was taken to be the dipolar field of the star.

Numerical work by Fitzpatrick & Mestel (1988a,b) suggested that the dipole approximation is inadequate. They developed a numerical technique for incorporating the modification to the poloidal magnetic field that is generated by the toroidal motions throughout the magnetosphere. They based their treatment on the hypothesis that those motions are such as to cancel the dipole field of the star, leaving a sextupole poloidal magnetic field at large distances.

A recent elaboration by Mestel & Shibata (1993) incorporates electron-positron pairs, created near and beyond S_L, into the outflowing stream. The gamma rays emitted by the rapidly accelerated electrons in that region result in copious pair production, and the outflowing stream becomes a ternary plasma, consisting of the primary electrons and a dense secondary electron-positron plasma. Mestel & Shibata located S_L well within the light cylinder.

The poster paper gave an interim report on my development of an approximate analytical technique for including the modification to the poloidal magnetic field that is generated by the toroidal motion of the outflowing stream. I am developing the technique in such a way as to retain as much as possible of the earlier formalism in which the poloidal magnetic field was taken to be dipolar. I expect to complete a paper presenting the technique later this year.

2. The Method

Any axisymmetric poloidal magnetic field can be expressed through a magnetic Stokes stream function P: With X denoting the cylindrical polar radial coordinate and Z the axial coordinate, the field components B_X and B_Z are respectively

$X^{-1}\,\frac{\partial P}{\partial Z}$ and $-X^{-1}\,\frac{\partial P}{\partial X}$. (With **t** the unit toroidal vector, the vector potential is $\frac{-P\mathbf{t}}{X}$.) Both X and Z will be dimensionless, normalized by the light cylinder radius. The auxiliary variables U and Q, defined as $X^{2/3}$ and $-P^{2/3}$, are often convenient. It is U (or X) and Q (or P), rather than X and Z, that I will regard as the independent variables, forming a non-orthogonal coordinate set in meridian planes. The poloidal field lines are lines of constant P or Q.

A dipole magnetic field is defined by $-P = \frac{X^2}{R^3} = \frac{(\sin^2\theta)}{R}$ or $QU = \frac{X^2}{R^2} = \sin^2\theta$, with R the dimensionless distance from the origin and θ the angle from the Z axis. The field lines have equations $-PR = \sin^2\theta = QU$. The dipolar form for P can be regarded as an equation for Z and re-arranged (*cf* MRΩ^2) to give $Z^2 = \frac{(1-QU)U^2}{Q}$. This enables Z to be eliminated, leaving U and Q as the coordinates.

The idea behind the distorted dipole approximation is to take a form for P which is sufficiently similar to the dipolar one that it readily allows the elimination of Z as a coordinate, but is more general in that it contains an undetermined function of distance from the axis. I will define distorted dipole fields by expressing the magnetic stream function by $-P = \frac{X^2}{D^3}$ or $QU = \frac{X^2}{D^2}$ with $D^2 = X^2 g(U) + Z^2$, where $g(U)$ is some function of distance from the symmetry axis. Just as in the dipolar case, this can be regarded as an equation for Z, yielding $Z^2 = \frac{U^2}{Q}\,\alpha(U,Q)$ with $\alpha(U,Q) = 1 - QUg(U)$. I will call $g\,(U)$ the magnetic structure function – its departure from unity determines the distortion of the poloidal magnetic field from the dipole form.

This form of magnetic stream function is algebraically attractive in that it enables Z to be eliminated analytically, leaving U and Q as the independent spatial variables – they seem to form a natural (though non-orthogonal) coordinate set.

Use of a magnetic Stokes stream function means that div $\mathbf{B} = 0$ is automatically satisfied. The distorted dipole stream function leads to a toroidally directed curl $\mathbf{B}$ containing $g(U)$ and its first and second derivatives. Steadily rotating axisymmetric systems have no displacement current, so Amperes law applies: this expression for the toroidal component of curl $\mathbf{B}$ must match the toroidal component of the electric current density in the region. The task becomes one of matching the toroidal electric current density, furnished by your favourite magnetospheric model, with the distorted dipole expression for curl $\mathbf{B}$.

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