DYNAMICAL CONSTRAINTS ON THE MASS FUNCTION

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1. Introduction

The aim of this paper is to summarise the dynamical constraints on the contribution of low luminosity stars. Important restrictions come from disk stability and the Oort Limit. These are reviewed elsewhere in these Proceedings in the contributions from Sellwood and Binney. Here, our attention is concentrated on two further pieces of evidence – constraints from (1) the stochastic heating of wide binaries and from (2) microlensing.

2. Wide Binaries

The physical principles underlying the constraint from the stochastic heating of wide binaries are easy to understand. Consider two stars in circular orbit with a combined mass of 1 $M_\odot$. Suppose they have a relative velocity of $\sim 0.1\,\mathrm{km\,s^{-1}}$. Perturbers encounter the binary at a speed some two orders of magnitude greater than this. As the binary gains energy, it expands and eventually it disrupts. The more perturbers, the fewer wide binaries. Therefore, the goal is to constrain the mass and the density of the unknown perturbers using the observed distribution of binaries and models of their evolution. This is a subject that starts with observational work by Bahcall & Soneira (1981) that appeared to detect a cut-off in the separation of wide binaries at about 0.1 pc. Retterer & King (1982) calculated that a binary with a semi-major axis of 0.1 pc will disrupt in roughly a Galactic age (10 Myrs). Bahcall, Hut & Tremaine (1985) then extended these arguments to include a population of dark matter perturbers of unknown mass. By fixing the density of the perturbers (at the value suggested by the kinematics of vertical tracer populations), they found the characteristic mass of the perturbers must be less than two solar masses. This work stimulated a major effort by Martin Weinberg and co-workers (1987, 1990), who provided a substantial theoretical framework with which to analyse the problem. They argued, first, that the observed break in the semi-major axes is consistent with being a sampling or selection effect, and, second, that no sharp cut-off is expected anyhow. As this spoilt the original claim, Weinberg (1987) then looked to see if a dynamical constraint on low mass objects could be restored somehow. Encounters are catastrophic if a single event disrupts the wide binary. Encounters are diffusive if many events are needed to disrupt the binary. Encounters with stars and low mass objects predominate and are typically diffusive, whereas encounters with Giant Molecular Clouds are catastrophic. Given that the dominant physical processes are diffusive, it is natural to treat the problem with a Fokker-Planck equation. Explicit forms for the diffusion coefficients are available, as are the Greens functions (Weinberg 1990). Given any assumed form for the binary birth-rate, the endpoint of the population can be determined. Of course, the binary birth-rate is not well-understood observationally, so it is not clear exactly what should be assumed for this unknown. However, a reasonable assumption is that the dependence is separable in energy and time, with a power-law dependence on energy (or equivalently semi-major axis). Weinberg actually examined models in which the binaries are created continuously and are created in bursts. The original idea of Bahcall, Hut & Tremaine (1985) was to use the shape of the distribution in semi-major axes – a sharp cut-off. However, Weinberg's analysis make it clear that this is untenable. Diffusion smoothes things out, so sharp cut-offs are not expected. The other possibility is a limit based on overall normalisation. For a particular model, the fraction of total number of binary pairs ever created may be computed from the binary birth rate. If we observe n_w

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wide binaries in a given volume of space containing n_c of stars in total, then we have the obvious limit that $n_c > 2n_w/f$. However, even here, it is sad to report that the best fit model does not give an interesting limit on the mass function of the dark objects. Weinberg (1990) found he could constrain the masses of perturbers in excess of twenty solar masses in some model forms for the binary birth rate, but not the low mass end of the mass function.

Given the recent resurgence of interest in careful astrometry, it is clearly worthwhile trying to build better catalogues of wide binaries. Whether the stochastic heating of wide binaries will ever give definite dynamical constraints on the mass function is less clear – the limitations seem partly to do with the scaling of the diffusion coefficients with the masses of the deflectors. Low mass deflectors are simply much less effective at heating than high mass ones.

3. Microlensing

Let me now pass to microlensing where the prospects are more upbeat! But first let me issue some cautionary words. Any inference concerning the mass function of deflectors depends on assumptions as regards the unknown distribution of deflectors along the line-of-sight and the unknown velocity distributions of the deflectors and sources. One way of gaining a better understanding of the uncertainties is to set up thought experiments. Suppose an observer and a source are separated by a single species of deflectors of known mass and of a known triaxial Gaussian velocity distribution with known dispersion. Suppose that all the microlensing observables – the optical depth, the rate and the histogram of events with respect to timescales – are known to arbitrary precision. Can the density of the deflectors be inferred uniquely? Another way to phrase this problem is, given a configuration that reproduces the observables, is it possible to change the density distribution along the line-of-sight without changing the observables? It is straightforward to show that densities which have no effect on all three of the microlensing observables – known as *microlensing konus densities* – do exist by expanding the quadratures as Fourier series. A microlensing konus density contains both positive and negative densities. It is useless in itself, but can be added to any model that reproduces the observables to generate further models that do so. In other words, it is a neat way to parametrise the degeneracy of the problem. So, the result of the first thought experiment is *the first microlensing theorem of ignorance.* Even given perfect accuracy, it is not possible to restrict the density of deflectors along the line-of-sight from microlensing data alone. Let us now pass to a second thought experiment. A microlensing group monitors the optical depth, rate and timescale histogram of a single species of deflectors of known density possessing a triaxial Gaussian distribution of velocities. The semi-axes of the velocity dispersion tensor are unknown. Can the microlensing group even in principle infer the mass of the species? This problem is also straightforward to solve - the rate and the timescale histogram depend on the unknowns not separately but only through the combination M/σ_{T}^2, where M is the characteristic mass and σ_{T}^2 is the square of the tangential velocity dispersion. Only this combination is knowable from the microlensing observables. To find the mass of the species is not possible, unless the tangential velocity dispersion is accurately known. So, any determination of the mass function of the deflectors must necessarily depend on uncertain assumptions as to the density distribution and the velocity distribution of the deflectors. The result of the second thought experiment is *the second microlensing theorem of ignorance.* It is not possible to infer the characteristic mass of deflectors from microlensing data alone. These theorems of ignorance tell us that only by combining the microlensing data-set with assumptions regarding the structure of the Milky Way can useful constraints on the density and mass of low luminosity stars be obtained.

There are two data-sets available thanks to the tenacity of the observers (e.g., Alcock et al. 1997a,b) . Let me first consider microlensing towards the Large Magellanic Cloud. This is less useful for the purposes of investigations into low luminosity stars, so I wish to deal with it very quickly. There is genuine uncertainty as to where the lenses lie along the line-of-sight. If we assume that the lenses are in the halo and have a roughly isotropic velocity dispersion, then we arrive at the conclusion that roughly a third or half of the halo is built of objects of around half a solar mass or so. But it is not clear that these assumptions are correct. In particular, the lenses may lie in the warped and flaring Milky Way disk (Evans et al. 1997) or in an intervening stellar population (Zaritsky & Lin 1997). Until the location of the lenses is known, it is not possible to come to any reliable conclusion regarding the masses of the lenses. The data-set towards the Galactic Centre is simpler to analyse, at least given a working model of the Galactic Bulge. There were two early analyses of the data-set of the nine OGLE events – one by Evans (1995) using an axisymmetric bulge and spheroid, and a second by Zhao, Spergel & Rich (1995) using a triaxial bar. Both came to the conclusion that mass functions dominated by brown dwarfs – such as Salpeter laws extrapolated down $0.01\,M_\odot$ – generate too many short timescale events to be consistent with the data-set. Han & Gould (1996) combined the OGLE data-set with the 45 events in the first year Macho data-set and produced an interesting discussion of the mass function of the deflectors. They used a double exponential disk, together with an analytic bulge model that is a reasonable approximation to the COBE infra-red light. The velocity distribution of disk objects is assumed to be Gaussian about the rotation velocity. The bulge is assumed to be non-rotating. The distribution of velocities is a triaxial Gaussian with components deduced from the flattening via tensor virial theorem. Han & Gould considered two possible mass functions for the deflectors The first is a Gaussian in the logarithm of the mass, with some unknown mean and unknown dispersion in logarithmic mass. The second is a power-law mass function of unknown index which is assumed to be valid down to some unknown lower cut-off. For each model, Han & Gould found the best fits to the unknowns by performing a maximum likelihood test. In the best fitting power-law, the mass function goes like mass$^{-2.1}$ down to a lower mass limit of $0.04\,M_\odot$. This is of course very close to a Salpeter mass function. The best-fitting Gaussian mass function is centered around $0.1\,M_\odot$ and has a broadish width, extending well into the brown dwarf régime. It seems that the lenses must have a mass function that is different to that of the local disk stars found by Kroupa, Tout & Gilmore (1993). It also seems from Han & Gould's analysis that a significant fraction of the events are caused by sub-stellar masses. This is rather surprising given the earlier analysis by Evans (1995) and by Zhao (1995) on the admittedly much smaller OGLE data set. One possible explanation is the neglect of over-all bulge rotation by Han & Gould. This causes the transverse velocity of the lenses to be reduced, and hence given events are produced by lower masses. Han & Gould make one further interesting point – there is a tail of long timescale events that are poorly fit by all the models. This led Han & Gould to speculate that they might be caused by a kinematically cold population with a low scale height. These would be observed near the Sun for sources near Baade's Window. Such a population would have a low transverse speed and so cause very long timescale events.

4. Conclusions

Microlensing is a more powerful method than stochastic heating of wide binaries for constraining the contribution of low luminosity stars. Han & Gould's (1996) nice analysis points the way forward. It would be very interesting to repeat it with the larger data-set soon to become available and with state-of-the-art models of the Galactic Bulge. The conclusion reached on the mass function clearly depend on the model adopted – and, in particular, on the velocities in the model. As readers of the Proceedings of Joint Discussion 15 will find out, such Bulge models will soon be available thanks to the work of my Oxford colleagues.

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