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Generic rank of Betti map and unlikely intersections

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Corrigendum

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(Compositio Math. 156 (2020), 2469–2509)

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1. Theorem 1.3(ii) of [Gao20] should read

$$\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}) = 2\dim \mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}) \quad \text{for all } m \geq \dim X \text{ if } \iota \text{ is quasi-finite.}$$

Indeed, Theorem 1.3(ii) is proved by applying Theorem 10.1(ii) to $t = 0$, which says

$$\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}) \geq 2\dim \iota^{[m]}(\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})) \quad \text{for all } m \geq \dim X.$$

If ι is quasi-finite, so is $\iota^{[m]}|_{\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]})}$, and hence $\dim \mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}) = \dim \iota^{[m]}(\mathscr{D}_m^{\mathcal{A}}(X^{[m+1]}))$.

This does not affect the applications of Theorem 1.3(ii) in this paper (Theorem 1.2′) or those in [DGH21, Theorem 6.2]. Indeed, in both cases ι is the identity map (or a quasi-finite morphism according to convention).

2. Theorem 1.7 should be weakened to be[1]: For each integer $l \leq \dim \iota(X)$, we have

$$\mathrm{rank}_{\mathbb{R}}(\mathrm{d}b_{\Delta}|_X) < 2l \Leftrightarrow X^{\deg}(l - \dim X) \text{ is Zariski dense in } X. \tag{1}$$

As a consequence, Theorem 1.1(ii) should be removed.

These modifications do not change the rest of the results stated in the Introduction or Theorem 10.1: First, these changes have no impact on Theorem 1.8 so they do not change the major result of the paper, which is the criterion to characterize the generic Betti rank (Theorem 1.1(i)), because the proof of this criterion in § 9.3 is unchanged (it uses Theorem 1.8 and this weaker version of Theorem 1.7). Thus, the consequences of this criterion (equation (1.4) and Theorems 1.2, 1.2′, 1.3, 1.4 and 10.1) remain unchanged. Finally, the proof of Proposition 1.10 in § 11 is unchanged as it does not use Theorem 1.7.

The reason for this modification of Theorem 1.7 lies in Proposition 6.1: the inclusion $\mathbf{u}(X_{<2l}) \subseteq X^{\deg}(l - d)$ does not hold in general. However, the statement in 'In particular' ('Conversely' in the current version) still holds true, and this statement together with the other inclusion $X^{\mathrm{sm}}(\mathbb{C})\mathcal{A}pX^{\deg}(l - d) \subseteq \mathbf{u}(X_{<2l})$ imply the equivalence (1) above; see the proof of Theorem 1.7 in § 9.2.

In the proof of this 'In particular' statement of Proposition 6.1, equation (6.1) should be changed to

$$(\dim_{\mathbb{R}})_{\tilde{x}}(\tilde{b}^{-1}(r)\mathcal{A}p\tilde{X}) > 2(d - l) \quad \text{for all } \tilde{x} \text{ in a non-empty open subset } \tilde{U} \text{ of } \tilde{X}.$$

Notice that $\mathbf{u}(\tilde{U})$ contains a non-empty open subset (in the usual topology) of $X^{\mathrm{sm,an}}$, so $\mathbf{u}(\tilde{U})$ is Zariski dense in X. The rest of the original proof of Proposition 6.1 then shows that $\mathbf{u}(\tilde{U}) \subseteq X^{\deg}(l - d)$. Thus, this establishes the statement in 'In particular'.

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