EVOLUTION OF STARS WITH $M \geqslant 8\,M_\odot$*

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Abstract. A review of research in the field of evolution of massive stars during the last three years is presented. The analysis of computed stellar models in the helium-burning stage provides evidence for the existence of neutrino emission predicted by the theory of universal Fermi-interaction. The criterion for convective stability in the zone of variable molecular weight still remains uncertain despite several possibilities for a unique choice of the stability criterion that were recently suggested.

Evolutionary computations of massive star models in the carbon, oxygen, neon and silicon-burning stages provide opportunities to obtain models of presupernovae of different types and to investigate the cause of instability responsible for the implosion as a function of the initial mass of the star.

If the neutrino emission is included in computations the core of a star with $M \gtrsim 90\,M_\odot$ loses stability due to the positron-electron pair formation in the oxygen-burning stage. A star with $16\,M_\odot \lesssim M \lesssim 90\,M_\odot$ collapses because of photodissociation of the iron nuclei but a core of a star with $9\,M_\odot \lesssim M \lesssim 16\,M_\odot$ collapses due to neutronization of silicon or iron nuclei.

Attention is paid to the possibility of circulation mixing of matter within the boundary of a carbon-oxygen core, due to the fast rotation of the core in advanced stages of evolution, if the angular momentum is conserved in the course of evolution. As the molecular weight barrier is low in late stages of chemical evolution, it can not prevent the penetration of circulation flows outside the convective core.

Difficulties of the present theory of convection in stars are stressed and the necessity of a two-dimensional approach to the solution of the stellar constitution equations is emphasized.

1. Introduction

Since the last meeting in Brighton a considerable increase of number of papers on evolution of massive stars may be noted. The main purposes for drawing attention to this problem are: (a) the still uncertain evolutionary status of blue and red supergiants; (b) a possibility to verify the theory of weak interactions; (c) problems connected with the synthesis of heavy nuclei and their penetration into the interstellar medium; (d) attempts to construct a realistic model of a presupernova; (e) the whole scope of new problems connected with the discovery of pulsars and neutron stars and the development of X-ray astronomy, particularly with the possibility of detecting collapsed bodies. Due to the development of computing techniques a large number of evolutionary sequences of stellar models has been constructed recently in many countries.

The lower mass range of stars we deal with in this review is determined in the title The upper limit is not so well defined. Ledoux (1941), Schwarzschild and Härm (1959) showed on the basis of the theory of linear pulsations that main sequence stars with $M \gtrsim 60\,M_\odot$ will be pulsationally unstable. This upper limit has been confirmed later by Stothers and Simon (1967). Recent computations of an evolutionary sequence for

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stellar models with $M = 130\,M_\odot$ carried out by Appenzeller (1970a, b) indicate however that pulsational mass loss might not be strong enough to affect considerably the evolution of a star with $M \lesssim 200\,M_\odot$. Still up to now detailed computations of advanced evolution of large mass stars have been carried out only for the mass range $M \lesssim 64\,M_\odot$.

The main feature of the evolution of stars with large masses is a rather 'calm' (noncataclysmic) consequent exhaustion of nuclear fuel in convective cores up to the iron group, followed by a collapse of the core caused by the photodisintegration of Fe-nuclei at very high temperatures.

The rather smooth change from one nuclear fuel to another (contrary to stars of low mass where it is accompanied by a flash) is caused by the relatively small degeneracy of stellar matter in the cores of massive stars.

A recent comparison of theoretical tracks with observations carried out by Stothers (1972a) led him to the conclusion that best agreement may be achieved for a chemical composition $X = 0.70$, $Z = 0.03$ in good agreement with earlier results (Ruben and Massevich, 1966). Similar values for X and Z have been derived recently by Barbaro and Chiosi (1972). Still the problem of an adequate comparison of theory and observations remains open. This is mainly due to the fact that such a comparison requires uncertain theoretical transformation of observational data – data that themselves are for most cases incomplete. This is particularly true for the effective temperature scale of red giant stars in very late stages of evolution which are the topic of the present Symposium. It should be noted that the present theory of stellar evolution has reached a state when it is possible in several cases to predict observable phenomena. So, for example, on the basis of evolutionary tracks for late stages of stellar evolution ratios of isotopes in stellar envelopes may be deduced.

The main difficulties that have to be dealt with when computing evolutionary models of massive stars are first the uncertainty of the criterion of convective instability in the intermediate layer with varying molecular weight and, second, the neutrino emission predicted by the theory of weak interactions.

A detailed review of papers on evolution of stars with large masses can be found in Stothers and Chin (1969, 1970), Ruben (1969), Dalloporta (1971), and Massevich and Schustov (1973).

2. The Stability Criterion in the Intermediate Layer with Varying Molecular Weight

Schwarzschild and Härm (1958) showed first that in models for stars of large masses a convective unstable layer (a 'semiconvective' region) develops at the boundary of the convective core. A 'semiconvective' region is one in which the stability criterion is initially violated, but which becomes almost stable if mixing of matter reduces the opacity, e.g. by mixing core helium with envelope hydrogen. A similar region develops in small mass stars on the stage of core-helium-burning. There have been some controversies as concerning the stability criterion in the semiconvective zone. Schwarzschild and Härm assumed:

$$\nabla_r = \nabla_a. \tag{1}$$

Sakashita and Hayashi (1959, 1961) introduced for the region of varying molecular weight the Ledoux criterion:

$$\nabla_r = \nabla_a + \frac{\beta}{4 - 3\beta} \frac{\mathrm{d} \ln \mu}{\mathrm{d} \ln P}, \tag{2}$$

where β – is the ratio of gas pressure P_g to the total pressure P, μ – molecular weight.

An important step was made by Kato (1966), who discovered that the zone where the conditions:

$$\nabla_a < \nabla_r < \nabla_a + \frac{\beta}{4 - 3\beta} \frac{\mathrm{d} \ln \mu}{\mathrm{d} \ln P} \tag{3}$$

is fulfilled is vibrationally unstable. The instability is caused by radiative heat exchange between convective elements and the surrounding medium. Vibrational instability leads to a mixing of matter until the Schwarzschild criterion of stability is fulfilled. The timescale of mixing is much shorter than the evolutionary timescale. Gabriel (1968) and Auré (1971) studied the vibrational stability of massive stars in a linear approximation. They showed that due to the stabilizing action of the radiative envelope no pulsations arise. This result is in favour of the Ledoux criterion of stability. It is however doubtful if the problem investigated by Gabriel and Auré is fully adequate for the problem of mixing in the layers of varying molecular weight of massive stars.

Dudorov and Tutukov (1972) investigated in detail the mixing problem using the linearized equation of motion of a convective element in a medium with varying molecular weight and with a radiative temperature gradient satisfying condition (3). They obtained analytically the roots of the characteristic equation for $\nabla_r - \nabla_a \ll \mathrm{d} \ln \mu / \mathrm{d} \ln P$ and velocities of growth of perturbations of different size elements. Elements with linear dimension 10^7 cm have the largest increment of perturbation growth under conditions existing in the hydrogen-helium layers of neutral stability of massive stars.

The growth of the amplitude of perturbations is limited by turbulent friction. The maximal amplitude is 10^{-5} of the linear dimensions of the semiconvective layer. The turbulent motion leads to a partial mixing of matter in the layer. The necessary mixing velocity is secured during the hydrogen-burning stage if $\nabla_r - \nabla_a$ is about 10^{-5}.

However in spite of important results achieved in the above mentioned investigations, the problem of stability of intermediate layers with varying molecular weight remains unsolved. The view has been expressed that numerous evolutionary computations might provide a rather simple solution to the question of what criterion of stability has to be chosen but these hopes were not realized.

The results of an experiment, studying heat transfer in a liquid with varying salt concentration seem to speak in favour of Schwarzschild criterion (Spiegel, 1969). But the identity of conditions in varying salinity liquids with conditions in stellar interiors has not been yet proved. The main difference between the experiment and reality is that heat conductivity coefficient in a liquid does not depend on salt concentration while in the stellar interiors it is determined by chemical composition (in massive stars).

Thus, up to now it seems necessary to perform two versions of evolutionary computations differing only in assumptions on the stability conditions. It can not be ruled out that both stability conditions might take place in reality. If a strong enough regular magnetic field which prevents turbulence of the main scale motions exists in the zone of varying molecular weight and if the amplitude of pulsations is limited by the stabilizing effect of the radiative envelope, then no mixing will occur and the Ledoux criterion of convective neutrality is realized. For stars without a magnetic field the Schwarzschild criterion remains valid.

The procedure of constructing evolutionary models for massive stars is described in details by Varshavsky and Tutukov (1972). Figure 1 shows how the structure of a star with 64 $M_\odot$ changes in the course of evolution if either the criterion given by Schwarzschild (Figure 1a) or by Ledoux (Figure 1b) is used. The intermediate convective layer develops in both cases as a result of high concentration of the energy sources in the hydrogen burning shell. The time of evolution in the hydrogen burning stage is for case 1b 10% shorter as compared with 1a (Tutukov, 1972). The most important difference between both sets of models shows in the resulting profiles of hydrogen distribution and in a consequent difference in hydrogen contents in the shell source. The distribution of hydrogen is for stars with $M \lesssim 50\, M_\odot$ (when the Ledoux criterion is used) smooth and similar to that for a star with $M \lesssim 10\, M_\odot$. The shell source develops in a region with low hydrogen content: $X \approx 10^{-2}$. If the Schwarzschild criterion is used the hydrogen distribution becomes discontinuous.

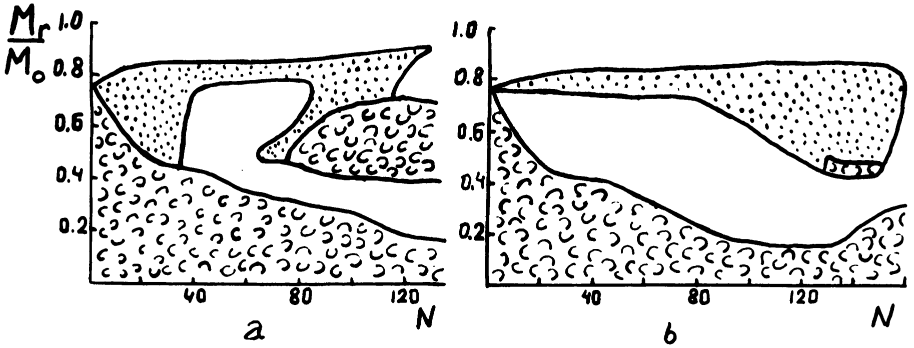

Fig. 1. Evolutionary changes in the structure of the 64 $M_\odot$ star (Varshavsky and Tutukov, 1972), N-number of the evolutionary model. On the left: $\nabla_r = \nabla_a$ in the semiconvective zone, on the right:

$$\nabla_r = \nabla_a + \frac{\beta}{4 - 3\beta} \frac{d \ln \mu}{d \ln p}$$

∽ – convective zones;

⁙ – semiconvective zones.

As a result of this the hydrogen content in the shell will be as large as 0.4 for a 16 $M_{\odot}$ star and about 0.2 for a 64 $M_{\odot}$ star (Varshavsky and Tutukov, 1972). The initial difference in the hydrogen profiles results in two different evolutionary tracks in the stage of helium burning.

Besides it should be noted that the hydrogen shell source situated at the discontinuity of the hydrogen profile will become unstable at early phases of helium burning for $M = 15 M_{\odot}$ (Stothers and Chin, 1972). The changes in luminosity caused by thermal flashes in the shell are too small to be used as a criterion for choice between the two stability conditions in the semiconvective region. The same instability was found also by Varshavsky (1972a) for 15 $M_{\odot}$ star and by Noels and Gabriel (1973) for 8 $M_{\odot}$.

3. The Stability Criterion and Evolution of Stars in the Helium Burning Stage

Starting with 1970, evolution of stars with large mass has been studied intensively in several countries (Chiosi and Summa, 1970; Paczyński, 1970; Simpson, 1971; Massevich *et al.*, 1971; Robertson, 1971). A number of computations have been performed and new results obtained mainly dealing with the helium burning stage.

First, the lower limit of masses for Population I stars has been estimated for which discrepancies connected with different approaches to the stability criterion are developing in the helium burning stage. According to Barbaro *et al.* (1972) the lower limit is 13 $M_{\odot}$. Stars with $M \lesssim 13 M_{\odot}$ evolve, notwithstanding which stability criterion is used, like stars of medium masses, e.g. helium burning starts in the region of red supergiants and continues in the blue supergiants region. The upper limit for the phenomena considered is about 40 $M_{\odot}$. As Barbaro *et al.* (1971b) and Varshavsky and Tutukov (1973a) have shown, stars with masses larger than 40 $M_{\odot}$ have no blue supergiant phase with a lifetime comparable with the time of helium burning in the core: helium is almost completely exhausted in the phase of a red supergiant.

For a mass range $13 M_{\odot} \lesssim M \lesssim 64 M_{\odot}$ the following cases should be distinguished.

Case A. If the Ledoux criterion is used, helium burning start in the phase of a red supergiant. The following evolution is completely defined by the depth of penetration of the outer convective envelope into the intermediate layer of varying chemical composition in the stage preceding helium burning (Lauterborn, *et al.*, 1971; Ziołkowski, 1972). Mixing at the inner boundary of the convective envelope causes a discontinuity in the profile of hydrogen distribution. As soon as the shell source developing outwards reaches the region of this discontinuity the star moves from the red supergiants region in the HR diagram to the region occupied by blue supergiants. This displacement occurs in a Kelvin timescale. Similar results have been obtained by Chiosi and Summa (1970) and Robertson (1972).

Case A1. If the convective envelope is not deep enough, the shell source will never reach the discontinuity in the hydrogen distribution during the time of helium burning. That means that the model of the star remains during its helium burning stage in the red supergiants region in the HR diagram (Paczyński, 1970; Varshavsky, 1972a, b; Varshavsky and Tutukov 1973a).

Case A1 can not be considered as the only possible way of evolution for stars of large masses because it can not explain the observed number of blue supergiants. It should be noted that in stars with $M \gtrsim 60\ M_\odot$ the occurrence of a discontinuity of hydrogen distribution caused by the development of a thin intermediate convective layer is possible even if the Ledoux criterion is used (Varshavsky and Tutukov, 1972). But as the shell source never approaches this discontinuity, the models remain in the red supergiant region.

Case B. When the stability criterion given by Schwarzschild is used, helium may be completely exhausted in the blue supergiants region (Case B1) or the star exhausts the main part of its helium fuel as a blue supergiant and only the remainder as a red supergiant (Case B2). Very massive stars, $M \gtrsim 40\ M_\odot$, notwithstanding the existence of an intermediate convective layer and a discontinuity in hydrogen distribution, will exhaust their core helium only in the red supergiants region (Case B3).

For Case A the ratio of lifetimes as blue and red supergiants τ_b/τ_r is highly dependent on uncertainties concerning efficiency of convection and the depth of its penetration. For Case B no such direct dependence has been noted. However we may note that the lifetimes in this case are functions of the profile of hydrogen distribution in the region where the shell source is located, of the absorption coefficient, the theory of convection, the initial chemical composition, possible mass loss and the initial mass of the model. A possibility of the existence of multiple solutions should not be excluded. For a model with $M = 32\ M_\odot$, Varshavsky and Tutukov (1973a) found that the transition from a blue to a red supergiant occurs in the Kelvin timescale (Case B has been investigated). The dependence of the ratio τ_b/τ_r on chemical composition has been studied by Robertson (1972) who found that a change of Z from 0.02 to 0.04 may change the ratio τ_b/τ_r by about three times. That means that even for a fixed stability criterion the lifetimes ratio cannot be determined uniquely. Stothers and Chin (1973a) showed that τ_b/τ_r is increasing with increasing Z, Y, $\varepsilon_{C+\alpha}$ and decreasing $\varepsilon_{3\alpha}$.

A thorough study of the behaviour of thermally stable models with varying chemical composition and different profiles of hydrogen distribution should shed more light on the influence of physical parameters (X, energy sources, convection, mass loss, rotation etc.) on models obtained at late stages of stellar evolution. Kozlowski (1971) computed models for $10\ M_\odot$ in the stage of helium burning with hydrogen profiles corresponding to Case A. He showed that for a certain rather small range of helium core masses two thermally stable solutions exist one for a blue and the other for a red supergiant. Frantsman (1973) and Frantsman *et al.* (1973) studied the influence of mass loss on the location of $15\ M_\odot$ and $30\ M_\odot$ models in various stages of helium burning. The hydrogen distribution profile was taken according to Case A. They showed, that for a He-content in the core about unity a continuous sequence of models with decreasing mass of the H-envelope can be obtained in the blue supergiants region. They did not succeed in obtaining a similar sequence for a much smaller He-content in the core (e.g. for a more evolved star).

The authors conclude that the necessary condition for obtaining a model of a blue supergiant is the activity of the hydrogen shell source.

The influence of the hydrogen distribution profile on the evolution of massive stars in the helium burning stage has been studied by Barbaro *et al.* (1971b). They introduced two characteristic times determining the prehelium burning evolution.

(1) $\tau_{sh} = X_{sh} \cdot E/\varepsilon$, where X_{sh} is the H-content in the H-shell source, E – energy output by burning 1 g of hydrogen and ε – rate of energy generation in the shell.

(2) τ_d – time of diffusion of the radiation from the shell to the surface, which is about the Kelvin time scale for the envelope. If $\tau_{sh} < \tau_d$, as it usually is the case for small X_{sh}, the change of thermal conditions in the shell occurs at such a rate that the envelope starts to expand due to the increasing energy flux. As soon as the decreasing surface temperature reaches several thousands of degrees, a convective envelope develops and decreases τ_d to a value comparable with τ_{sh}. It should be noted that earlier computations of late stages of evolution, not taking into account convection in the outer layers lead usually to extremely large values of radii.

If the shell source developing outwards approaches the discontinuity in the hydrogen profile, e.g. caused by the outer convective envelope, τ_{sh} increases sharply and becomes larger than τ_d. As a consequence the model is transfered to the region of blue supergiants at a rate comparable to the Kelvin timescale for the envelope.

If in the model considered when leaving the main sequence $\tau_{sh} > \tau_d$ (because of a high H-content in the shell source), thermal stability of helium-burning model will be reached in the blue supergiants region. It should be noted also that the development of an intermediate convective layer and a layer of neutral stability will decrease the value of τ_d in this case.

All mentioned above is true only if the times τ_{sh} and τ_d are smaller than the characteristic time of complete He-exhaustion in the core. The activity of the H-shell source is another necessary condition as has been already mentioned. Briefly summarizing, for thermally stable stellar models with Population I composition the model will be a red supergiant if the H-content in the H-shell source is small and a blue supergiant if it is relatively large. The possibility that two stable solutions (for red and blue stars simultaneously) may exist for a range of X_{sh} values was proved by Kozlowski and Paczyński (1973).

It is interesting to note that Trimble *et al.* (1973) found that stars with $M = 12\ M_\odot$ and 30 $M_\odot$ (for an abundance of heavy elements $Z = 0.001$) reach thermal stability in the helium burning stage in the blue supergiants region even if the Ledoux criterion of stability is applied. Similar results have been obtained by Varshavsky (1973) for models with 16 $M_\odot$ and 32 $M_\odot$ (for $Z = 0$). The decrease of opacity for small Z values is probably sufficient to decrease τ_d even if the H-content in the shell is low. Fricke and Strittmatter (1972) and Höppner and Weigert (1973) point out the importance of the gravitational field of the star core in defining of the position of a core-helium burning model in the HR diagram.

4. Blue and Red Supergiants and the Stability Criteria and the Hypothesis of Electron-Neutrino Interaction

The main uncertainties in the theoretically derived value of the ratio n_b/n_r from evolutionary computations of massive stars arise from the uncertainties concerning the stability criterion, efficiency of convection and the reality of weak interactions. From the observational point of view the uncertainties in deriving this ratio are caused by the difficulty to distinguish blue supergiants from main sequence stars in the stage of hydrogen exhaustion in the core and by uncertainties in the scale of effective temperatures and bolometric corrections for very hot stars. Theoretical values of n_b/n_r are discussed by Stothers and Chin (1969), Chiosi and Summa (1970), Simpson (1971), Robertson (1972), Barbaro *et al.* (1972b), Tutukov and Varshavsky (1973), and Robertson (1973).

As both the theoretical and the observational values appear to be rather uncertain, no final conclusion can be made at present concerning the stability criteria or the existence of weak interactions. This is one of the characteristic features of the development of the theory of stellar evolution. The values of such (important for stellar evolution) parameters as the opacity coefficient or the nuclear energy sources have been obtained either by numerical computations using known physical processes or by extrapolation of experimental results and cannot in general be verified in Earth conditions. The role of the number of various physical phenomena on stellar evolution such as magnetic fields, rotation, convection and semiconvective diffusion is at present not yet fully known. Observations provide an integral effect of all these possible factors and there are almost no possibilities at present to distinguish between them. There are only very few well established observational data that could be used as criteria for various theories of evolution. And regretfully almost for every positive argument there exists a negative one too.

The estimate for n_b/n_r obtained from observational results by Schild (1970) for the solar surroundings is about 1.6. Humphreys (1970) gives $n_b/n_r \approx 2.3$ for a mass range $10\,M_\odot \lesssim M \lesssim 20\,M_\odot$ and $n_b/n_r \approx 10$ for masses $M \gtrsim 20\,M_\odot$. According to Hartwick (1970) n_b/n_r decreases with the distance from the galactic centre from 10 for distances $R < 10$ kpc to about 3 at a distance of 12–13 kpc. These are the main results that can be derived from observations.

Evolutionary tracks for stars in the mass range $11\,M_\odot \lesssim M \lesssim 64\,M_\odot$ (Case B) are plotted on Figure 2 together with the Humphreys data.

Recent investigations have shown that it might be possible to discuss the problems concerning stability criteria and weak interactions separately. Let us summarize the results.

The Role of Neutrino Emission

(1) According to Stothers (1972), if neutrino emission is not taken into account, the ratio n_b/n_r never exceeds 3 due to the long time of depletion of carbon, oxygen, neon and silicon. Chiosi and Summa (1970) took into consideration the binary nature of

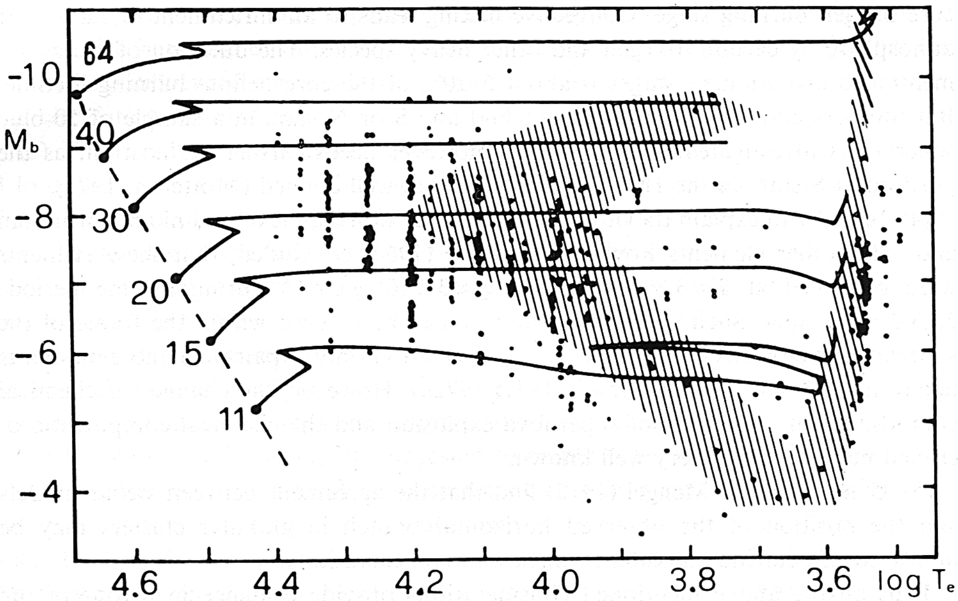

Fig. 2. Evolutionary tracks of 11, 15, 20, 30, 40 $M_\odot$ (Barbaro *et al.*, 1971a) and 64 $M_\odot$ stars (Varshavsky and Tutukov, 1972, 1973a) on the HR diagram. Version B. The position of supergiants after Humphreys (1970) are marked by dots. The hatched areas mark the position of core helium burning models.

a fraction of massive stars, which prevents them from evolving into red supergiants. If neutrino emission is absent then duplicity of massive stars allows us to increase n_b/n_r about one and a half times, but the ratio still remains less than 2 (Barbaro *et al.*, 1971). In the case neutrino emission is present $n_b/n_r \gtrsim 7$. This is considered as an argument in the favour of the existence of neutrino emission.

(2) As core carbon- and oxygen-burning inevitably occurs in the red supergiants region (Stothers and Chin, 1969; Varshavsky and Tutukov, 1973b), the absence of very high luminosity red supergiants provides evidence for acceleration of advanced evolution due to neutrino emission. However core helium burning in stellar models with $M \gtrsim 40\ M_\odot$ (Case B) usually occurs in the red supergiants region, where the influence of neutrino emission is negligible. It is quite possible that these stars never reach the blue supergiants region (Barbaro *et al.*, 1971; Varshavsky and Tutukov, 1973b). Moreover Bisnovaty-Kogan and Nadyozhin (1972) have shown that all stars with $M \gtrsim 20\ M_\odot$ lose rapidly the main part of their envelopes, while evolving in the red supergiants region. As a result of this mass loss the models are shifted into the Wolf-Rayet stars region of the HR diagram.

(3) Stothers and Chin (1969), Sugimoto (1970a), and Varshavsky and Tutukov (1973b) showed, that in absence of neutrino emission the outer convective zone in stars with $M \gtrsim 20\ M_\odot$ penetrates subsequently into the hydrogen and helium shell sources during the core carbon burning stage and for less massive stars during the

core oxygen burning stage. Convective mixing leads to an enrichment of the stellar atmosphere by carbon, oxygen and other heavy species. The duration of the above mentioned evolutionary stages is about 5–20% of the core helium burning lifetime. But Stothers and Chin (1969) did not find any S or N stars in a sample of 50 blue supergiants investigated. This argument however needs further verification, as the position of S-stars in the HR diagram is not yet well defined (Motteran, 1971).

(4) In order to explain the observed abundance of iron, nickel, titanium, chromium and some other elements Fowler and Hoyle (1964) concluded, that these elements were produced at $T \approx 3.8 \times 10^9$ K and $\varrho \approx 3 \times 10^6$ g cm^{-3} during a time period: $3_{10}3 \lesssim \tau(\mathrm{s}) \lesssim 8_{10}4$. Such short time intervals can be secured within the frame of the supernova explosions theory of elements formation only if pair-neutrino emission is taken into account (Ikeuchi *et al.*, 1971, 1972a). However, the changes of chemical composition in the course of supernova explosion and the chemical composition of ejected matter are not very well known.

(5) Demarque and Mengel (1972) find that the agreement between stellar models and the position of the observed horizontal branch in globular clusters may be improved, if neutrino emission is taken into consideration.

Thus all the above mentioned considerations provide evidence in favour of the existence of neutrino emission, but a number of uncertainties in estimation of both the theoretical and the observed ratio n_b/n_r prevents some authors (Barbaro *et al.*, 1972) from definite conclusions concerning the validity of universal weak interaction theory.

The problem of choice between two stability criteria for semi-convection remains very complicated even if neutrino emission is granted. This problem was discussed in detail by Ziołkowski (1972), Varshavsky and Tutukov (1973), and Robertson (1973). Let us describe the main points of this discussion.

(1) Blue supergiants occupy the effective temperature region $3.9 \lesssim \log T_e \lesssim 4.35$ in the HR diagram (see Figure 2). The effective temperatures of core helium burning models are $3.8 \lesssim \log T_e \lesssim 4.2$ for Case B and $4.15 \lesssim \log T_e \lesssim 4.25$ for Case A. (Ziołkowski, 1972). It was shown (Frantsman, 1973; Frantsman *et al.*, 1973) that mass loss (up to 30% of the initial stellar mass) in the red supergiants region is able to extend the interval of effective temperatures of models sufficiently to reach an agreement with observations (for Case A). However no appropriate mechanism of this mass loss is proposed. Besides, there are some difficulties in obtaining models with $\log T_e \approx 4.3$ and $M_b \approx -9$ for Case B. At the same time, for Case A it is possible to compute models with still higher effective temperatures, if the penetration of the convective envelope is accurately enough taken into account. Thus, effective temperatures do not provide a possibility to make a definite choice between the two stability criteria, although Case A seems to agree better with observations if mass loss is taken into account.

(2) If the red supergiant evolutionary stage is preceding the blue supergiant stage, the relative abundance of the CNO-group elements should be changed: the ratio N/C increases during evolution. According to Ziołkowski (1972) N/C is about

5 for α Cyg, but other blue supergiants show normal N/C ratios. Besides several O-stars also have higher ratios N/C than normal stars. It may be possible, that the N/C ratio is influenced not only by convection but also by a number of other processes: e.g. circulation and diffusion.

(3) The variation of periods of long period cepheids (Ziołkowski, 1972). There are 3 long period cepheids in LMC and 3 in SMC. If stellar evolution corresponds to Case A, a part of the cepheids should show the increase of periods and the other part the decrease of periods. In Case B all cepheids should have increasing periods. But in spite of the fast rate of evolution of massive stars in the cepheid strip (it is crossed in some hundreds of years only), it is impossible to detect changes of periods as the periods are themselves very long. Nevertheless, this method appears a very promising one and it might allow in future to come to a definite conclusion concerning the direction of stellar evolution in the Hertzsprung gap.

(4) Stothers and Evans (1970) suggested to employ the statistics of binaries with one blue supergiant component. It is necessary to use short period binaries in order to be sure that the blue supergiant has not evolved into a red one. But the number of such binaries is too small to allow any definite conclusion (Ziołkowski 1972). Moreover even in Case A the primary component of a binary appears after mass loss as a blue supergiant in the core carbon and oxygen burning stages (Tutukov and Yungelson, 1973, see Figure 3). If neutrino emission is absent, lifetime in these evolutionary stages is long enough for the star to be observable.

(5) The evolution of remnants of primary components of massive binaries suffering mass exchange depends on the stability criterion used. As Barbaro *et al.* (1969) have shown for a 30 $M_{\odot}$ star evolving according to Case B, if mass exchange starts before the helium burning stage in the blue supergiant region, the effective temperature of the remnant does not exceed 4×10^4 K in core-helium burning stage. Tutukov and Yungelson (1973) studied the evolution of close binaries with mass exchange for primary stars with $10\ M_{\odot} \leqslant M \leqslant 64\ M_{\odot}$, using the Ledoux criterion for convective stability. Their evolutionary tracks are plotted on Figure 3. The effective temperatures of the remnants, which are usually regarded as models of Wolf-Rayet stars are of the order of 8×10^4 K. So it seems that there is a considerable difference between the two cases of evolution. The temperatures of Wolf-Rayet stars are not well known, but it is most agreed that they are very high: Cherepashchuk (1972) obtained ~80000 K for V444 Cyg. This result is an evidence in favour of Case A. But it is necessary to mention, that mass loss which is of great importance for Wolf-Rayet stars may increase their effective temperatures also for Case B.

(6) Different authors obtain different theoretical estimates for the ratio n_b/n_r for different stability conditions, e.g. Chiosi and Summa (1970) came to practically equal values of n_b/n_r for both stability conditions for a 20 $M_{\odot}$ star. To clarify this problem it is necessary to study the properties of models in thermal equilibrium and their dependence on the H-profile, chemical composition and input physics.

(7) Barbaro and Chiosi (1972) estimated the dependence of chemical composition on the galactocentric distance R of stars using a set of evolutionary tracks com-

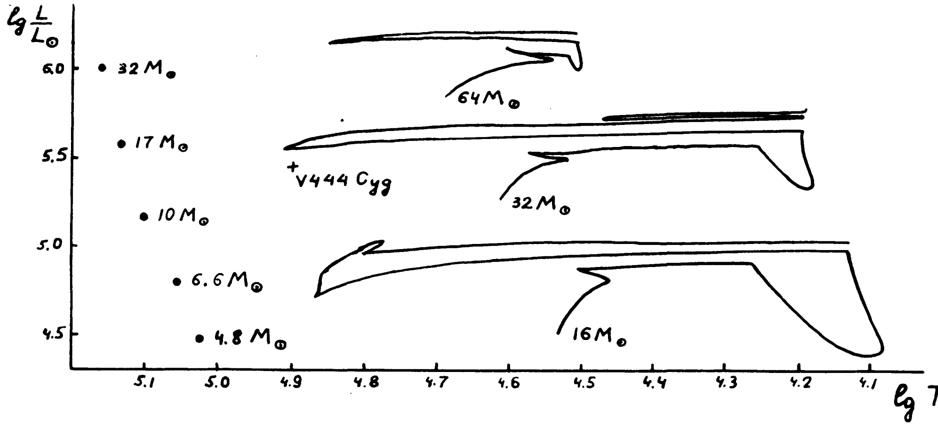

Fig. 3. Evolutionary tracks of primary components of massive binary stars on the HR-diagram. ● – models of homogeneous helium stars ($Y = 0.956$, $Z = 0.044$); + – position of WR-component of V444 Cygni (Cherepashchuk, 1972).

puted with the Schwarzschild stability criterion. The obtained result is somewhat striking at first sight. The abundance of heavy elements grows with increasing R: $Z = 0.02$ for $R \approx 7.5$ kpc and $Z = 0.04$ for $R \approx 10.5$ kpc. This result may be a consequence of uncertainties in the value of n_b/n_r obtained from evolutionary computations. But it appears that the abundance of heavy species must not necessarily monotonically increase with increasing age. Ikeuchi *et al.* (1973b) found that if mass loss by stars with small masses is taken into account the heavy element abundances in the interstellar matter should have grown during the early history of the Galaxy, as long as the amount of matter lost by supernovae exceeds the flow of mass lost by stars of small masses. The maximal value of Z is attained after about 10^9 yr. After that the value of Z is decreasing, as the flow of matter from supernovae becomes smaller than the flow from stars with small masses. This allows to explain the chemical composition of stars with high metal contents studied by Van den Bergh and Sackman (1964), Spinrad and Taylor (1969), and Taylor (1970). The results of Barbaro and Chiosi can be also explained if we assume that the rate of chemical evolution of the periphery of the Galaxy is slower than that in the central parts owing to a lower star density. The absence of active mixing leads then to big values of Z for those remote regions.

5. Chemical Composition of Stellar Matter After the Depletion of Helium

Conditions defining the chemical composition of stellar matter after core helium depletion were examined by Uus (1970), Arnett (1972), Ferrari *et al.* (1972), and Varshavsky and Tutukov (1973a). An analysis of the chemical composition variation equations showed that the final composition is defined by temperature and density of stellar matter in the course of helium burning. On Figure 4 are plotted constant carbon abundance lines for the final configuration on the $\log T - \log \varrho$ diagram according

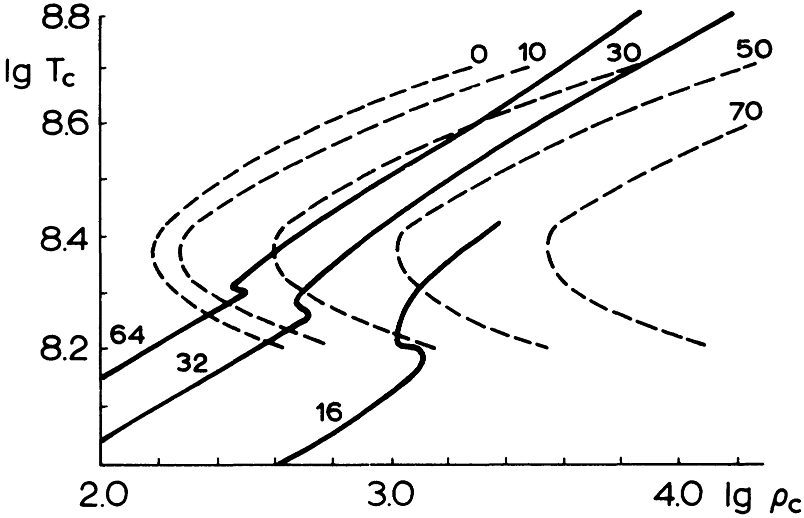

Fig. 4. Evolutionary tracks of stellar centres in the $\lg T - \lg \varrho$ plane (heavy lines). $M = 16$, 32, 64 $M_{\odot}$. Constant final carbon abundance curves are marked (dashed lines) after Varshavsky and Tutukov (1973a). Numbers near the dashed lines show the percentage of carbon.

to Varshavsky and Tutukov (1973). The values of the concentrations can be obtained by the numerical solution of a very simple equation. Helium burning occurs mainly near the centre of the star, but the position of this centre on the $\log T - \log \varrho$ diagram varies. It follows from evolutionary computations, that the main part of helium is depleted in the upper part of the 'S-like' bend of the path in Figure 4. This allows us to explain the variation in composition of the final helium burning products with stellar mass. This method can be also used to estimate the variations of nuclear reaction rates. E.g. Austin *et al.* (1971) varied the rate of the 3α-reaction, and Weisser *et al.* (1972) – the rate of the $C + \alpha$ reaction. The variations of the reaction rates shift the set of constant carbon abundance lines in the $\log T - \log \varrho$ diagram and lead to slight deformations of their upper parts. Tutukov and Varshavsky (1973a) showed that due to the above mentioned variations the final carbon content changes from 0.45 to 0.2 for a 16 $M_{\odot}$ star and from 0.3 to 0.10 for a 64 $M_{\odot}$ star.

It is possible to estimate in the same way the neon formation efficiency. After helium depletion the abundance of core neon is 0.003 for a 16 $M_{\odot}$ star and 0.03 for a 64 $M_{\odot}$ star (Varshavsky and Tutukov, 1973a). Similar values were obtained also by Arnett (1972b). It is worthwhile to mention that for new neon-formation reactions rates given by Toevs *et al.* (1972) the estimated final neon abundance does not change significantly.

6. Core Carbon, Oxygen, Neon, Silicon and Nickel-Burning Stages of Evolution of Massive Stars

Direct computations of evolution of massive stars from the main sequence up to the iron core formation stage are heavily embarrassed by the necessity to deal with very complex models consisting of many layers with a number of nuclear burning shells. The large number of meshpoints necessary for precise computations also does not facilitate the task. One possible way to overcome some of those difficulties is to compute a number of models in thermal equilibrium for regions in the HR diagram representing late stages of evolution. In such a way Stothers and Chin (1969) computed core carbon, oxygen and neon-burning models for red supergiants region of the HR diagram.

The evolution of helium models, representing cores of normal stars up to the oxygen depletion stage was studied by Sugimoto (1970a, b) for 3 $M_\odot$ and 10 $M_\odot$ stars. Arnett (1972a, b, c) also studied carbon and oxygen burning starting from pure helium models. Core oxygen burning was studied also by Woosley *et al.* (1972), Rakavy and Shaviv (1967), Barkat *et al.* (1967), and Fraley (1968).

Varshavsky and Tutukov (1972, 1973a, b) computed evolutionary tracks for 32 $M_\odot$ and 64 $M_\odot$ stars starting from hydrogen burning on the main sequence up to oxygen depletion in the core. The computations were performed for both the Ledoux and Schwarzschild criteria of convective stability in the zone of variable molecular weight. Their results confirmed the conclusion of Stothers and Chin (1969) that the core carbon and oxygen-burning models appear as red supergiants in the HR diagram. Tutukov and Varshavsky also found that evolution after the start of carbon-burning does not noticeably depend on assumptions concerning stability conditions in the layer of variable molecular weight.

Evolution of carbon-oxygen cores with 1.5 $M_\odot$, 2.6 $M_\odot$, 5 $M_\odot$, 10 $M_\odot$, 30 $M_\odot$ up to formation of an iron core was studied by Ikeuchi *et al.* (1971, 1972a). Evolutionary tracks in the $\log T_c - \log \varrho_c$ plane for 5 $M_\odot$, 10 $M_\odot$ and 30 $M_\odot$ cores with neutrino emission taken into account are plotted on Figure 5. Regions of dynamical instability

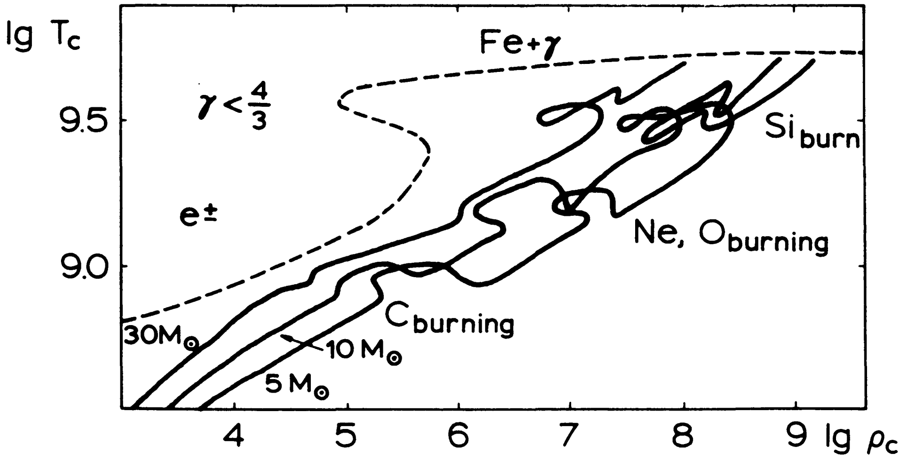

Fig. 5. Evolution of central values of temperature and density of carbon-oxygen stars (Ikeuchi *et al.*, 1971, 1972a).

due to pair formation and iron photodissociation are also marked on the same figure. A comparison of results obtained by Varshavsky and Tutukov (1972, 1973a, b) and by Ikeuchi *et al.* (1971, 1972a) shows that carbon-oxygen cores with 2.5 $M_\odot$, 10 $M_\odot$ and 30 $M_\odot$ correspond to cores of main sequence stars with 16 $M_\odot$, 32 $M_\odot$ and 64 $M_\odot$. Thus, the two papers are supplementing each other in some respect. The main shortcoming of the carbon-oxygen cores evolution computations is the impossibility of taking into account the decrease of mass of the core caused by penetration of the outer convective zone.

Evolutionary changes of the structure of a 10 $M_\odot$ star are shown on Figure 6 (Ikeuchi *et al.*, 1971). The upper drawing corresponds to the case, when no neutrino emission is taken into account, the lower corresponds to evolution with pair- and

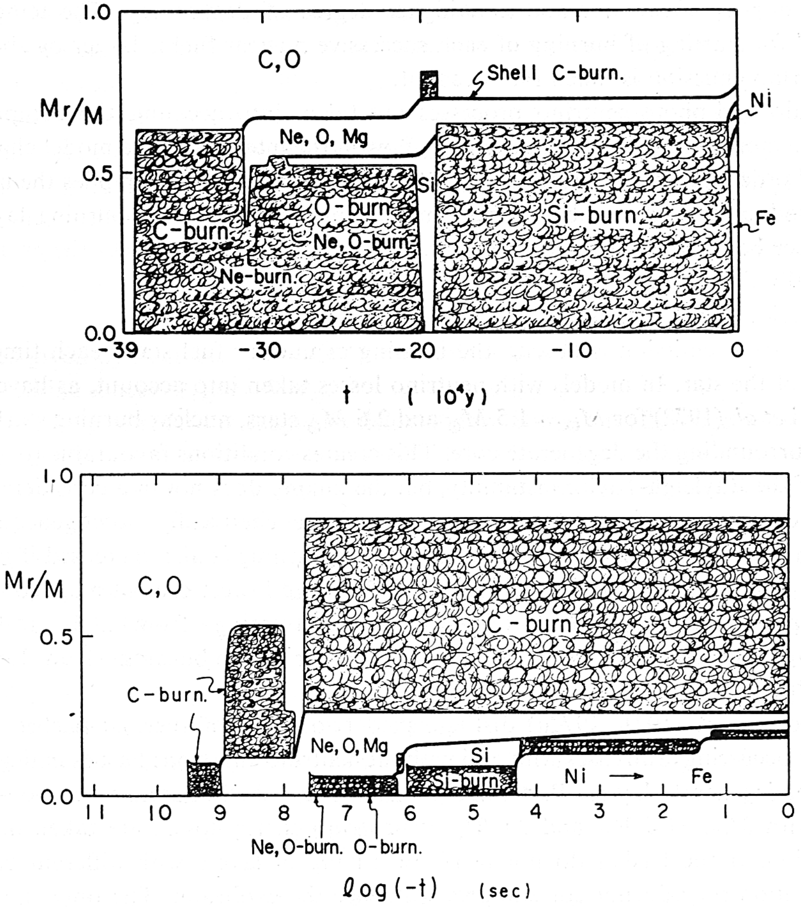

Fig. 6. Evolutionary changes of the structure of carbon-oxygen 10 $M_\odot$ star (Ikeuchi *et al.*, 1971). Upper drawing corresponds to evolution without neutrino emission, lower one – to evolution with neutrino emission.

photoneutrino emission. The main structural differences are the following: models with neutrino emission have small convective cores and thick convective nuclear burning shells, whereas models without neutrino emission have large convective cores and nearly always radiative shell sources. Neutrino emission accelerates the evolution: the carbon depletion time shortens about 10^2 times, the oxygen and neon depletion time $\sim 10^4$ times, the silicon depletion time $\sim 10^7$ times.

As Varshavsky and Tutukov (1973a) have found, only one or two nuclear burning shells nearest to the core are active, all outer shell sources are dying out in succession and getting absorbed by the convective envelope if neutrino emission is absent. For models with neutrino emission all shell sources are active during the computed evolutionary stages. Evolution without neutrino occurs nearly homologously, the point representing the centre of the star moves in the $\log T - \log \varrho$ plane along an almost straight line, for stars with a core $M_{\mathrm{CO}} \geqslant 5\ M_{\odot}$. Neutrino emission leads to higher density of the core and to a higher degree of degeneracy. The temperature needed for starting of burning of each successive nuclear fuel is larger by about 25% if neutrino emission is taken into account.

If pair- and photo-neutrino processes are taken into account, the computations become more complicated, as the energy flow in the interiors of the model changes by several orders on the way from one shell source to another. This implies the necessity to introduce a large number of new meshpoints in the nuclear burning layers. At the inner boundaries of the carbon, oxygen and silicon shell sources the energy flow is usually directed inwards, so that it compensates the neutrino energy losses from the interior parts of the star.

If neutrino emission is absent, the burning of nuclear fuel starts each time in the centre of the star. In models with neutrino losses taken into account, as have shown Ikeuchi *et al.* (1972) for $M_{\mathrm{CO}} = 1.5\ M_{\odot}$ and $2.6\ M_{\odot}$ stars, nuclear burning starts in the layer surrounding the degenerate core. This creates conditions favourable for appearance of the Rayleigh-Taylor instability, but the mixing does not in a considerable way influence the evolution. The lowest mass of the chemically homogeneous core, necessary to start the following stage of nuclear burning is also increased if neutrino energy losses are present (Ikeuchi *et al.*, 1972). The lowest critical mass for carbon burning grows from $0.75\ M_{\odot}$ to $1.06\ M_{\odot}$, for neon burning – from $0.9\ M_{\odot}$ to $1.1\ M_{\odot}$, for oxygen burning – from $0.93\ M_{\odot}$ to $1.15\ M_{\odot}$, for silicon burning – from $1.4\ M_{\odot}$ to $1.55\ M_{\odot}$.

Ikeuchi *et al.* (1971, 1972a) distinguished two kinds of thermal flashes in shell sources occurring in all cases with and without neutrino emission: flashes in degenerate and nondegenerate layers. Particularly unstable are shell sources in carbon-oxygen stars with $M_{\mathrm{CO}} = 1.5\ M_{\odot}$ and $2.6\ M_{\odot}$ if neutrino energy losses are taken into consideration. If the flashes do not result in mixing of layers with different chemical composition they do not considerably influence the evolution. The only effect is an increase of the burning time of the given nuclear fuel. Stothers and Chin (1973) found that the helium burning shell in a $15\ M_{\odot}$ star is thermally unstable with a characteristic time of the instability growth about some tens of years.

Let us discuss the reasons for collapse of cores of different mass stars. If neutrino emission is absent, carbon-oxygen cores with mass $M_{CO} \gtrsim 30\ M_\odot$ (this corresponds to main sequence stars with mass $M \gtrsim 64\ M_\odot$) are collapsing owing to formation of electron-positron pairs (Rakavy and Shaviv, 1967; Barkat *et al.*, 1967; Fraley, 1968). The explosion of the star in this case was studied by Fraley (1968). The evolution of carbon-oxygen cores with $1.4\ M_\odot \lesssim M_{CO} \lesssim 30\ M_\odot$ (corresponding to main sequence stars with masses $9\ M_\odot \lesssim M \lesssim 64\ M_\odot$) can be followed without neutrino emission still to the stage of formation of an iron core with $M_{Fe} = 0.5$–$0.7\ M_{CO}$. The iron core collapses owing to photodissociation of the iron nuclei.

If pair- and photoneutrino cooling is taken into consideration, all aforementioned estimates of limiting masses will be increased. Collapse caused by pair formation in carbon-oxygen cores will occur for $M_{CO} \gtrsim 40\ M_\odot$ (main sequence stars with masses $M \gtrsim 90\ M_\odot$). For collapse caused by photodissociation of iron nuclei the mass of the carbon-oxygen cores will be $2.6\ M_\odot \lesssim M_{CO} \lesssim 40\ M_\odot$ ($16\ M_\odot \lesssim M \lesssim 90\ M_\odot$ on the main sequence). The lowest mass of a collapsing iron core will be $\sim 3\ M_\odot$ for a carbon-oxygen star with $M_{CO} = 30\ M_\odot$ and $M_{Fe} \approx 1.3\ M_\odot$ for $2.6\ M_\odot \lesssim M_{CO} \lesssim 10\ M_\odot$. And finally carbon-oxygen cores $1.4\ M_\odot \lesssim M_{CO} \lesssim 2.6\ M_\odot$ ($9\ M_\odot \lesssim M \lesssim 16\ M_\odot$ on the main sequence) collapse owing to neutronization of silicon or iron (Ikeuchi *et al.*, 1972). The upper limit $\sim 2.6\ M_\odot$ is not well defined as, because of the complexity of interaction of different energy sources, the evolutionary path of the stellar centre in the $\log T - \log \varrho$ plane is very sensitive to the input physics. In the course of computation of the evolution of carbon-oxygen stars with $M_{CO} = 1.5\ M_\odot$ and $2.6\ M_\odot$ Ikeuchi *et al.* (1972) have increased the neutrino energy loss rate 5 to 20 times as compared to values given by Beaudet *et al.* (1967). This might have led to an overestimate of the upper limiting mass: $M_{CO} \approx 2.6\ M_\odot$.

The influence of rotation, magnetic fields and mass loss on advanced evolution of massive stars has not yet been studied in details. It is evident however that even the loss of almost the whole hydrogen envelope in the red supergiant stage cannot change noticeably the evolution of the core. The role of rotation in advanced evolution has up to now attracted little attention. If we assume, that the local angular momentum is conserved in the course of evolution, the angular velocity of the core would increase considerably and the ratio of centrifugal force to gravitation α approaches unity. Under such conditions the velocity of circulation is strongly enhanced and the molecular weight barrier for late stages reactions becomes low (Varshavsky and Tutukov 1973b). As Varshavsky and Tutukov showed, if $\alpha_0 \gtrsim 0.06$ on the surface of the main sequence star, total depletion of carbon within the carbon-oxygen core becomes possible. If there exists a mechanism supporting the carbon-oxygen core on the verge of rotational stability, it is possible for all successive reactions to occur also within the original core borders.

The distribution of chemical elements of the presupernova will in this case be considerably changed. The evolutionary lifetimes will be also increased several times.

Conclusion

Summarizing we may state with some satisfaction that at present the evolutionary path of massive stars, starting from the zero age main sequence up to the occurrence of dynamical instability in the core of a star in a very late stage of evolution actually preceding a presupernova can be followed not only in a general outline but also in some details. The main results in this field mainly for advanced stages have been achieved in the last 3–4 years and the amount of work (particularly computations) implied is amazing. It has also to be stated that at present we are much better aware of the difficulties and 'weak points' of the present theory then perhaps 3–4 years ago and that in some aspects we even do not see a way out how to overcome or avoid these difficulties. This should by no means lead us to a pessimistic view for the future. It is not the first time that the theory of stellar evolution faces awkward problems for the solution of which new physical theories or quite new observational data have been needed. There were even worse situations in the past.

What really is very badly needed to improve our present views on advanced stages of evolution of stars of large masses is a better theory of convective envelopes (one possible solution may be a theory of nonlocal convection and time dependent convection) a relevant mechanism of mass loss, improved bolometric corrections and spectral or effective temperature of blue and red supergiants, improved nuclear reaction rates for heavy nuclei etc. A very important up to date problem is the study of the influence of rotation and magnetic fields on stellar evolution. This problem is of interest for the theory of evolution of stars of all masses. A two-dimensional approach to the solution of the stellar constitution equations seems to be the most promising way in this respect.

There is another problem that deserves thorough consideration. Recently a large amount of work has been carried out on studying supernovae explosions. This is the topic of one of the following reviews at this Symposium. Now the problem consists in that there remains a gap between the most advanced stage of evolution obtained by means of the present theory of stellar evolution with nuclear energy sources (a 'last' stellar model preceding a presupernova, as stated above) and the 'presupernova' model that is the starting point for the theory of supernovae explosions. The discrepancies between these two models are very large, particularly because the theory of supernova explosions starts usually with a very simplified model. We have tried to show above how very important for advanced stages are details concerning the structure of models at early stages of evolution: initial chemical composition, hydrogen profiles, intermediate convective layers, layers of varying chemical composition etc. The same is doubtless true for the initial presupernova model. There remains now to cover this gap and to achieve a fit between both these theories. This is by no means an easy task but an absolutely necessary step towards understanding of the latest stages of evolution of a star of large mass.

References

Appenzeller, I.: 1970a, *Astron. Astrophys.* **5**, 355.
Appenzeller, I.: 1970b, *Astron. Astrophys.* **9**, 216.
Arnett, W. D.: 1972a, *Astrophys. J.* **173**, 393.
Arnett, W. D.: 1972b, *Astrophys. J.* **176**, 681.
Arnett, W. D.: 1972c, *Astrophys. J.* **176**, 699.
Arnett, W. D.: 1973, *Astrophys. J.* **179**, 249.
Auré, J.-L.: 1971, *Astron. Astrophys.* **11**, 345.
Austin, S. M., Trentelman, G. F., and Kashy, E.: 1971, *Astrophys. J.* **163**, 79.
Barbaro, G., Giannone, P., Giannuzzi, M. A., and Summa, C.: 1969, in *Proc. of the 2nd Trieste Colloq. on Astrophys.* p. 217.
Barbaro, G., Chiosi, C., and Nobili, L.: 1971a, in *Proc. of the 3rd Colloq. on Astrophys.*, p. 313.
Barbaro, G., Chiosi, C., and Nobili, L.: 1971b, in *Proc. of the 3rd Colloq. on Astrophys.* p. 334.
Barbaro, G., Chiosi, C., and Nobili, L.: 1972, *Astron. Astrophys.* **18**, 186.
Barbaro, G. and Chiosi, C.: 1972, in *Proc. of the IAU Colloq.*, No. 17, Meudon, Section XV.
Barkat, Z., Rakavy, G., and Sack, N.: 1967, *Phys. Rev. Letters* **18**, 379.
Beaudet, G., Petrosian, V., and Salpeter, E. E.: 1967, *Astrophys. J.* **150**, 979.
Bisnovatyi-Kogan, G. S. and Nadyozhin, D. K.: 1972, *Astrophys. Space Sci.* **15**, 353.
Cherepashchuk, A. M.: 1972, *Astron. Tsirk.* **739**, 1.
Chiosi, C. and Summa, C.: 1970, *Astrophys. Space Sci.* **8**, 478.
Dallaporta, N.: 1971, in *Proc. of the 3rd Colloq. on Astrophys.*, p. 250.
Demarque, P. and Mengel, J. G.: 1972, *Nature Phys. Sci.* **239**, 55.
Dudurov, A. and Tutukov, A.: 1972, *Nauch. Inform. Moscow* **21**, 3.
Ferrari, A., Gillino, R. and Masani, A.: 1972, *Mem. Soc. Astron. Ital.* **43**, 731.
Fowler, W. A. and Hoyle, F.: 1964, *Astrophys. J. Suppl.* **91**, 201.
Fraley, G. S.: 1968, *Astrophys. Space Sci.* **2**, 96.
Frantsman, Ju. L.: 1973, *Nauch. Inform. Moscow* **26**, 62.
Frantsman, Ju. L., Popova, E. I., and Ziolkowski, J.: 1973, *Nauch. Inform. Moscow* **27**, 54.
Fricke, K. J. and Strittmatter, P. A.: 1972, *Monthly Notices Roy. Astron. Soc.* **156**, 129.
Gabriel, M.: 1968, *Astron. Astrophys.* **1**, 321.
Hartwick, F. D. A.: 1970, *Astrophys. Letters* **7**, 151.
Höppner, W. and Weigert, A.: 1973, *Astron. Astrophys.* **25**, 99.
Humphreys, R. M.: 1970, *Astrophys. Letters* **6**, 1.
Ikeuchi, S., Nakazawa, K., Murai, T., Hōshi, R., and Hayashi, C.: 1971, *Prog. Theor. Phys. Kyoto* **46**, 1713.
Ikeuchi, S., Nakazawa, K., Murai, T., Hōshi, R., and Hayashi, S.: 1972a, *Prog. Theor. Phys. Kyoto* **48**, 1870.
Ikeuchi, S., Sato, H., Sato, T., and Takeda, H.: 1972b, *Prog. Theor. Phys. Kyoto* **48**, 1885.
Kato, S.: 1966, *Publ. Astron. Soc. Japan* **18**, 374.
Kozlowski, M.: 1971, *Astrophys. Letters* **9**, 65.
Kozlowski, M. and Paczyński, B.: 1973, *Acta Astron.* **23**, 65.
Lauterborn, D., Refsdal, S., and Roth, M. L.: 1971, *Astron. Astrophys.* **13**, 119.
Ledoux, P.: 1941, *Astrophys. J.* **94**, 537.
Ledoux, P.: 1947, *Astrophys. J.* **105**, 305.
Massevich, A. G.: 1970, *Stellar Constitution*, Presidents Report, 1970, Draft Reports IAU, Brighton.
Massevich, A. G., Tutukov, A. V., Dluzhnevskaya, O. B., Varshavsky, V. J., Uus, U., Ergma, Eh. V., Popova, E. I., and Rodionova, G. G.: 1971, *Nauch. Inform. Moscow* **19**, 45.
Massevich, A. G. and Schustov, B. M.: 1972, *Physics and Evolution of Stars*, Astronomy Vol. 8, VINITI Reviews on Science and Technics, Moscow.
Motteran, M.: 1971, in *Proc. of the 3rd Colloq. on Astrophys.*, p. 292.
Noels, A. and Gabriel, M.: 1973, *Astron. Astrophys.* **24**, 201.
Paczyński, B.: 1970, *Acta Astron.* **20**, 195.
Pontekorvo, B. M.: 1959, *Zh. Exp. Theor. Phys.* **36**, 1915.
Rakavy, G. and Shaviv, G.: 1967, *Astrophys. J.* **148**, 803.
Robertson, J. W.: 1971, *Proc. Astron. Soc. Australia* **2**, 23.
Robertson, J. W.: 1972, *Astrophys. J.* **177**, 473.

Robertson, J. W.: 1973, *Astrophys. J.* **180**, 425.
Ruben, G.: 1969, *Nauch. Inform. Moscow* **14**, 3.
Ruben, G. and Massevich, A. G.: 1966, *Nauch. Inform. Moscow* **3**, 36.
Sakashita, S. and Hayashi, C.: 1959, *Prog. Theor. Phys. Kyoto* **22**, 830.
Sakashita, S. and Hayashi, C.: 1961, *Prog. Theor. Phys. Kyoto* **26**, 942.
Schild, R. F.: 1970, *Astrophys. J.* **161**, 855.
Schwarzschild, M. and Härm, R.: 1958, *Astrophys. J.* **128**, 348.
Schwarzschild, M. and Härm, R.: 1959, *Astrophys. J.* **129**, 637.
Simpson, E. E.: 1971, *Astrophys. J.* **165**, 295.
Spiegel, E. A.: 1969, *Comments Astrophys. Space Phys.* **1**, 57.
Spinrad, H. and Taylor, B. J.: 1969, *Astrophys. J.* **157**, 1279.
Stothers, R. and Simon, N.: 1968, *Astrophys. J.* **152**, 233.
Stothers, R. and Chin, C.-W.: 1969, *Astrophys. J.* **158**, 1039.
Stothers, R. and Evans, T. L.: 1970, *Observatory* **90**, 186.
Stothers, R.: 1972a, *Astrophys. J.* **175**, 431.
Stothers, R.: 1972b, *Astrophys. J.* **175**, 717.
Stothers, R. and Chin, C.-W.: 1972, *Astrophys. J.* **177**, 155.
Stothers, R. and Chin, C.-W.: 1973a, *Astrophys. J.* **179**, 555.
Stothers, R. and Chin, C.-W.: 1973b, *Astrophys. J.* **182**, 209.
Sugimoto, D.: 1970a, *Prog. Theor. Phys. Kyoto* **44**, 375.
Sugimoto, D.: 1970b, *Prog. Theor. Phys. Kyoto* **44**, 599.
Taylor, B. J.: 1970, *J. Suppl., Ser. Kyoto* **22**, 177.
Toevs, J. W., Fowler, W. A., Barnes, C. A., and Lyons, P. B.: 1971, *Astrophys. J.* **169**, 421.
Trimble, V., Paczyński, B., and Zimmerman, B. A.: 1973, *Astron. Astrophys.* **25**, 35.
Tutukov, A. V.: 1972, *Proc. of the IAU Colloq.*, No. 17, Meudon, Section XIV.
Tutukov, A. V. and Yungleson, L. R.: 1973, *Nauch. Inform. Moscow* **27**, 1.
Uus, U.: 1970, *Nauch. Inform. Moscow* **17**, 35.
Van den Bergh, S. and Sackman, I. J.: 1965, *Astron. J.* **70**, 133.
Varshavsky, V. I.: 1972a, *Nauch. Inform. Moscow* **21**, 25.
Varshavsky, V. I.: 1972b, *Astron. Zh. Akad. Nauk SSSR* **49**, 1055.
Varshavsky, V. I. and Tutukov, A. V.: 1972, *Nauch. Inform. Moscow* **23**, 47.
Varshavsky, V. I. and Tutukov, A. V.: 1973a, *Nauch. Inform. Moscow* **26**, 35.
Varshavsky, V. I. and Tutukov, A. V.: 1973b, *Nauch. Inform. Moscow* **27**, 73.
Varshavsky, V. I.: 1973c, *Nauch. Inform. Moscow* **27**, 96.
Weisser, D. C., Morgan, J. F., and Thompson, D. R.: 1972, in press.
Woosley, S. E., Arnett, W. D., and Clayton, D. D.: 1972, *Astrophys. J.* **175**, 731.
Ziolkowski, J.: 1972, *Acta Astron.* **22**, 327.