CORRESPONDENCE

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The Journal of the Institute of
Actuaries Students' Society

Actuarial Science in the Middle Ages

Sirs,

An interesting application of actuarial principles during the Middle Ages is related in a recent book by Dom David Knowles (*The Religious Orders in England*, Cambridge, 1948).

It had at one time been the custom when an abbacy fell vacant for the abbot's revenues to be diverted to the Crown. As these revenues were part of the regular income on which the whole establishment depended, the death or preferment of its abbot threw a heavy financial burden on the community over which he ruled. And this recurrent liability, falling as it did at irregular intervals, proved a serious embarrassment to monastic economy.

Certain abbeys, which were known as 'exempt houses' because they had gained exemption from episcopal jurisdiction, had an additional burden to meet. In their case, a newly appointed abbot had to make a costly journey to Rome to obtain confirmation of his appointment and to pay certain fees to the papal Curia. This naturally increased the liability incurred by a vacancy.

Thus every abbey was faced with the prospect that, without warning, it might lose a large part of a year's income. To meet this contingency a solution was devised which is best described in Dom David Knowles's own words:

> Later, it became common to take out a form of insurance against frequent vacancies by paying a fixed annual premium. This was set at fifty marks at St Albans, which would profit the abbey if a series of abbots ruled for less than twenty years; St Augustine's Canterbury, likewise put the matter on an actuarial basis in 1392. At almost the same time the Curia allowed similar policies to be taken out by exempt houses; thus St Albans, for an annual premium of twenty marks, were excused from sending their abbot out to Rome and he might receive confirmation from a bishop of his choice. Evesham

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received a similar privilege in 1363. It will be noticed that while the premium paid to the king did no more than put the recurring large expenses upon an actuarial footing, the annual payment to the papacy was advantageous to all parties: to the Curia it gave a fixed annual rent with which to budget, while to the abbey it was a considerable saving to pay no more than the Curial fee, with none of the expenses of travelling.

As the author has emphasized to me in a private letter, there was in fact no third party intervening in these transactions as actuary. Nevertheless, I feel that they have a definite place in actuarial history and may, perhaps, be of some interest to readers of the *Journal*.

Yours faithfully,

R. D. CLARKE

26 Rossetti Garden Mansions,
London, S.W. 3

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Sirs,

A useful approximate method by H. A. Gosden for calculating the net single premium at rate of interest i for an assurance to cover the amount outstanding under a Building Society Mortgage subject to a rate of interest j appeared in Vol. VII of the *Journal* (p. 174). While that method is sufficiently accurate for most practical purposes some actuaries may prefer to use a theoretically accurate method if this is possible without much extra work. As will be seen this can be done by constructing special commutation columns.

The formula for the true net single premium for an assurance to cover a loan of initial amount unity repayable over n years by level annual instalments in arrear is given by Gosden (p. 176) and is

$$\frac{(1+j)\sum_{r=1}^{n} C^{i}_{x+r-1}\, a^{j}_{\overline{n-r+1|}}}{a^{i}_{\overline{n|}}\, D^{i}_{x}},$$

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where i = the rate of interest at which the premium is to be calculated,
j = the rate of interest on the loan.

We may write this expression as:

$$\frac{(1+j)}{a_{\overline{n}|}^{j}\,D_x^i}\,\frac{1}{j}\left[\sum_{r=1}^{n} v_i^{x+r}\,d_{x+r-1}\,(1-v_j^{n-r+1})\right]$$

$$=\frac{(1+j)}{a_{\overline{n}|}^{j}\,D_x^i}\,\frac{1}{j}\left[\sum_{r=1}^{n} C_{x+r-1}^i-\sum_{r=1}^{n} v_i^{x+r}\,d_{x+r-1}\,v_j^{n-r+1}\right]$$

$$=\frac{(1+j)}{a_{\overline{n}|}^{j}\,D_x^i}\,\frac{1}{j}\left[M_x^i-M_{x+n}^i-\frac{v_j^{n+1}\sum_{r=1}^{n} v_i^{x+r}\,(1+j)^{x+r}\,d_{x+r-1}}{(1+j)^x}\right]$$

$$=\frac{1}{a_{\overline{n}|}^{j}}\,\frac{(1+j)}{j}\left[\frac{M_x^i-M_{x+n}^i}{D_x^i}-\frac{v_j^{n+1}(M_x'-M_{x+n}')}{D_x'}\right],$$

where
$$C_x'=(1+j)^{x+1}\,C_x^i,$$
$$M_x'=C_x'+C_{x+1}'+C_{x+2}',\ \text{etc.},$$
$$D_x'=(1+j)^x\,D_x^i.$$

It is only necessary to construct columns of C_x', M_x' and D_x' for the range of ages over which these policies extend and with a modern calculating machine this does not take long.

Yours faithfully,
G. J. KNAPMAN

7 Cavendish Square,
London, W. 1