Multiple Periodic Variable Stars
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MULTIPLE PERIODICITIES IN CATACLYSMIC VARIABLE AND WHITE DWARF STARS

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1. Dwarf Novae

The normal dwarf nova (DN) outburst is characterized by a rapid rise, a brief stay near maximum and a slower decline to minimum light; the whole sequence lasts in general for 4 to 6 days, but can take as long as 10 or 12 days in some stars. While not strictly periodic, the DN outbursts are quasi-periodic in the sense that the mean periods derived from intervals sufficient to contain at least 50 outbursts (i.e. a few years for the shortest period systems, up to 30 years for longer period systems) remain sensibly constant. It should be added, however, that in the case of U Gem there is evidence for an increase in its recurrence period which may indicate some evolutionary change: for the first 50 years of observation of U Gem the mean period was 96.5 days and for the second 50 years it was 107.6 days (Mayall, 1957); during the past four years the mean interval has been 291 days (with only one outburst in 1974).

The shortest period systems - those with normal recurrence periods $P_n \lesssim 30$ days - can almost all be divided into two subgroups. These are the well-known Z Cam subgroup in which standstills occur at apparently random intervals, and a subgroup which we propose to call the SU UMa stars which show supermaxima. In supermaxima, the star rises initially to a peak brightness about 0.5 magnitude brighter than the normal maximum, and then (often with large fluctuations) hovers around normal maximum brightness for between 10 and 20 days (corresponding to factors $\sim$ 4 times longer than the duration of normal maxima) before declining rapidly to minimum brightness.

We suspect that those stars with $P_n < 30$ days that are not yet known to fall into either the Z Cam or SU UMa subgroups are poorly observed and that further studies may well result in all such objects being classifiable into the two subgroups. The Z Cam or SU UMa characteristic occasionally occurs in stars with $P_n > 30$ days; for example BS Cep is a Z Cam star with $P_n \sim 40$ days and Z Cha is a SU UMa star with $P_n \sim 96$ days.

Bateson (1975a) has pointed out that the shorter the mean interval between supermaxima, the more regular and predictable they become. For these regular supermaxima we can therefore define a "super-period" P_s, and we can classify such stars as multiple quasi-periodic systems. Information on SU UMa stars (mostly drawn from Circulars of the Variable Star Section of the Royal Astronomical Society of New Zealand and from the Variable Star Section reports appearing in the Journal of the British Astronomical Association, plus private correspondence with the present Directors: F. M. Bateson and J. E. Isles) is given in Table 1.

Table 1

The SU UMa Stars

Star	P_n	Supermaxima
SU UMa	13	P_s = 180 days
AY Lyr	31	P_s = 200 days
VW Hyi	28	P_s = 180 days
WX Hyi	29	P_s = 165 days
Z Cha	96	P_s = 313 days
V436 Cen	25:	Random, mean interval $\sim$ 630 days
AT Ara	70	Random, mean interval $\sim$ 500 days
CU Vel	139	Scattered
OY Car	188	$P_s \sim$ 300 days

There are four stars with $P_s = 180 \pm 20$ days and these have the shortest periods and most regular behaviour. Bateson (1975b) finds that $P_s = 179.6 \pm 12.2$ days for VW Hyi, which means that supermaxima are predictable to at least the same accuracy as the most regular Mira variables. In fact, the scatter may be even less than that quoted: there is some ambiguity as to when maximum brightness occurs in the supermaxima – they stay bright for many days and can have large fluctuations. Timings made on the rising branch (e.g. passing through a particular magnitude) should be investigated to see if they improve the predictability. However, some supermaxima (in VW Hyi) are known to occur soon after a normal outburst, or even while one is in progress. This confusion must be eliminated before timings of the rising branch are analysed.

A statistical analysis of possible relationships between normal and super-maxima needs to be made – preferably in several objects. The occasional occurrence of supermaxima that begin while a normal maximum is underway suggests that the supermaxima may be triggered by normal outbursts. At the moment we have no idea of the cause of the multiple periodicity in the SU UMa stars. It has been shown (Warner 1975) that the time-averaged energy radiated in the visible during the normal outburst cycle and the supermaximum cycle is roughly the same implying that the same energy source is being tapped with the same efficiency in the two cases. The light curves should be analysed to see if a supermaximum has any significant effect on subsequent normal maxima; if this is the case then this would afford more evidence that the two periodicities are modulations of the same energy-generating process.

At maxima of VW Hyi, the colours are B-V = -0.02, U-B = - 0.75 (Marino and Walker 1973, Eggen 1968) and during supermaximum they are B-V = -0.08, U-B = -0.75 (Vogt 1975a). Spectra taken during a supermaximum of VW Hyi (Vogt 1975b) are similar to those obtained during normal maxima of other systems (Joy 1960). There do not, therefore, seem to be any large differences in the physical properties of dwarf novae at normal and supermaxima.

Of the stars listed in Table 1, only three have orbital periods determined; these are VW Hyi (P = 107 mins), Z Cha (P = 107 mins) and V436 Cen (P = 92 mins). It may be significant that these three stars are ultra-short period binaries: this may be necessary for the SU UMa phenomenon. None of the Z Cam stars is known to have such short orbital periods; those known are EM Cyg ($P = 7^h\,00^m$), Z Cam ($P = 6^h 56^m$), RX And ($P = 5^h 05^m$), WW Cet ($P = 3^h 50^m$) and CN Ori ($P \sim 6^h$). Furthermore, the other ultra-short period systems all have peculiarities: WZ Sge (P = 82 mins) is a dwarf nova with a recurrence period of 33 years, EX Hya (P = 99 mins) has outbursts every $\sim$465 days but many more frequent low amplitude outbursts (the larger outbursts associated with the 465 day timescale do not have the characteristics of supermaxima but nevertheless may be loosely related) and VV Pup (P = 100 mins) which has never been seen to outburst but may be like WZ Sge with a much longer recurrence timescale.

2. Rapid Oscillations in Cataclysmic Variable Stars

The rapid oscillations of brightness discovered in cataclysmic variable stars, especially in dwarf novae during outbursts (Warner and Robinson 1972a), have been seen in a total of fourteen stars (Warner 1975). Among these, some evidence for multiple periodicities exists.

Firstly, however, it should be stated that the original claim for the existence of multiple discreet periodicities in dwarf novae and for transitions between these, has largely been discounted by the more detailed analysis of Warner and Brickhill (1975). Apparent multiple periodicities can arise in periodogram analyses if there is amplitude modulation of the periodic signal, and the characteristic timescale of the modulation is comparable with the data length used for computation of the periodograms. Under these circumstances, large side lobes can appear to the principal peak of a periodogram and these give the impression of multiple periodicities. As a result, the various apparent multiplicity of periodicities that occur in some of the periodograms of the cataclysmic variables must be treated with reservation.

Several types of periodic modulation of the rapid oscillations in dwarf novae are, however, present. The orbital period is seen to modulate the rapid oscillations in VW Hyi and V436 Cen (Warner and Brickhill 1975) with a total period range $\sim$5 percent. A smaller modulation, displayed as a phase shift around the orbit (excluding the region near eclipses) is seen in DQ Her (Warner et al 1972) and UX UMa (Nather and Robinson 1974). Still unexplained, these modulations place some restrictions on what mechanisms may be responsible for the rapid oscillations.

Longer term modulations of the rapid oscillations may exist. The oscillations seen in UX UMa (Warner and Nather 1972a, Nather and Robinson 1974) have shown a range of 28.55 to 30.03 secs, which may be cyclical with a period of weeks. Observations of CD $-42^{\circ}14462$ (Warner 1973, Hesser et al 1974, Warner unpublished) range from 29.1 to 32.0 secs and could be showing the same phenomenon as in UX UMa. Whether these longer term variations will be explicable in terms of orbital motion due to the presence of a third body remains to be seen.

A 413 sec modulation of the 30 sec oscillations present during a normal maximum of VW Hyi has been found (Warner and Brickhill 1975). This period is of the right order of magnitude to be attributed to rotation of an obscuring cloud near the outer edge of the accretion disc that surrounds the white dwarf primary in this system. It could also be identified with oscillatory motion of the disc as a whole, periodically obscuring the oscillating source near the centre of the disc.

As can be seen, the cataclysmic variables possess a multitude of periodic phenomena. To summarise the properties of just two: UX UMa shows a 29 year variation of its 4^h43^m orbital period (Nather and Robinson 1974) and has an orbital modulation and a possible modulation with a period of weeks of its $\sim$30 sec oscillations. VW Hyi shows outburst quasi periodicities on timescales of 180 days and 28 days, has a 1^h47^m orbital period which can modulate its $\sim$30 sec oscillations and has also on one occasion shown a 413 sec periodicity.

3. Variable White Dwarfs

Generalities

The first of the variable white dwarfs, HL Tau-76 (V411 Tau) was discovered in 1964 by Landolt (1968) while conducting a general photometric program. At about the same time, a special search was started by the Princeton group using power spectrum techniques to look for coherent periodicities in white dwarfs in the period range 1 - 1000 secs. This initial survey was not fruitful which served to show that variability is a rare occurrence among white dwarfs picked at random (Lawrence et al 1967, Hesser et al 1969). Extension of this survey by observations made at Cerro Tololo resulted in the discovery of two low-amplitude white dwarfs out of a total of 23 observed; a summary of this program has been given by Hesser and Lasker (1972). The two new variables, G44-32 (CY Leo; Lasker and Hesser 1969) and R548 (ZZ Cet; Lasker and Hesser 1971) have amplitudes of only $\sim$0.01 - 0.02 mag, whereas HL Tau-76 has $\sim$0.3 mag variations. Smak (1967) discovered variability in HZ 29 (AM CVn), a star with a spectrum resembling a DB white dwarf, but this object has subsequently been considered to be an ultra-short period binary (Warner and Robinson 1972b).

During the past two years, discoveries of variable white dwarfs have increased dramatically. Partly this is due to the larger number of groups searching for variability, but it is more a consequence of noting (Lasker and Hesser 1971) that HL Tau-76, G44-32 and R548 lie close together in the two-colour plane near B-V = 0.25, U-B = -0.55, with the resulting concentration on stars in that region. Richer and Ulrych (1974) claimed variability in G117-B15A and G169-34, confirmation of variability in the former has been given but G169-34 appears to have constant brightness (McGraw and Robinson 1975a). Schulov and Kopatskaya (1973) discovered that G29-38 is variable, which has been confirmed and a further star, G38-29, added by McGraw and Robinson (1975b). Three further objects, GD99, R808 and G207-9 have recently been found (McGraw and Robinson 1975 a, c). We therefore have a total of nine known variable white dwarfs; these are listed in Table 2. It is a significant fact that all of these are multi-periodic.

The first four white dwarf variables (V411 Tau, ZZ Cet, CY Leo and AM CVn) have been given the new classification of ZZ Cet stars in the Second Supplement to the General Catalogue of Variable Stars. To judge from the amplitudes of variability listed in Table 2, there may be two distinct types, with ZZ Cet unfortunately representing the minority group. We propose a more general classification, the DV stars, which (d.v.) will suffice to cover all possible classes of variable white dwarfs and allow of subgroups such as ZZ Cet or V411 Tau stars.

Table 2

DV Stars

EG	Name	Spectrum	B-V	U-B	Amplitude (mags)	Periods (secs)
10	R548	DA	0.20	-0.54	0.01	212.86, 273.0
34	G38-29	DAs	0.16	-0.53	0.20	925, 1022
65	G117-B15A	DA	0.20	-0.56	0.08	216, 272, 308
72	G44-32	DC	0.29	-0.58	0.02	1638, 822, 600
115	R808	DA	0.17	-0.56	0.15	833 + others
127	G207-9	DAn	0.17	-0.60		
159	G29-38	DA	0.20	-0.65	0.23	613, 671, 816, 930 1000
219	GD99	DA	0.19	-0.59	0.10	229 + others
265	HL Tau-76	DA	0.20	-0.50	0.30	746, 494

Individual Stars

R548 (ZZ Cet) Lasker and Hesser (1971) found two nearly sinusoidal variations with periods of 213 and 273 secs, whose relative amplitudes vary on a time scale of hours, both being up to $0^m.01$. Hesser and Lasker (1972) have additional photometry suggesting that at least the 213 sec oscillation is probably coherent on a long timescale. More extensive observations are required on this star.

G44-32 (CY Leo) This star also has low and variable amplitude oscillations (Lasker and Hesser 1969). Two periods and a probable harmonic are present.

HL Tau-76 (V411 Tau) Landolt's (1968) discovery observations were made in poor photometric conditions but a periodogram analysis showed the presence of a periodicity near 12.5 mins. The photometry by Warner and Nather (1972b) clarified the nature of the light curve. The brightness variations have a range up to $0^m.3$, are highly non-sinusoidal and exhibit a large range of pulse shapes. Further observations were made by Fitch (1973). Power spectrum analyses of these observations (Warner and Robinson 1972a; Page 1972; Fitch 1973) show a fairly stable spectrum of oscillation components including a rich series of harmonics and cross components of the principal frequencies. We omit further discussion as further details may be found in the contribution by Desikachavy and Tomazewski later in this Colloquium.

G29-38 Observations by McGraw and Robinson (1975b) have extended those of the discoverers (Shulov and Kopatskaya 1973). The light curve is strikingly similar to that of HL Tau-76. The power spectrum shows a qualitatively similar pattern from night to night, but there are small variations in the peaks and their relative amplitudes which can occur on a timescale of hours. There is a nearly equal spacing of the major spectral peaks in the frequency plane.

G38-29 This star also has a light curve like that of HL Tau-76 (McGraw and Robinson 1975b). Power spectra of different runs again show some instability in the periods and relative amplitudes.

G117-B15A Richer and Ulrych (1974) obtained rather poor-quality observational data on this star and an analysis by the Maximum Entropy Method indicated a period of 1311 secs with an amplitude of 0.01 mag. However, observations by McGraw and Robinson (1975a) show G117-B15A to have an almost sinusoidal light curve with period 216 secs and amplitude 0.08 mag. Two smaller peaks in the power spectra are found at 272 and 308 secs with amplitudes ~ 0.001 mag. Power spectra of different runs are almost identical.

R808 With a light curve similar to that of HL Tau-76, R808 shows complex structure in its power spectrum, with many peaks in the range 770-1250 secs (McGraw and Robinson 1975a). The power spectra are unstable.

GD99 This star has a very irregular light curve and at times may be quiescent for intervals of up to an hour (McGraw and Robinson 1975a). Only one peak, at 229 secs, in the power spectrum remains constant in frequency; other peaks vary in frequency and amplitude.

G207-9 Recently discovered by Robinson (McGraw and Robinson 1975c) this object is a low-amplitude variable.

Discussion

Several features of significance to the interpretation of the variability in white dwarf stars emerge from these studies. We note that, with the exception of G44-32 (which is faint and for which further spectra should be obtained) all DV stars are of spectral type DA. From their colours, they lie near the cool end of the observed DA sequence. The range of colours is small; although this may be affected by observational selection, a survey of white dwarfs lying outside the two-colour limits of the stars listed in Table 2 has failed to find any variables (Hesser and Lasker 1972, McGraw and Robinson 1975c). Not all of the stars lying within the range of colours of the DV stars are variable. This may be partly due to photometric errors scattering some non variables into the area of the variables, but it may also imply differences in internal structure of the DV and non-variable stars. The region of the two-colour diagram that the DV stars occupy, when transformed to the M_{bol} - log T_e diagram with the aid of model atmosphere calibrations, lies in the same region as that where calculated white dwarf evolutionary sequences (and DA stars with known distances) pass through an extension of the Cepheid instability strip (Vauclair 1971).

The low amplitude ($\Delta m < 0.10$ mag) DV stars possess sinusoidal light variations with constant periods. The larger amplitude systems have highly non-sinusoidal variations, with cross-terms in their frequency spectra, and have unstable periods. The long periods of the DV stars excludes radial oscillations and suggests non-radial modes of oscillation (Brickhill 1975). The presence of many closely-spaced eigenvalues supports this contention. Instability of the eigenfrequencies may be a result of interaction between the g-mode oscillations and the convective layer: the g-mode eigenfunctions corresponding to pulsations having periods of hundreds of seconds will have large amplitude in the convective region of the envelope.

Further theoretical work is clearly required. Some simplification may be offered from the fact that the DV stars have DA atmospheres which are probably the best understood. However, for a full understanding of the light curves it will probably be necessary to develop the non-linear, non-adiabatic, non-radial theory, with additional complications such as interaction with convection. In the meantime, extensions of the survey for DV stars, especially to the southern hemisphere, should result in many further discoveries, the systematics of which may provide some further clues to the nature of the variability.

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Discussion to the paper of WARNER

COX: I had thought that the case of DQ Her was settled in favor of rotation by Kemp and coworkers. Yet you still say the 71 second variation is pulsation rather than a 142 second rotation. Could you comment?

WARNER: Power spectra analysis of DQ Her by Nather and Kiplinger show no power at 142 second period. This throws some doubt on the various claims for a 142 second variation in circular polarization of DQ Her, on which the conclusion that rotation is favored was based. This does not, of course, exclude rotation as a still possible mechanism.

BATESON: Regarding Warner's reference to my work, I would like to add some comments on observations in New Zealand. In 1954, we commenced to observe U Gem stars in the southern sky. With a wide spread of observers in latitude (S5° to S46°) and a very wide spread in longitude in southern countries, the best observed have resulted in such stars being covered on most nights each year. Two decades of observations, much already published, now enable a discussion of the gross behavior of such stars to be published, and these will be in a series of memoirs, the first due late in 1975. In 1970, photoelectric observations were commenced at the Auckland Observatory both during outbursts of U Gem stars and during minima. Now three groups have photoelectric equipment. In 1974 at Carter Observatory a program commenced of observation of U Gem stars fainter than magnitude 14 at maximum, using photographic photometry. This will result in finding and detailed charts for these faint members as part of the Chart Series of the V.S.S., R.A.S.N.Z. Charts for most of the brighter stars have already been published. Observational results appear in the Publications, V.S.S., R.A.S.N.Z. During the course of the present meeting, Janet Mattei and myself have agreed on a cooperation program to avoid duplication of observations between the A.A.V.S.O. and V.S.S., R.A.S.N.Z.

BATESON: Were your observations of RR Pic of the shell or of the remnant?

WARNER: The remnant.

MATTEI: Gordon in 1950, Walker and others later have reported irregular variations within seconds, minutes, and hours, during minima of SS Cyg. Would you comment if these irregular variations are of the same nature as those variations you have observed?

WARNER: The photometric observations of those workers, and unpublished observations that I have made, show that in SS Cyg there is rapid flickering activity. Our analyses show that this flickering is nonperiodic, i.e., it is random or stochastic. We have not, however, observed SS Cyg during an outburst.

WOLFF: You indicated that the variable white dwarfs occupy a restricted range in colors. If you observe a sample of DA white dwarfs with these colors, what fraction have detactable variations?

WARNER: One third.

BATH: Regarding Cox's question, a rotation model of DQ Her 71 second oscillation does not necessarily lead to significant power in a periodogram analysis at 142 seconds. Even a non-sinusoidal oscillation could give negligible power at the 142 second period, but spread the power over <u>many</u> higher harmonics. If it is indeed a pure sine wave, then that is no argument at all in favor of pulsation as opposed to rotation as an explanation.

With reference to Dr. Cox's point, there has been frequent use of the word pulsation in the discussions, to describe periodicities observed in many systems. To a theoretician, pulsation is a heavily loaded word. Could I plead that observers do not use the word pulsation to describe observed phenomena that may have nothing to do with stellar pulsation.

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