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The Frequency Distribution of Semimajor Axes of Wide Binaries: Cosmogony and Dynamical Evolution

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Abstract. The frequency distribution *f(a)* of semimajor axes of double and multiple systems, and their eccentricities and mass ratios, contain valuable fossil information about the process of star formation and the dynamical history of the systems. In order to advance in the understanding of these questions, we made an extensive analysis of the frequency distribution *f(a)* for wide binaries ($a > 25$ AU) in various published catalogues, as well as in our own (Poveda *et al.* 1994; Allen *et al.* 2000; Poveda & Hernández–Alcántara 2003). Based upon all these studies we have established that the frequency distribution *f(a)* is a function of the age of the system and follows Öpik's distribution *f(a)* $\sim 1/a$ in the range of 100 AU $< a < a_c(t)$, where $a_c(t)$ are the critical semimajor axes beyond which binaries have been dissociated by encounters with massive objects. We argue that the physics behind the distribution *f(a)* $\sim 1/a$ is a process of energy relaxation, analogous to those present in stellar clusters (secular relaxation) or in the early stages of spherical galaxies (violent relaxation). The existence of runaway stars indicates that both types of relaxation are important in the process of binary and multiple star dynamical evolution.

Keywords. binaries: wide; Galaxy: kinematics and dynamics; stars: proper motions.

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1. Introduction

The distribution of semimajor axes (separations) of double and multiple stars is a fossil record of the conditions at star formation, as well as of the processes of dynamical evolution, including the dissociation of wide binaries produced by encounters with massive objects: molecular clouds, spiral arms, etc. Our long-standing interest in these topics has led us to investigate the frequency distribution of semimajor axes (separations) of wide binaries as a function of age. In the past, two main distributions of semimajor axes have been proposed:

(1) a power-law frequency distribution *f(a)* $\sim a^{-\alpha}$
(2) a Gaussian distribution in $\log P$ or $\log a$.

When $\alpha = 1$ in the power-law distribution, we have the well known Öpik (1924) distribution (OD). On the other hand, Kuiper (1935, 1942) proposed the Gaussian distribution, which was further elaborated by Heintz (1969). More recently, Duquennoy & Mayor (1991, DM) again proposed a Gaussian distribution in log P, valid throughout the interval $1 < \log P$ (days) < 10.

Our interest in the subject led us to construct a catalogue of wide binaries in the solar vicinity, based on the catalogue of nearby stars of Gliese & Jahreiss (1991, GJ); see Poveda *et al.* (1994) for details. We have also constructed a list of common-proper-motion binaries in the Orion Nebula Cluster (age 10^6 years), extracted from the Jones & Walker (1988) catalogue of proper motions. (Poveda & Hernández-Alcántara 2003). In our search for evolutionary effects in the observed distributions we have also looked at the oldest stars in the Galaxy. For this purpose we constructed a catalogue (Allen *et al.* 2000) of common-proper-motion companions to the lists of high velocity metal-poor stars of Schuster *et al.* (1988; 1989a; 1989b), with ages of about 10^{10} years.

In all our catalogues, as well as in the *Luyten Double Star Catalogue* (1940–1987, LDS) and in Chanamé & Gould's catalogue (2004, CG), we confirm our previous findings (Poveda *et al.* 1997; Poveda & Allen 2004), that the separations follow Öpik's distribution in an interval that is bounded at the lower end ($a \sim 100$ AU) by the process of close binary and protoplanetary disk formation, and at large separations ($a > 2500$ AU) by the dissociation effects produced by encounters with massive objects.

It can be shown that Öpik's distribution in the plane (N, log P) is a horizontal straight line which is quite consistent (within its error bars) with the DM distribution in the interval $2.44 < \log P$ (years) < 5.44, which corresponds to $53 < a$ (AU) < 5500. (Poveda *et al.* 2004). Since a great number of binaries from many different and largely independent sources confirm the validity of Öpik's distribution, and since there is no stellar formation or single physical process able to produce a Gaussian distribution valid in the interval $1 < \log P$ (days) < 10, we propose to abandon the Gaussian representation for $a >$ 100 AU. On the contrary, Öpik's distribution, which is equivalent to a surface density of secondaries $\rho(a) \sim a^{-2}$, has a physical interpretation. In fact, this distribution is similar to the run of surface brightness in globular clusters (King 1962) or to that of elliptical galaxies (Hubble's 1930 or de Vaucouleurs' 1953 distributions). In both cases the physics behind such distributions is well known: it is the result of energy relaxation. The similarity of OD to the surface brightness in clusters and elliptical galaxies indicates that binaries are not born alone; at birth, they must be subject to a process of energy relaxation which cannot be produced by two-body encounters, i.e., stars must be formed in groups of multiplicity $n \geqslant 3$.

This paper is organized as follows. In Section 2 we study the distribution *f(a)* for a volume–complete sample extracted from our catalogue (Poveda *et al.* 1994). Section 3 examines the distribution *f(a)* in our catalogue of very young ($T < 10^6$ years), common-proper-motion binaries in the Orion Nebula Cluster (Poveda & Hernández-Alcántara 2003). In Section 4 we examine another sample of wide binaries, namely Chanamé & Gould's (2004) common-proper-motion binaries from the revised NLTT (Salim & Gould 2003; Gould & Salim 2003). Again, the binaries in this catalogue follow OD. Section 5 examines the physics behind Öpik's distribution, and Section 6 presents our conclusions.

2. A Catalogue of Nearby Wide Binaries and a Volume–Complete Sample of these Objects

With the purpose of detecting the effects of dissociation of weakly-bound binaries with the passage of time, we constructed a catalogue of wide binaries (305 double systems, 26 triples and 3 quadruples) with $a > 25$ AU; (Poveda *et al.* 1994).

A sub-sample of this catalogue, i.e., all those systems with primaries of luminosity class V or IV and brighter than absolute magnitude $M_V = 9$ is very important, as it is volume-complete. To show this, in Figure 1 we plot $N(\log r)$ vs. $3 \log r$, where $N(\log r)$ is the number of systems (M_V <9) out to a distance r. As can be seen, with the exception of a few very close systems, the great majority follow the relation $N \sim r^3$ right to the limit of the catalogue ($r = 22.5$ pc), as expected for a volume-complete sample.

Having a volume-complete catalogue, we proceed to investigate the frequency distribution of semimajor axes. It can be shown that an equivalent representation of OD is the cumulative distribution $N(< \log a) \sim \log a$. In the plane $N(< \log a) - \log a$, OD is a

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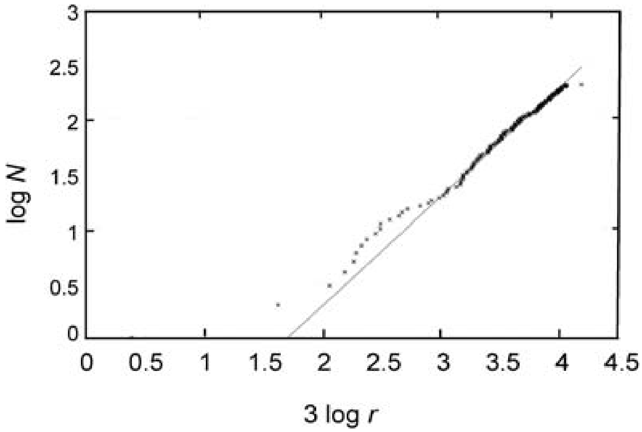

Figure 1. Completeness of the systems in the catalogue of nearby wide binary and multiple systems (Poveda *et al.* 1994). The straight line corresponds to $N(r) \sim r^3$, i.e., a volume-complete sample.

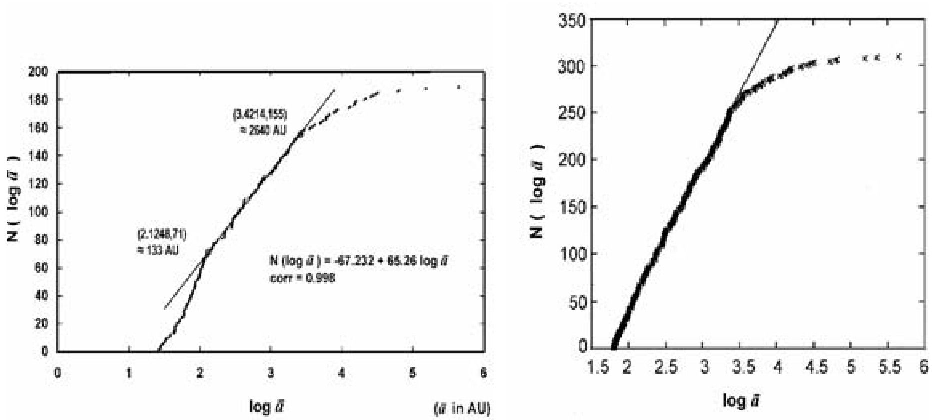

Figure 2. (a)Left. Cumulative distribution of $\log \bar{a}$ for the 189 binaries of the complete sample in Poveda *et al.* (1994), i.e., for systems with $M_V < 9$ and luminosity class IV-V. The straight line is a fit for Öpik's relation. The KS test for this fit gives a value of $Q = 0.99$ in the interval (133AU, 2640AU). (b)Right. Comparison between Öpik's distribution and the cumulative numbers of $\log \bar{a}$ for the binaries in the Poveda *et al.* (2004) catalogue. The straight line is a fit to Öpik's distribution for systems satisfying $60 < \bar{a}$ (AU) < 2965. The KS test gives a value of $Q = 0.96$ for the interval $60 < \bar{a}$ (AU) < 2965.

straight line. In general, we will favor the analysis of the cumulative distribution *f(a)* in order to reduce the noise introduced by small number sampling fluctuations. Since most of the wide binaries (in our catalogues) do not have reliable orbits because of their long periods, we have to use angular separations s and distances to give projected separations in astronomical units. However, a statistical relation between the average value of a for a given projected separation s, namely $\bar{a} = 1.41s$ (Couteau 1960), can be used to estimate for each binary a value of $\bar{a}$.

Figure 2a shows for our volume-complete sample the run of the cumulative distribution $N(\log \bar{a})$ vs. $\log \bar{a}$. An inspection of Figure 2a shows that this sample closely follows Öpik's distribution in the interval $2.12 < \log \bar{a} < 3.42$, i.e., $133 < \bar{a}$ (AU) < 2640. To evaluate quantitatively how reliably Öpik's distribution represents the data, we shall use the Kolmogorov–Smirnov (KS) test, which was developed precisely for cumulative distributions and where no arbitrary binning of the data is required, (as is the case in the popular χ^2 test). In Figure 2a each binary is plotted, thus $N(\log \bar{a})$ increases one by one. By least-squares we fitted to the data points a number of straight lines, each one defined in the interval ($\log \bar{a}_i$, $\log \bar{a}_j$), until we found the best straight line that minimized the residuals and maximized the interval ($\log \bar{a}_i$, $\log \bar{a}_j$), i.e., the one giving the largest interval ($\log \bar{a}_i$, $\log \bar{a}_j$) in which Öpik's distribution reliably represents the cumulative distribution of separations.

In Figure 2a we give the equation of the straight line that best represents the data, in the interval $\log \bar{a}_i = 2.1248$, $\log \bar{a}_j = 3.4214$ ($\bar{a}_i \approx 133$ AU, $\bar{a}_j \approx 2640$ AU). Having found Öpik's distribution for the wide binaries in the interval $133 < \bar{a}(\mathrm{AU}) < 2640$ we now proceed to test, via KS, what is the level of significance of the theoretical distribution OD. The closer to 1 the estimator Q is, the better the theoretical representation (Press *et al.* 1990). For the present case we find $Q = 0.99$; i.e., we can accept Öpik's distribution at a very high level of confidence in the interval 133–2640 AU. For the interval 133–3100 AU we find $Q = 0.96$, also representing a high level of confidence.

In Figure 2b, we plot in the same plane as in Figure 2a, all the binaries (305) from our 1994 catalogue. Even though this sample is not volume-complete, one can argue that this does not seem to introduce an important bias in the distribution of separations. In fact, as can be seen from Figure 2b, the cumulative distribution $N(< \log \bar{a})$ again defines a straight line. Repeating the statistical analysis for this catalogue of 305 binaries we find that the KS test gives a value $Q = 0.96$ for $\bar{a}$ in the interval between 60 AU and 2965 AU.

We now seek an explanation for the limits of validity of the OD as shown in Figures 2a and 2b, particularly for the volume-complete sample. The following hypothesis is proposed: (1) at the short end of the distribution ($\bar{a} \sim 100$ AU), any primeval OD will be quickly modified by the presence of close binaries and protoplanetary disks; (2) at the wide end ($\bar{a} > 3{,}000$ AU), encounters with massive objects (molecular clouds, spiral arms, black holes, MACHOS, etc.) will gradually dissociate the widest binaries. In fact, in a theoretical paper, Weinberg *et al.* (1987) estimated that a binary with a semimajor axis smaller than 2000 AU has a probability of one to survive (against dissociation by giant molecular clouds) during 10 billion years, yet a binary with $\bar{a} \approx 12{,}600$ AU after two billion years has a probability of survival of only 0.5. The consistency between the results of Weinberg *et al.* and the distributions shown in Figures 2a and 2b gives support to the hypothesis that Öpik's distribution is primeval, valid up to $\bar{a} \approx 45{,}000$ AU (see next section), but with the passage of time it gets truncated at the large separations. To further examine this hypothesis, we analyze the distribution of semimajor axes of the binaries in a very young group ($T < 10^6$ years).

3. Wide Binaries in a Very Young Group

Taking advantage of the Jones & Walker (1988) catalogue of proper motions of the stars in the Orion Nebula Cluster, we have identified 68 candidate common-proper-motion binaries (Poveda & Hernández-Alcántara, 2003). Jones & Walker's determinations of proper motions and infrared magnitudes for 1053 stars of the Orion Nebula Cluster are appropriate for our work, because of the proper motions accuracy ($\sigma_\mu < 0.1$ arcsec/century) and also because the faint limiting magnitude ($I < 13$). Moreover, the one-million year age of the cluster allows us to look into the 'almost' primeval distribution of semimajor

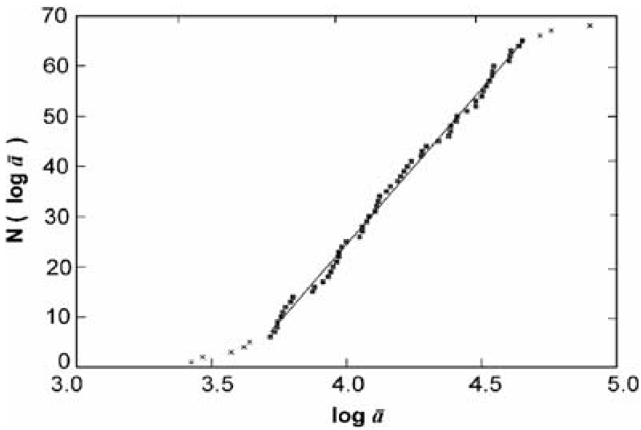

Figure 3. Cumulative distribution of the logarithms of the expected value of a for the common-proper-motion binaries of the Orion Nebula Cluster (Poveda & Hernández-Alcántara 2003). The straight line represents Öpik's distribution. For the interval fitted with a straight line, $5180 < \bar{a}(\mathrm{AU}) < 44800$ (60 binaries) the KS test gives a value of $Q = 0.99$.

axes. The cumulative distribution of semimajor axes for this sample of Orion binaries follows very neatly Öpik's distribution (see Figure 3). The KS analysis of the data plotted in Figure 3 indicates that OD fits the data up to semimajor axes of 45,000 AU with a $Q = 0.99$.

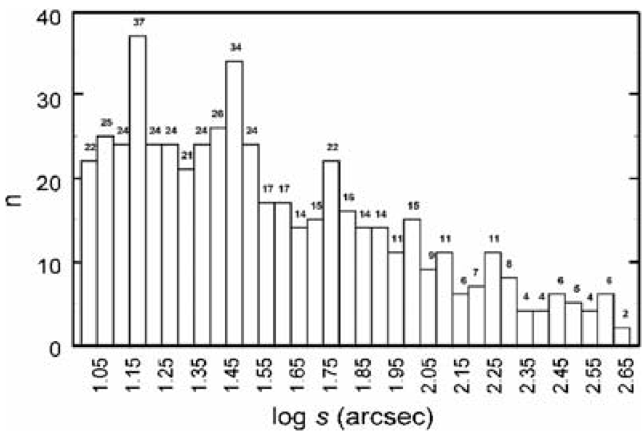

Figure 4. Frequency distribution of the separations s for the disk binaries of Chanamé & Gould (2004) for binaries with $s > 10''$ (523 binaries). Note that in the interval $1 < \log s < 1.55$ ($10'' < s < 36''$) which includes 285 binaries, this distribution is consistent with Öpik's, which in this plane would be a horizontal straight line. For larger separations, the distribution becomes depopulated similarly to what we found to occur for other samples of binaries (see text). We interpret this departure as due to the dissociation of wide binaries by encounters with massive perturbers.

4. Another Sample of Wide Binaries

Chanamé and Gould (2004, CG) have assembled a catalogue of wide binaries based on the revised *Luyten NLTT Catalogue* by Gould & Salim (2003) and Salim & Gould (2003). CG identified 999 common-proper-motion pairs. Good photometry allowed them

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to construct a reduced proper motion diagram for their binaries. The position of the binary components in this diagram helps to separate disk main-sequence pairs (801) from halo subdwarfs (116). The large number of binaries in the CG Catalogue offers an independent sample to test the validity of OD for main-sequence and halo binaries, respectively.

According to CG, their catalogue is incomplete for separations $s < 10''$. Since we are trying to establish the frequency distribution of separations of binaries in CG we extracted from their list a sample of disk binaries with separations $s > 10''$, i.e., we rejected 276 binaries closer than $10''$ out of 800 disk binaries. For the remaining 524 pairs we plot in Figure 4 the frequency distribution of separations $f(s)ds$. The large number of binaries allows to display this frequency distribution with small sampling fluctuations. In this figure we see clearly that $f(s)ds$ is essentially constant in the interval $1 \leqslant \log s \leqslant 1.55$. For larger separations, i.e., $s > 35''\!.5$, $f(s)$ is a decreasing function of s. CG noticed this behavior, which they found statistically significant. The constancy of $f(s)$ in the interval $10'' < s < 35''\!.5$ is just what we expect from OD. Since CG estimate the mean distance of their disk binaries to be 60 pc, we can transform into astronomical units the separations listed by CG. The angular interval where OD holds transforms into: $600 \leqslant s$ (AU) $\leqslant 2129$, which is equivalent to $840 \leqslant \bar{a}$ (AU) $\leqslant 2981$.

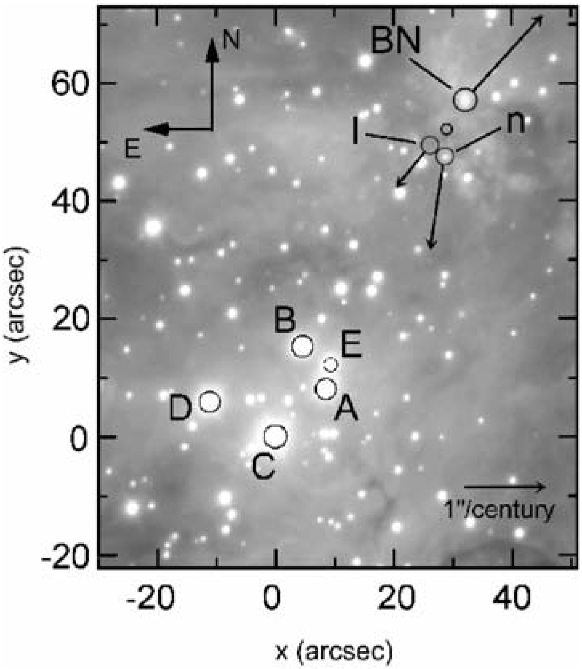

Figure 5. An example of a recent and nearby case of violent relaxation (Rodríguez *et al.* 2005; Gómez *et al.* 2006) Next to the Orion Trapezium, in the Becklin–Neugebauer, Kleinman–Low region we show 3 sources: BN, I, n, which are moving away from a center with transverse velocities of the order of 26, 8 and 28 km s^{-1}, respectively. These velocities are much larger than those expected for secular relaxation. Indeed, they correspond to violent relaxation.

5. The Physics Behind Öpik's Distribution

The analysis of the various samples of wide binaries, as shown in Figures 2–4, shows convincingly that Öpik's distribution holds for $a > 100$ AU. The distribution *f(a)* $\sim a^{-1}$ is equivalent to a surface density of secondaries $\rho(a) \sim a^{-2}$. This surface density distribution is reminiscent of the surface density of stars in globular clusters and elliptical galaxies (King 1962; Hubble 1930; de Vaucouleurs 1953). The physics involved in these distributions is well understood. In the case of globular clusters we have what we may call secular relaxation, i.e., a process in which two-body encounters gradually modify the energy of a given star over a time scale much greater than the crossing time. In the case of elliptical galaxies, what we have is a case of initially violent relaxation, in which the energy of the stars changes rapidly because the potential of the stellar system (galaxy) changes rapidly due to an initial stellar collapse (van Albada 1982) or because of galaxy mergers. In the first case (secular relaxation) there is equipartition of energy and therefore a tendency for the lighter stars to diffuse to the outer parts of the globular cluster (this has been observationally confirmed); on the contrary, in the case of violent relaxation there is no equipartition of energy and hence no segregation of stellar masses, which indeed is the case in elliptical galaxies (no color gradients are observed).

The process of energy relaxation involves the interaction of several close stars ($n \geqslant 3$), very early in the stellar history. In fact, the wide binaries in the Orion Nebula Cluster follow OD even though they are still in the pre-main sequence phase (T $< 10^6$ years). We conclude that the process of energy relaxation is not related to interactions with stars in the cluster environment, but rather it is the result of very early interactions in multiple systems. This suggests that stars are formed in multiple systems that quickly relax and assume OD, with the possible ejection of one or more single stars. The common proper motion binaries in Orion show that there is not enough time for the reverse process to take place, i.e., the formation of double and multiple stars by capture from stars in the cluster.

In the process of star formation, the transition from gas to stellar dynamics may lead to a virialized multiple system where $2T + \Omega \approx 0$, or to a non-virialized one, depending on the initial conditions prevailing in the transition phase. If initial condensations (proto-stars) are not virialized, $2T + \Omega \ll 0$, then the proto-stars will collapse towards the center of mass of the multiple systems. Here we meet the conditions of violent relaxation, i.e., the collapse of the proto-stars will produce a rapid change in the gravitational potential experienced by the stars. In this conditions some stars will be accelerated to velocities larger than those associated with a virialized multiple system. An n-body simulation of this scenario was realized by the present authors many years ago (Poveda *et al.* 1967) with the purpose of finding an alternative explanation for the formation of runaway stars. In those simulations we found not only runaway stars but also that the binaries formed followed Öpik's distribution (Allen 1968; Poveda *et al.* 2004). Figure 5 shows a recent and nearby case of violent relaxation (Rodríguez *et al.* 2005; Gómez *et al.* 2006). Over an image of the Orion Trapezium region (McCaughrean 2001) we have superposed the proper motion vectors of objects BN, I, and n. The proper motions of the heavily obscured B star (Becklin-Neugebauer Object), as well as of the infrared objects I and n imply transverse velocities much larger than those expected to be produced in a virialized multiple system.

6. Conclusions

(1) The study of a large number of wide binaries, from mostly independent sources confirms that the frequency distribution of semimajor axes is *f(a)* $\sim a^{-1}$, i.e. precisely Öpik's distribution. This distribution is truncated at the short end ($a \approx 100$ AU) by the presence of close binaries and protoplanetary disks; at the large separations the distribution is depopulated by the process of dissociation produced by encounters with

massive objects. Figure 2 clearly exhibits these effects for the volume-complete sample in the solar vicinity.

(2) Öpik's distribution in the plane $N(\log P)$ is a constant, which is entirely consistent (within the error bars) with Duquennoy & Major's Gaussian distribution in the interval 42$<\bar{a}$ (AU) $<$ 4213.

(3) Since there is no single astrophysical process that would generate a Gaussian distribution in $\log P$ (or in $\log a$) over such a wide interval (1 day $< P < 10^{10}$ days) and since, on the other hand, Öpik's distribution can be explained by the process of energy relaxation in few-body interactions, we propose to abandon the Gaussian representation for $\log a$, in favor of Öpik's distribution (for $a > 100$ AU.)

(4) Öpik's distribution suggests that the process of star formation produces multiple stars which evolve towards binaries after ejecting one or more single stars.

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