

CELEBRATING 50 YEARS OF THE APPLIED PROBABILITY TRUST

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Part 1. Historical reflections

APPLIED PROBABILITY BEFORE 1964, AND AFTER 2014

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Abstract

This paper is an edited version of a talk given in Sheffield as part of the celebration of fifty years of the Applied Probability Trust and its journals. I sought to sketch the background to the Trust's formation in the development of different applications of probability during the previous century, and to draw lessons for the future of the discipline and therefore of its journals.

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1. The emergence of applied probability

Applied probability awaits its historian. There have been many studies of the history of probability (and statistics), but few of these relate to what we now call applied probability. As a striking exception, the history of the Bienaymé–Galton–Watson branching process by Kendall [15] and Seneta and Heyde [12] stands virtually alone. Yet the late nineteenth and early twentieth centuries saw many different uses of probabilistic ideas in pure and applied sciences, of which important examples are the work of Galton [10] on natural inheritance, of Bachelier [1] and later Einstein and Smoluchowski on Brownian motion, of Erlang [4] on congestion in teletraffic systems, of Campbell [5] on shot noise, of Rutherford on radioactive decay, of Lotka on many areas of quantitative biology, and (if pure mathematics be regarded as a science) of Dickman [6] on large prime factors.

It is clear that these were, to a large extent, independent of one another. Their authors knew of the classical work of Fermat, Pascal, and Laplace, since they used the language of probability and independence, and the normal distribution of de Moivre and Gauss was familiar to some from the use of least squares methods in astronomy, but the Poisson distribution was often rediscovered, and there seems to have been little interplay between different applications before about 1930.

In 1928 Thornton C. Fry, an engineer at the Bell Telephone Laboratories, published a book on *Probability and Its Engineering Uses* [9]. Almost all of his account was about different probability distributions and curve fitting, crude data analysis, but in Chapter X he described the work of Erlang and others on queues, and Chapter XI is about the kinetic theory of gases.

How then did almost unrelated developments come together in an interacting mixture of ideas? One possibility is that it was a result of the exodus of scholars fleeing continental Europe with the rise of the Nazis, and that it then accelerated during the Second World War because diverse groups of scientists were brought together to address technical problems of urgent importance. Indeed, there were in Britain alone several high-powered groups, famously at Bletchley Park [13], but also in the Admiralty where OR originated, and in the rocket propulsion

group in Wales which included Maurice Bartlett and David Kendall [3]. But wartime security meant that there was little if any communication between the different groups.

After the war many of the young scientists found jobs in universities, and began to interact through learned societies such as the Royal Statistical Society (RSS) and exchange of seminars. But what was perhaps even more significant was the publication of a number of remarkable books. Paul Lévy's 1948 treatise [17] on Brownian motion showed how central the Wiener process was to probability theory, and laid the foundations for stochastic calculus. The first volume of Feller's introduction [8] in 1950 produced a remarkable array of problems that could be addressed with simple discrete probability. Doob's majestic *Stochastic Processes* [7] made it possible rigorously to analyse processes in continuous time, and demonstrated the power of the martingale concept in pure and applied probability.

It was however a more modest book that may have proved decisive. Maurice Bartlett had visited the University of North Carolina shortly after the end of the war, and developed the notes of his lectures there into his 1955 book on stochastic processes [2]. He ranged (more systematically than Feller) over many different applications, and described methods for analysing them in a more applied spirit than other authors. His book showed very clearly that diverse applications depended on a few key ideas, such as the Markov property, so that techniques developed for one application could often be applied more widely.

In the 1950s there was little separation between the construction and analysis of stochastic models on the one hand, and statistical methodology and data analysis on the other. Lively statistics groups grew up in the USA, the UK, and Australia, which typically worked in both statistics and applied probability as aspects of a single discipline. But a natural process of specialisation led in the 1960s to a rising consciousness of both the beauty and the usefulness of probability theory, and to a feeling of community among probabilists.

It was not all good. There was in some areas a retreat into a pure mathematical view of probability, with a level of abstraction which distanced the subject from its applications. Bartlett [2] expressed it succinctly:

As a statistician I find it at times exasperating when the mathematics of stochastic processes tends to become so abstract; time spent wrestling with it can hardly be spared . . .

But he went on with characteristic honesty and insight:

. . . unless, as of course mathematics is best fitted to do, it deepens one's perception of the over-all theoretical picture in the probabilistic and statistical sense.

There was another danger, which was explained trenchantly to me in 1961 by David Kendall when he became my research supervisor and I told him of my attempts to work on queues. He felt that, following his 1951 paper [14], too many had jumped on the Erlang bandwagon by producing solutions of unusable complexity to problems of little applied interest. (An excellent example can be found in [16].) There was therefore a need for rigorous peer review such as can be provided by specialist journals with knowledgeable and discerning editors.

2. Applied probability publications

Cometh the hour; cometh the man. Joe Gani described in [11] how, after an adventurous early life, he found himself in Pat Moran's lively research group in Canberra, how he became convinced of the need for a journal dedicated to applied probability, how he gained support both in the USA and the UK, how Kendall smoothed the path to financial help from the London Mathematical Society, and how both the Applied Probability Trust (APT) and the *Journal of*

Applied Probability (JAP) were founded in 1964. The new journal was an immediate success, attracting a strong group of editors and advisers, and a large number of fine papers. It raised a flag for applied probability, and gave a focus to a community which had previously lacked a clear identity.

This early success was no ephemeral phenomenon. In its first half century, JAP and its younger sibling *Advances in Applied Probability* (AAP) have established themselves as major journals on the world stage. Under successive editors, they have maintained a high scientific standard and a distinctive flavour, and by publishing papers from many different countries have encouraged international scientific collaboration. Imitation is the sincerest form of flattery, and even the august Institute of Mathematical Statistics set up the *Annals of Applied Probability* in 1991; the RSS has recently formed an Applied Probability Section.

The history of these fifty years is not part of my remit. If you seek a monument, look around you at the volumes of JAP and AAP, at the many seminal papers they contain. But one remark is in order. For the whole of this period, its founder Joe Gani has been involved, formally as the first Editor-in-Chief, and later as an informal influence. It must be almost unique for a single scientist to be closely concerned with a journal for such a long period, and this must have contributed to the consistency of JAP and AAP to the present day. It is very satisfying that he is able to share in the celebration of this golden jubilee.

3. Quo vadis?

But what of the future? A question mark hangs over scientific publication as a whole, as scientists increasingly use electronic media rather than print on paper. What this change has shown is that the real value-added of a journal lies in the refereeing and editing of scientific output, and ways must be found to preserve this in a new era. Established journals cannot assume that they can survive on past achievements, and sharp questions will be asked about relevance to the advance of science.

In thinking about the future place of the APT, we need therefore to ask the obvious question: what is applied probability and why is it important? Joe Gani was always rather reluctant to commit to a definition, but in [11] he used the phrase ‘the application of probability methods to real-life problems’. That however is too wide to be useful; it encompasses the whole of statistics and much of OR, and nowadays vast tracts of pure and applied sciences, not to mention disastrous activities in the financial world.

To focus on the essence of applied probability, one approach might be to note that probability theory is a branch of mathematics, so applied probability is a branch of applied mathematics. Thus, a definition might be: *applied probability is that part of applied mathematics which is concerned with probability*. Of course, that is only useful if we know what applied mathematics is, but applied mathematics is an older discipline of which a more considered view can be taken.

Although applied mathematics goes back to Newton, the explicit use of the phrase seems to be much later. The first ‘public’ use may have been in 1826, when Crelle founded his great journal, the first devoted specifically to mathematics, in Berlin. Although this is always known as *Crelle’s Journal*, its proper title is *Journal für die Reine und Angewandte Mathematik*. Ten years later, Liouville started a similar journal in Paris, and gave it a title which is almost an exact translation of Crelle’s: *Journal de Mathématiques Pures et Appliquées*.

The interesting thing about these names is that they bracket pure and applied mathematics. Crelle and Liouville are not creating separate journals for pure and applied work, they are recognising that mathematics is done both for its own sake and in order to be used in real-life problems. Some mathematicians are motivated by the intrinsic elegance of mathematical

concepts and by the deep challenges they offer, while others look to mathematics to understand the world and to solve pressing problems of everyday life. Many mathematicians do both, and some of the greatest defy classification.

William Morris [18] once said that we should have nothing in our houses that we do not know to be useful, or believe to be beautiful. In mathematics usefulness may not become apparent for many years, while beauty seems to be rather readily recognisable. Thus we should invert Morris's dictum, and argue that we should have nothing in our mathematics which we do not know to be beautiful, or believe to be, or be likely to become, useful.

Applied mathematics is then mathematics where the emphasis is on actual or potential use in real-life problems. It is not the application of that mathematics, which lies within physics or biology or engineering or economics or whatever, but the mathematics which practitioners of those fields can use. Its great power lies in the ability to abstract the essence of the application so that either existing mathematics can be brought to bear, or new mathematics can be developed for the purpose.

The record of four hundred years shows that particular bits of mathematics prove relevant to apparently diverse applications. Thus, for instance, Laplace's equation or Fourier transforms crop up in many different contexts. Even if the mathematician cannot completely solve a differential equation, he/she can point to isomorphisms between different fields, so that insight in one can transfer to another.

Until well into the twentieth century (and even now in some backward universities) applied mathematics was concerned with deterministic situations, but all that has been said above is equally valid when probability enters. Probability theory is an exciting branch of mathematics that can well be studied for its own sake, but it is nourished by the challenges that arise from applied fields as different as population genetics and teletraffic engineering. Applied probability links the pure theory with the different applications, and links one application with another, to mutual benefit.

As time goes on, concepts and methods from probability theory become commonplace in applications where their relevance has been accepted. But the advance of science and technology throws up new challenges. For example, genomics poses difficult problems for the probabilist, and the Internet is like a gigantic queueing network. There will always be a need to develop new mathematics, and to recognise novel applicability of old mathematics. And whatever the new era in scientific publication brings, there will be a need for rigorous and perceptive peer review and editorial judgement. So long live the Applied Probability Trust, and may it long maintain the high standards which it has set over the past fifty years.

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