ON THE SOLUTION OF A PROBLEM RECENTLY GIVEN BY MR. TUCKER.

To the Editor of the Assurance Magazine.

SIR,—At page 255 of your Number for *April* appears a communication from Mr. Tucker embodying a problem and its solution, on which latter I beg to be allowed a few remarks.

First: For the purpose apparently of confining himself to the employment of Mr. Thomson's tables, Mr. Tucker approximates to the value of one of the quantities required in the solution; but I submit whether it is a legitimate mode of approximation which for its application presupposes a knowledge of the *true* value: for how else does Mr. Tucker ascertain that 50 is the age of the single life most nearly equivalent to the joint lives 32, 40, but by first ascertaining the true value of these joint lives? Having done this, does it not seem trifling to throw away this correct value, and to employ an erroneous one instead?

But, secondly: it appears to me that Mr. Tucker is wrong in the value he assigns for the portion of the annuity remaining after the first five years. The value of the n^{th} payment for those years is $p_{32,n}(1-p_{40,5})v^n$, in which it will be noticed that the factor $1-p_{40,5}$ is *independent of n*; it is therefore *constant in all the terms* composing the value of the *deferred* annuity. The value of the latter portion of the annuity is therefore $_{5|}a_{32}(1-p_{40,5})$, or $_{5|}a_{32}-{}_{5|}a_{32}\cdot p_{40,5}$; and the value of the former portion being $_{\overline{5}|}a_{32}-{}_{\overline{5}|}a_{32,40}$, the total value is

$$_{\overline{5}|}a_{32}-{}_{\overline{5}|}a_{32,40}+{}_{5|}a_{32}-{}_{5|}a_{32}p_{40,5}.={}a_{32}-{}_{\overline{5}|}a_{32,40}-{}_{5|}a_{32}p_{40,5}.$$

This expression proves itself. It shows that the annuity in question is equivalent to a whole life annuity on 32, the payments of which during the first five years are to be repaid if 40 be alive at the time they are made, and also during the remainder of 32's life if 40 shall have attained the age of 45.

Mr. Tucker's value is, in the notation I use, $_{\overline{5}|}a_{32}-{}_{\overline{5}|}a_{32,40}+a_{37}\cdot{}_{\overline{5}|}A_{40}$, $_{\overline{5}|}A_{40}$ denoting the present value of a short term assurance on 40; and I believe it will be found that this expression does not admit of an interpretation in accordance with the conditions of the problem.

The numerical solution is as follows:—

$N_{32,40}$ (Jones)	123087546			
$N_{37,45}$,,	86373791			
	36713755	Log.	7·5648289	
$D_{32,40}$	8600330	,,	6·9345151	
$_{\overline{5}|}a_{32,40}$	4·26889		0·6303138	
$_{5|}a_{32}$ (Thomson)	14·69049	,,	1·1670363	
$p_{40,5}$,,		,,	$\bar{1}$·9691496	Table V.
$_{\overline{5}|}a_{32}\cdot p_{40,5}$	13·68314	,,	1·1361859	
$_{\overline{5}|}a_{32,40}$ (above)	4·26889			
	17·95203			
a_{32} (Thomson)	19·13521			
	1·18318; or, for £100 annuity, £118. 6*s*. 4*d*.			

For the annual premium, divide the single premium by $\frac{N_{31,39}-N_{36,44}}{D_{32,40}}$, which gives ·26260, or, for £100, £26. 5*s*. 2*d*.

These values do not differ greatly from those given by Mr. Tucker; but they *might* have differed very much more, since it so happens that his three errors nearly neutralize each other. The problem, of course, admits of ready solution by the commutation method. The formula for this mode of solution is

$$\frac{100[(N_{32}l_{40}-N_{37}l_{45})v^8+N_{37,45}-N_{32,40}]}{N_{31,39}-N_{36,44}},$$

which gives the annual premium. The formula for the single premium has the same numerator as the above, and for denominator $D_{32,40}$.

I am, Sir,

Your most obedient Servant,

Aberdeen, 4th June, 1855. H. A. S.

Note.—We understood Mr. Tucker's communication as proposing merely an approximate method, convenient on the score of its conciseness, and sufficiently accurate for practical purposes. Our correspondent, nevertheless, does well to show that the exact method of solution is attended with not much more labour.—Ed. *A. M.*