XV

The Higgs boson

On July 4, 2012, the LHC collaborations ATLAS and CMS announced the discovery of a resonance which, despite limited statistics, seemed to have characteristics expected of a Standard Model Higgs boson. Mass determinations presented at the 2013 Lepton-Photon Conference are

$$M_H(\text{GeV}) = \begin{cases} 125.5 \pm 0.2\ (\text{stat})\ {}^{+0.5}_{-0.6}(\text{syst}) & [\text{Ja (ATLAS collab.) 13}] \\ 125.7 \pm 0.4 & [\text{De (CMS collab.) 13}]. \end{cases} \tag{1.1}$$

Since this resonance has a nonzero branching fraction for decay into two photons, it must be a boson, one not having spin-one. In fact, current spin/parity analyses are compatible with $J^P = 0^+$ but not with $J^P = 0^-, 1^+, 1^-, 2^+$ [Aa *et al.* (ATLAS collab.) 13*b*], [Ch *et al.* (CMS Collab.) 13]. Its couplings to bosons and fermions appear to be consistent with Standard Model expectations, in particular that the Higgs should couple to mass. At present, the overall precision is limited to about 25%, so an extended period of careful study will be necessary to reveal the anomalous properties, if any, of this particle. In this chapter, we will consider the basics of the Standard Model Higgs, with the intent of describing its phenomenology and also addressing certain theoretical issues.

XV–1 Introduction

A central feature of the Standard Model is the spontaneous symmetry breaking in the electroweak sector which gives mass to fermions and to the $W^\pm$ and Z^0 gauge bosons. Although a complex doublet of Higgs fields is initially introduced in the Weinberg–Salam model, there remains following spontaneous symmetry breaking precisely one *physical* Higgs state, a neutral scalar particle H^0. That is, if we define the number of degrees of freedom for Higgs and gauge-boson states, respectively, as N_H and N_G, then before the symmetry breaking we have $N_H = 4$, $N_G = 8$

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whereas afterwards we find $N_H = 1$, $N_G = 11$. To obtain these values, recall that massive vector particles have three spin components whereas massless vector particles have just two. Although the total of Higgs and gauge-boson degrees of freedom remains fixed ($N_H + N_G = 12$), there is a transfer of three states from the Higgs sector to the gauge-boson sector. These Higgs states become the longitudinal spin modes of the $W^{\pm}$, Z^0 particles.

This transfer can be displayed analytically by first performing a contact transformation to cast the two complex Higgs states φ^0, φ^+ in terms of four real fields H^0 and $\boldsymbol{\chi} = \{\chi_i\}$ $(i = 1, 2, 3)$

$$\Phi = U^{-1}(\boldsymbol{\chi}) \begin{pmatrix} 0 \\ (v + H^0)/\sqrt{2} \end{pmatrix}, \tag{1.2}$$

where

$$U(\boldsymbol{\chi}) = \exp(i\boldsymbol{\chi} \cdot \boldsymbol{\tau}/v), \tag{1.3}$$

and we recall that $v = 1/\sqrt{2^{1/2} G_F} \simeq 246$ GeV. One completes the procedure with the gauge transformation,

$$\begin{aligned} \Phi' &= U(\boldsymbol{\chi})\Phi = \begin{pmatrix} 0 \\ (v + H^0)/\sqrt{2} \end{pmatrix}, \\ \psi_L' &= U(\boldsymbol{\chi})\psi_L, \quad \psi_R' = \psi_R, \quad B_\mu' = B_\mu, \\ \frac{\boldsymbol{\tau}}{2} \cdot \mathbf{W}_\mu' &= U(\boldsymbol{\chi})\frac{\boldsymbol{\tau}}{2} \cdot \mathbf{W}_\mu U^{-1}(\boldsymbol{\chi}) + i g_2^{-1} \partial_\mu U(\boldsymbol{\chi}) \cdot U^{-1}(\boldsymbol{\chi}), \end{aligned} \tag{1.4}$$

for all fermion weak isodoublets ψ_L and weak isosinglets ψ_R. Within this unitary gauge, the physical content of the theory is manifest, and the quantity Φ' is seen to contain *a single Higgs field* H^0.[1] In the following, we shall employ this gauge but with the primes in Eq. (1.4) suppressed.

XV–2 Mass and couplings of the Higgs boson

We have already specified in Chap. II how the Higgs boson H fits into the Standard Model. The various lagrangians written down there provide the basis for a complete phenomenological portrait to be drawn for the H boson. In this section, and the ones to follow, we present the theory for this program.

[1] For notational simplicity, we shall hereafter omit the superscript ‘0’ and denote the Higgs field simply as H.

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Higgs mass term

Consider first the Higgs potential of Eq. (II–3.19) which, when expressed in terms of the field H, becomes

$$V = -\frac{\mu^2 v^2}{4} + \mu^2 H_0^2 + \lambda v H_0^3 + \frac{\lambda}{4} H_0^4, \tag{2.1}$$

where the parameters μ, λ are *a priori* unknown. The term quadratic in the Higgs field determines the Higgs mass to be

$$M_H = \sqrt{2}\,\mu = v\sqrt{2\lambda}. \tag{2.2}$$

This does not provide a numerical value for the Higgs mass M_H because only the quantity v, but not λ, is phenomenologically determined.

This fact places the burden of determining the Higgs mass on experiment. We will interpret the LHC finding of an unstable boson as indeed the Standard Model Higgs boson and for definiteness adopt the value

$$M_H = (126.0 \pm 0.5)\ \text{GeV} \tag{2.3}$$

for subsequent discussion. If so, the remaining parameters in Eq. (2.1) become

$$\mu = 89.1 \pm 0.3\ \text{GeV} \qquad \text{and} \qquad \lambda = 0.131 \pm 0.001. \tag{2.4}$$

The naturalness problem

Radiative corrections to the Higgs mass raise a question of the 'naturalness' of the Standard Model. To motivate the discussion, let us first consider one-loop electromagnetic corrections to the electron mass. If we impose a cut-off Λ_e on the momentum flowing through the loop, the mass shift,

$$m_e = m_{e,0}\left[1 + \frac{3}{2}\frac{\alpha}{\pi}\ln\frac{\Lambda_e}{m_{e,0}} + \cdots\right], \tag{2.5}$$

is obtained. The magnitude of this first-order correction, although cut-off dependent, is generally tiny. Taking for Λ_e the entire mass of the observable universe, $\Lambda_e \simeq 10^{79}$ GeV, results in only the modest mass shift $m_e \simeq 1.7 m_{e,0}$. This teaches us that, with logarithmic behavior, the renormalization program of absorbing divergences into renormalized parameters is not implausible.

However, radiative corrections to the Higgs mass are not as tame. We display in Fig. XV–1 one-loop self-energy processes which shift the Higgs boson mass.

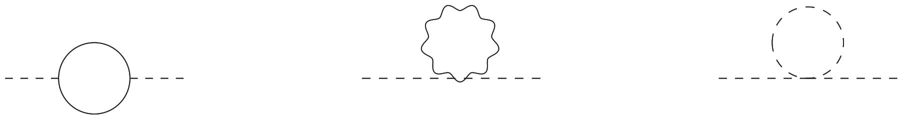

Fig. XV–1 Some quadratically divergent Higgs self-energy diagrams.

Considering for definiteness diagram (c), which involves a Higgs loop with quartic self-coupling, we have

$$-i\Sigma_H(p) = -3i\lambda \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - M_{H,0}^2 + i\epsilon}. \tag{2.6}$$

This expression is quadratically divergent, $\Sigma_H \sim \Lambda_H^2$, where Λ_H is the cut-off parameter for the above integral, and leads to a shift of the Higgs mass,

$$M_H^2 = M_{H,0}^2 + \frac{3}{16} \frac{\lambda}{\pi^2} \Lambda_H^2. \tag{2.7}$$

If Λ_H is as large as, say, the Planck mass $E_{\text{Planck}} \simeq 10^{19}$ GeV, then in order to obtain a renormalized mass as given by Eq. (2.3), the parameter $M_{H,0}^2$ must be negative and have a magnitude which equals the correction up to 31 decimal places! This is referred to as *fine tuning*. While technically possible, it is surely unnatural. Including the other contributions of Fig. XV–1 we obtain the Higgs mass shift

$$M_H^2 = M_{H,0}^2 + \frac{3}{16} \frac{\lambda}{\pi^2} \Lambda_H^2 \left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2 \right]. \tag{2.8}$$

It is possible to cancel this mass shift by arranging the value of M_H contained within the brackets in Eq. (2.8). This strategy gives $M_H \simeq 314$ GeV, which is ruled out by experiment.

The inability to make sense of Higgs mass corrections is perhaps the most serious flaw in the fabric of the Standard Model. At present, there are no known compelling mechanisms for curing this ailment. Accordingly, many physicists have been motivated by this ‘unnaturalness problem’ to search for alternatives to the Standard Model description, and to suggest that New Physics must exist not very far above the weak scale $v \sim 250$ GeV.

Higgs coupling constants

There are a variety of ways that the Higgs can interact, including vacuum energy, Higgs self-couplings, Higgs couplings to massive particles, and finally Higgs couplings to massless particles.

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Table XV–1. *Higgs-boson coupling constants.*

$g_{\bar{f}fH}$	g_{WWH}	g_{WWH^2}	g_{ZZH}	g_{ZZH^2}	g_{H^3}	g_{H^4}
$\dfrac{m_f}{v}$	$\dfrac{2M_W^2}{v}$	$\dfrac{2M_W^2}{v^2}$	$\dfrac{2M_Z^2}{v}$	$\dfrac{2M_Z^2}{v^2}$	$\dfrac{3M_H^2}{v}$	$\dfrac{3M_H^2}{v^2}$

Vacuum Higgs energy: The first term in V, the Higgs potential of Eq. (2.1), is a constant energy density, which can be interpreted as a contribution $\Lambda^{(\text{Higgs})}$ to the full cosmological constant Λ. Inserting known values for μ and v, we have

$$|U^{(\text{vac})}_{\text{Higgs}}| = \Lambda^{(\text{Higgs})} = \frac{\mu^2 v^2}{4} \simeq 1.2 \times 10^8\ \text{GeV}^4, \tag{2.9a}$$

which is huge compared to the observed value [RPP 12],

$$|\Lambda^{(\text{obs})}| \simeq (2.3 \times 10^{-3}\ \text{eV})^4 = 2.8 \times 10^{-47}\ \text{GeV}^4. \tag{2.9b}$$

This should not, however, be viewed as a defect of the Higgs mechanism, as there are many such contributions to the vacuum energy. Presumably, there is some overriding issue of physics which forces the suppression or cancelation of the vacuum energy by so many orders of magnitude.

Higgs coupling to massive particles: Next, we express couplings of the Higgs boson to particles which have nonzero mass. In cases where n identical fields appear, a numerical factor $1/n!$ is introduced to account for the number of identical fields. The set of all such coupling constants is collected in Table XV–1.

The Higgs potential of Eq. (2.1) contains cubic and quartic Higgs interactions, which we express as

$$\mathcal{L}_{\text{self}} = -\frac{g_{H^3}}{3!}\,H^3 - \frac{g_{H^4}}{4!}\,H^4. \tag{2.10a}$$

There are also couplings of the Higgs to massive fermions. From Eq. (2.3) and Eq. (II–3.20), we find for the interaction to fermion f,

$$\mathcal{L}_{f\bar{f}H} = -g_{\bar{f}fH} H \bar{\psi}_f \psi_f. \tag{2.10b}$$

The catalog of Higgs particle interactions is extended by presenting its couplings to the $W^{\pm}$ and Z^0 bosons, including both trilinear and quadrilinear terms for each,

$$\begin{aligned} \mathcal{L}_{WWH} &= W^-_\mu W^\mu_+ \left[\frac{g_{WWH^2}}{2!} H^2 + g_{WWH} H\right], \\ \mathcal{L}_{ZZH} &= Z_\mu Z^\mu \left[\frac{g_{ZZH^2}}{(2!)^2} H^2 + \frac{g_{ZZH}}{2!} H\right], \end{aligned} \tag{2.10c}$$

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where we have employed Eqs. (II–3.18), (II–3.29), (II–3.32). Observe that each of the couplings $g_{H^4}, g_{\bar{f}fH}, g_{WWH^2}, g_{ZZH^2}$ are pure numbers whereas $g_{H^3}, g_{WWH}, g_{ZZH}$ have the unit of energy.

Higgs coupling to massless particles: The coupling between the Higgs boson and a particle depends on the particle's mass. This means that at the basic level of the Higgs lagrangian, there is no coupling to photons and gluons because these particles are massless. However, such couplings are induced through quantum effects. This is a phenomenon we have seen already in Chap. IV, in which the photon-photon interaction, $\gamma\gamma \to \gamma\gamma$, although zero at a fundamental level, is described to one-loop order by the Euler–Heisenberg effective lagrangian of Eq. (IV–8.5).

Higgs–photon–photon vertex: A Higgs boson will couple to a two-photon final state through $W^\pm$-boson and charged-fermion loops. The decay rate

$$\Gamma_{H\to\gamma\gamma} = \frac{M_H^3}{4\pi} \cdot \left| \frac{\alpha}{8\pi v} \left[\mathcal{A}_1(x_W) + \sum_{f=q,\ell} N_c q_f^2 \mathcal{A}_{1/2}(x_f) \right] \right|^2 , \tag{2.11}$$

contains the loop functions $\mathcal{A}_1(x)$ and $\mathcal{A}_{1/2}(x)$,

$$\begin{aligned} \mathcal{A}_1(x) &= -\frac{1}{x^2}\left[2x^2 + 3x + 3(2x-1)f(x)\right], \\ \mathcal{A}_{1/2}(x) &= \frac{2}{x^2}\left[x + (x-1)f(x)\right] , \\ f(x) &= \begin{cases} \arcsin^2(\sqrt{x}) & (x \le 1) \\ -\dfrac{1}{4}\left(\ln\left[\dfrac{1+(1-1/x)^{1/2}}{1-(1-1/x)^{1/2}}\right] - i\pi\right)^2 , & (x > 1) \end{cases} \end{aligned} \tag{2.12}$$

where x is the dimensionless variable $x \equiv M_H^2/(4m^2)$ and the subscripts on $\mathcal{A}_1(x)$ and $\mathcal{A}_{1/2}(x)$ denote the respective spins of the loop particles. The sum over fermions f in Eq. (2.11) is taken over both quarks q and leptons ℓ.

The above procedure is based on calculating the decay amplitude from Feynman diagrams as in Fig. XV–2. It is worthwhile to consider the possibility of an alternative approach. Throughout this book, we have emphasized the use of effective field theories. Can we employ this method here, via a local effective lagrangian, to describe the Higgs–photon–photon vertex? Note that the function $f(x)$ defined in Eq. (2.12) develops an imaginary part for $m < M_H/2$, which is the case for all the loop fermions except the t quark. The imaginary part signals that H would be able to physically decay into any of the light fermion–antifermion loop pairs. If so, the conversion of a Higgs into two photons is nonlocal and cannot possibly be described with a local lagrangian defined at scale $\mu = M_H$. Although the $W^\pm$ and t quark evade such a prohibition, the issue remains whether it would be a good

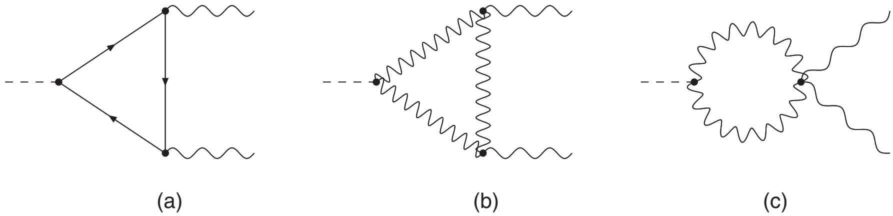

Fig. XV–2 $H \to \gamma\gamma$ via (a) charged fermion, (b)–(c) W boson.

numerical approximation to use a local lagrangian for either. Let us compare the loop functions $\mathcal{A}_1$ and $\mathcal{A}_{1/2}$ evaluated both in the heavy mass limit $x \to 0$ and also using the physical values $x_W \simeq 0.60$ and $x_t \simeq 0.13$,[2]

$$\left|\frac{\mathcal{A}_1(0) - \mathcal{A}_1(x_W)}{\mathcal{A}_1(0)}\right| \simeq 0.16 \qquad \text{vs.} \qquad \left|\frac{\mathcal{A}_{1/2}(0) - \mathcal{A}_{1/2}(x_t)}{\mathcal{A}_{1/2}(0)}\right| \simeq 0.03.$$

Since the difference between the infinite-mass and physical t-quark amplitudes is only 3%, most would agree that an effective lagrangian description for the t-quark contribution is appropriate, and we write

$$\mathcal{L}_{\text{eff}} = g^{(\text{t})}_{\gamma\gamma} H F^{\mu\nu} F_{\mu\nu} \qquad \text{with} \qquad g^{(\text{t})}_{\gamma\gamma} = \frac{2\alpha}{9\pi v}, \tag{2.13}$$

where α is the fine-structure constant and $F_{\mu\nu}$ is the electromagnetic field strength tensor (cf. Eq. (I–5.9)). Note that the heavy top quark evades the decoupling theorem of Sect. IV–2 because the $t\bar{t}H$ vertex is proportional to the large mass parameter m_t.

An alternate derivation of Eq. (2.13) begins by considering the contribution of a $t\bar{t}$ loop to the photon vacuum polarization [ShVVZ 79],

$$\Pi^{\mu\nu}(q)\Big|_{\text{t-quark}} = (q^\mu q^\nu - q^2 g^{\mu\nu})\left[\frac{N_c q_t^2 \alpha}{3\pi} \ln\frac{\Lambda^2}{m_t^2} + \cdots\right], \tag{2.14}$$

where $q_t = 2/3$ is the top-quark electric charge in units of e and we have chosen regularization with cut-off Λ here (instead of the dimensional approach used elsewhere in this book) to keep the notation compact. The photon vacuum polarization of Eq. (2.14) can equivalently be expressed via the effective lagrangian,

$$\mathcal{L}^{(\text{t-loop})}_{\text{ph. vac. pol.}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \cdot \frac{q_t^2 \alpha}{\pi} \ln\frac{\Lambda^2}{m_t^2}, \tag{2.15}$$

[2] For reference we note the expansions about $x = 0$: $\mathcal{A}_{1/2}(x) \simeq 4/3(1 + 7x/30 + 2x^2/21 + \cdots)$ and $\mathcal{A}_1(x) \simeq -7 - 22x/15 - 76x^2/105 + \cdots$.

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as can be shown by taking its photon-to-photon matrix element. Now, writing the top-quark mass term together with its Higgs interaction,

$$\mathcal{L}_{Ht\bar{t}} = -\left(m_t + \frac{m_t}{v}H\right)\bar{t}t = -m_t\left(1 + \frac{H}{v}\right)\bar{t}t, \tag{2.16}$$

suggests treating the Higgs field as a constant and thus formally extending the top-quark mass as $m_t \to m_t\,(1 + H/v)$. Inserting this into Eq. (2.15) and considering only the term linear in H yields precisely the effective lagrangian of Eq. (2.13). This 'background field' derivation is valid if the momenta involved are small compared to the top-quark mass, which is not perfect but a good first approximation.[3]

Higgs–Z^0–photon vertex: This process, too, occurs first as a loop amplitude ([CaCF 79], who assume $M_H < M_Z$ and study $Z^0 \to H\gamma$; see also [BeH 85]) via triangle diagrams dominated by $W^\pm$-boson and t-quark contributions. We refer the reader to the literature for the explicit, somewhat cumbersome, analytic form of the vertex.

Higgs–gluon–gluon interaction: The Higgs two-gluon amplitude has similarities with the Higgs two-photon interaction. One calculates Feynman amplitudes for triangle diagrams, although now summed over only quarks $\{q\}$ since gluons couple neither to leptons nor to the electroweak gauge bosons, leading to

$$\Gamma_{H\to gg} = \frac{2M_H^3}{\pi}\cdot\left|\frac{\alpha_s}{16\pi v}\left[\sum_q \mathcal{A}_{1/2}(x_q)\right]\right|^2. \tag{2.17}$$

The top-quark amplitude is by far the largest in the above sum, and so we can again turn to the effective lagrangian description. The contribution of a $t\bar{t}$ loop to the gluon vacuum polarization in cut-off regularization is

$$\Pi^{\mu\nu}(q)_{ab}\Big|_{\text{t-quark}} = \delta_{ab}(q^\mu q^\nu - q^2 g^{\mu\nu})\left[\frac{\alpha_s}{6\pi}\ln\frac{\Lambda^2}{m_t^2} + \cdots\right], \tag{2.18}$$

which leads, as explained earlier, to

$$\mathcal{L}^{\text{(t-loop)}}_{\text{gl. vac. pol.}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}\cdot\frac{\alpha_s}{6\pi}\ln\frac{\Lambda^2}{m_t^2}, \tag{2.19}$$

and finally, from Eq. (2.16), to

$$\mathcal{L}_{\text{eff}} = g^{(\text{t})}_{gg} H F^{a\mu\nu}F^a_{\mu\nu} \qquad \text{with} \qquad g^{(\text{t})}_{gg} = -\frac{\alpha_s}{12\pi v}, \tag{2.20}$$

[3] For completeness, we take note of yet another derivation [ElGN 76] of Eq. (2.13) which uses the *QED* trace anomaly (see Eq. (III–4.16) for the *QCD* version),

$$\theta^\mu_\mu = \frac{\alpha_s}{12\pi}F_{\mu\nu}F^{\mu\nu} + m_t\bar{t}t,$$

taking into account only the t-quark part of the fermion contribution.

where $F^{a\mu\nu}$ is the chromodynamic field strength tensor of Eq. (II–2.2a). This represents the linear term in an expansion in powers of the Higgs field H. Higher powers provide the two-gluon coupling to an arbitrary number of Higgs bosons. The quadratic term in this expansion would be a prediction for $gg \to HH$. There, in addition to the direct coupling of Eq. (2.20), one encounters a pole diagram (i.e. $gg \to H \to HH$) which contains the triple Higgs coupling. The direct and pole contributions cancel exactly at threshold and, more generally, the residual effect remains small.

XV–3 Production and decay of the Higgs boson

Following the discovery of the top quark, finding the Standard Model Higgs boson became a primary goal of experimental particle physics. The search strategy was based on Standard Model predictions of both production and decay amplitudes. We discuss each of these in turn, beginning with the topic of Higgs decay.

Decay

One begins calculation of a Higgs decay mode with the lowest-order amplitude, and then incorporates higher-order *QCD* and electroweak (EW) corrections. These higher-order effects are described, with many references, in [Dj 08]. Here, we display branching fraction predictions in Table XV–2 [He *et al.* 13], but restrict our presentation here to only the lowest-order analysis (except for two decays $H \to b\bar{b}$ and $H \to gg$, which have especially large corrections). The major two-body Standard Model decay branching fractions in Table XV–2 correspond to a total width,

$$\Gamma_H^{(\mathrm{tot})} \simeq 4.21\ (\pm 3.9\%)\ \mathrm{MeV}. \tag{3.1}$$

The individual branching fractions in Table XV–2 are purely theoretical quantities. An experimental reality at LHC is that detection of the modes $b\bar{b}$, gg, $c\bar{c}$ is greatly inhibited by huge hadronic backgrounds. As a consequence, other modes (e.g. $\gamma\gamma$) can play a central role in Higgs phenomenology at the LHC, despite their smaller branching fractions.

Table XV–2. *Two-body Higgs branching fractions.*[a]

$\bar{b}b$	WW^{*b}	gg	$\tau^+\tau^-$	$\bar{c}c$	ZZ^{*b}	$\gamma\gamma$	γZ	$\mu^+\mu^-$
56.1	23.1	8.48	6.15	2.83	2.89	0.23	0.16	0.02

[a] All branching fractions are in % and the value $M_H = 126.$ GeV is assumed.
[b] The asterisk denotes a virtual vector boson.

Decay into fermion–antifermion pairs: For transitions of the type $H \to f\bar{f}$, the leading-order (LO) decay rate is

$$\Gamma^{(\mathrm{LO})}_{H\to f\bar{f}} = \frac{N_c}{8\pi}\frac{m_f^2}{v^2} M_H \left(1 - 4x_f^2\right)^{3/2}, \tag{3.2a}$$

where m_f is the fermion mass (which arises from the Yukawa coupling), $x_f \equiv m_f/M_H$ and $N_c = 1$ for leptons and $N_c = 3$ for quarks. We already know that in the Standard Model the Higgs coupling to a fermion–antifermion pair is linear in the fermion mass m_f. The factor of m_f^2 in Eq. (3.2a) reflects this and ensures that the $b\bar{b}$ mode is largest amongst all fermions with $2m_f < M_H$ (the mode $H \to t\bar{t}$ is kinematically forbidden).

Let us consider the $H \to b\bar{b}$ mode in a bit more detail. If Eq. (3.2a) is used to determine the $b\bar{b}$ decay rate and Eq. (3.1) is used for $\Gamma_H^{(\mathrm{tot})}$, then a branching fraction $\simeq$ 104% is predicted. This unphysical result is disconcerting to say the least! The flaw in our numerical exercise is that we have ignored corrections to the tree-level prediction of Eq. (3.2a). Ordinarily, one expects a 'correction' to be no more than a few tens of percent and usually much smaller. This case is not like that; it turns out that the most important correction is to replace the m_f^2 factor by the squared running mass $\overline{m}_f^2(\mu)$ with $\mu = M_H$,

$$\Gamma_{H\to f\bar{f}} = \frac{N_c}{8\pi}\frac{\overline{m}_b^2(M_H)}{v^2} M_H \left(1 - 4x_f^2\right)^{3/2}\left[1 + 5.67\frac{\alpha_s(M_H)}{\pi} + \cdots\right], \tag{3.2b}$$

where the $\mathcal{O}(\alpha_s)$ correction is also displayed. For the b quark, we have already found below Eq. (XIV–1.12) that $\overline{m}_b(M_H) \simeq 0.665\, \overline{m}_b(\overline{m}_b)$, implying a corrected $H \to b\bar{b}$ branching fraction of 56%. This means that all the remaining corrections for this mode amount to a rather more modest effect. The moral of this lesson is to not place unwarranted trust in tree-level estimates.

Decay into three-body states: Although the Higgs boson couples to the electroweak gauge bosons, a Higgs with mass $M_H \simeq 126$ GeV is too light to decay into WW and ZZ final states. However, a transition like $H \to WW^* \to Wf\bar{f}'$ (or $H \to Zf\bar{f}$) can occur, e.g., $H \to W^+d\bar{u}$ or $H \to W^-c\bar{s}$ and so on. We shall consider this possibility here. If dependence on fermion mass (such as m_f/M_H or m_f/M_W) is ignored, the energy distribution of the final state W is [KeM 84]

$$\frac{d\Gamma^{(\mathrm{LO})}_{H\to Wf\bar{f}'}}{dx} = \frac{1}{192\pi^3}\left(\frac{M_W}{v}\right)^4 M_H \frac{(x^2 - 4\epsilon^2)^{1/2}}{(1-x)^2}\left(x^2 - 4\epsilon^2 x + 8\epsilon^2 + 12\epsilon^4\right), \tag{3.3}$$

where $x = 2E_W/M_H$ and $\epsilon = M_W/M_H$. Integration over the W-boson energy yields

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$$\Gamma^{(\mathrm{LO})}_{H\to W f\bar f'} = \frac{1}{192\pi^3}\left(\frac{M_W}{v}\right)^4 M_H F(\epsilon)$$

$$F(\epsilon) = \frac{3(1-8\epsilon^2+20\epsilon^4)}{(4\epsilon^2-1)^{1/2}}\arccos\left[\frac{3\epsilon^2-1}{2\epsilon^3}\right] - (1-\epsilon^2)\left[\frac{47}{2}\epsilon^2-\frac{13}{2}+\frac{1}{\epsilon^2}\right] - 3\left(1-6\epsilon^2+4\epsilon^4\right)\ln\epsilon. \quad (3.4)$$

Thus far, we have kept the final state fixed as $W f\bar f'$. To obtain the inclusive rate $\Gamma_{H\to W^\pm X}$, we sum over all distinct final states (like the ones displayed above Eq. (3.3)) to find

$$\Gamma^{(\mathrm{LO})}_{H\to W^\pm X} = \frac{3}{32\pi^3}\left(\frac{M_W}{v}\right)^4 M_H F(\epsilon). \quad (3.5)$$

The case of $H \to Z f\bar f$ is obtained from the above relations via insertion of a factor $\eta_Z = \frac{7}{12} - \frac{10}{9}\sin^2\theta_{\mathrm{w}} + \frac{40}{9}\sin^4\theta_{\mathrm{w}}$.

Decay into four-body states: The degrees of freedom appearing in Table XV–2 are those occurring at the primary vertex, at which the Higgs decay process begins. However, these are often not the final states which are actually detected. For example, the quark–antiquark states will hadronize into jets whereas the vector gauge bosons will quickly decay and be observed as four-fermion final states, e.g., as in final states containing leptons and antileptons. We do not display analytic formulae here for such modes, but numerical examples are displayed in Table XV–3. The leptons and neutrinos there are summed respectively over $\ell = e, \mu, \tau$ and $\nu = \nu_e, \nu_\mu, \nu_\tau$.

Decay into massless final-state particles: The general leading-order $H \to \gamma\gamma$ decay rate is given in Eq. (2.11). Approximating this with the W-boson and top-quark contributions gives

$$\Gamma^{(\mathrm{LO})}_{H\to\gamma\gamma} \simeq \frac{\alpha^2}{256\pi^3}\cdot\frac{M_H^3}{v^2}\left|\mathcal{A}_1(x_W) + N_c q_t^2 \mathcal{A}_{1/2}(x_t)\right|^2, \quad (3.6)$$

where the quantities $\mathcal{A}_1(x_W)$ and $\mathcal{A}_{1/2}(x_t)$ are the loop functions defined in Eq. (2.11), with arguments $x_W = M_H^2/(4M_W^2)$ and $x_t = M_H^2/(4m_t^2)$. Although the

Table XV–3. *Four-body Higgs branching fractions.*[a]

$(qqqq)$	$(qq\ell\nu_\ell)^b$	$(qq\nu\nu)$	$(qq\ell^+\ell^-)$	$(\ell^+\ell^-\ell^+\ell^-)$
11.8	3.38	0.81	0.27	0.03

[a] All branching fractions are in % and the value $M_H = 126.$ GeV is assumed.
[b] Here, $\ell = e, \mu$.

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top-quark contribution dominates that of the other fermions (due to its much larger Yukawa coupling to the Higgs), that of the W-boson is even larger, $|N_c q_t^2 A_{1/2}(x_t)/\mathcal{A}_1(x_W)| \simeq 0.22$.

For the transition $H \to gg$, decay products would appear as jets consisting of light hadrons. The $H \to gg$ decay rate has already been given in Eq. (2.17). Approximating this with the dominant top-quark contribution in the heavy m_t limit yields the tree-level expression,

$$\Gamma^{(\mathrm{LO})}_{H\to gg} = \frac{\alpha_s^2 M_H^3}{72\pi^3 v^2}. \tag{3.7}$$

Virtual gluon exchanges will modify the above. Unlike the case for $H \to \gamma\gamma$ the next-to-leading-order $H \to gg$ amplitude will experience gluon self-interactions such as triple-gluon vertices and turns out to have a large numerical effect [SpDGZ 95],

$$\Gamma_{H\to gg} \simeq \Gamma^{(\mathrm{LO})}_{H\to gg}\left[1 + \left(\frac{95}{4} - \frac{7}{6}n_f\right)\alpha_s(M_H) + \ldots\right] \simeq 1.64\ \Gamma^{(\mathrm{LO})}_{H\to gg}, \tag{3.8}$$

with $n_f = 5$ and $\alpha_s(M_H)$ given previously in Eq. (II–2.79).

Production

Next, we consider the most important of the mechanisms at LHC energies for producing the Higgs boson in the inclusive process $p + p \to H + X$, where X represents a sum over all the other final-state particles. The scattering which yields the Higgs production will involve the basic degrees of freedom (*partons*) occurring within a proton, the quarks and gluons. Because the partons are not physical entities, the cross section must be expressed as

$$\sigma = \sum_{i,j}\int_0^1 dx_1\, dx_2\ f_i(x_i, Q) f_j(x_2, Q)\hat{\sigma}_{ij}, \tag{3.9}$$

where the indices i, j refer to the two initial-state protons and the quantities f_i and f_j are parton distribution functions (‘PDFs’). A hadron’s PDF $f(x, Q)$ gives the probability density for finding a parton carrying a fraction x of the hadronic longitudinal momentum at momentum reference scale Q. Given the difficulty presented by nonperturbative *QCD*, a PDF is commonly inferred from experimental data, e.g., as with

$$f_i(x, Q) = N x^{\alpha_i}(1-x)^{\beta_i} g_i(x). \tag{3.10}$$

where α_i, β_i are fit parameters. The function $g_i(x)$ is defined to approach constants at $x = 0, 1$ e.g., $g_i(x) = 1+\epsilon_i\sqrt{x}+D_i x+E_i x^2$ and itself contains the fit parameters

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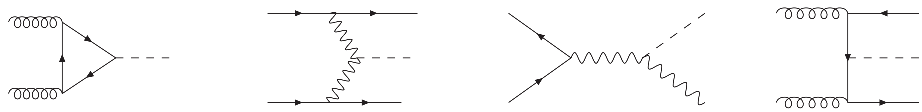

Fig. XV–3 Higgs production via: (a) gg fusion, (b) VBF, (c) HV, (d) $t\bar{t}H$.

ϵ_i, D_i, E_i. The parton cross section $\hat{\sigma}_{ij}$ is calculated at leading order from various Standard Method processes and corrected by both *QCD* and EW perturbations.

Within this phenomenological framework, one has at the *Higgs mass scale and LHC energies* the following Standard Model mechanisms, depicted to leading order in Figure XV–3 and listed here according to cross-section magnitude:

(1) Gluon–gluon fusion (gg fusion): $gg \to t\bar{t} \to H$
(2) Vector–boson fusion (VBF): $qq \to qq + V^*V^* \to qq + H$
(3) Vector–boson-associated production (HV): $q\bar{q} \to V^* \to H + V$
(4) $t\bar{t}$ associated-production ($t\bar{t}H$): $gg \to t\bar{t} + H$,

Numerical values [He *et al.* 13] for each of these contributions at the energies $\sqrt{s} = 8$, 14 TeV appear in Table XV–4. Table XV–4 contains not only cross-section values but also uncertainties for each, given numerically in per cent. These arise mainly from aspects of *QCD*, such as uncertainties in *QCD* parameters (e.g. α_s, m_c, etc.), parton PDFs and a significant uncertainty from the uncalculated higher-order *QCD* corrections.

The gluon–gluon fusion reaction proceeding via top-quark loops is the dominant component of the $p + p \to H + X$ cross section.[4] It also has the interesting property of being sensitive to certain types of virtual heavy particles. We saw in the derivation of Eq. (2.20) that the top-quark contribution to the triangle graph for $H \to gg$ does not decouple, despite having $4m_t^2 \gg M_H^2$, because the coupling

Table XV–4. *Standard Model Higgs production cross sections.*[a]

$\sqrt{s}$ (TeV)	gg Fusion	VBF	HW	HZ	$t\bar{t}H$
8	$18.97\ (^{+7.2\%}_{-7.8\%})$	$1.57\ (^{+0.3\%}_{-0.1\%})$	$0.69\ (\pm 1.0\%)$	$0.41\ (\pm 3.2\%)$	$0.13\ (^{+3.8\%}_{-9.3\%})$
14	$49.85(^{+19.6\%}_{-14.6\%})$	$4.18(^{+2.8\%}_{-3.0\%})$	$1.50(^{+4.1\%}_{-4.4\%})$	$0.88(^{+6.4\%}_{-5.5\%})$	$0.61(^{+14.8\%}_{-18.2\%})$

[a] All cross sections are in pb units; the value $M_H = 126$ GeV is used for $\sqrt{s} = 8$ (TeV) and $M_H = 125$ GeV for $\sqrt{s} = 14$ (TeV).

[4] The next most important contribution, that of the b-quark loop, is estimated at leading order to be at most a 10% effect.

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between $t\bar{t}H$ is proportional to m_t. Thus, what if there were a very heavy fourth generation of Standard Model fermions (a situation often denoted as SM_4) with all else the same (i.e. same Standard Model couplings, only one physical Higgs boson) as with the known fermions? The new generation would contain two new, very heavy quarks, say u_4, d_4, which likewise would not decouple in the $H \to gg$ vertex. The $H \to gg$ amplitude would then be about a factor three larger than in the Standard Model case, and the gluon–fusion production cross section about nine times as large. Moreover, using LHC and Tevatron data as input, it has been concluded from an analysis of Higgs decay modes that SM_4 is excluded at more than 5σ [EbHLLMNW 12].

Earlier, in the discussion following Eq. (3.1), we pointed out that detection of final states like $b\bar{b}, gg, c\bar{c}$ at the LHC, where the $gg \to H$ is the dominant production mechanism, is greatly hindered by hadronic backgrounds. However, a $b\bar{b}$ final state can be relatively more accessible if the Higgs particle is predominantly produced in association with a vector boson ($V = W, Z$) or a $t\bar{t}$ pair, a strategy which has been pursued by the detectors CDF and D0 (Tevatron) and ATLAS and CMS (LHC). This can lead to detection of $H \to b\bar{b}$ via more easily identifiable configurations like

$$HW \to b\bar{b}\ell\nu_\ell, \qquad HZ \to b\bar{b}\ell\bar{\ell}, \qquad HW, HZ \to \not{E}_T b\bar{b}$$

where $\ell = e, \mu$ and $\not{E}_T$ represents missing transverse energy. Some promising results have been obtained thus far, e.g., a reported excess of events at 3.1σ with $M_H = 125.$ GeV [Aa *et al.* (CDF and D0 Collabs.) 13] and a $> 3\sigma$ significance in the combined $\tau\bar{\tau} + b\bar{b}$ channels reported by the CMS collaboration at the 2013 Lepton–Photon Conference.

Comparison of Standard Model expectations with LHC data

Statistical data analyses have been performed to test the extent to which collected data agree with the Standard Model Higgs boson scenario. Such testing can be done directly by experimental collaboration or as a theoretically motivated exercise:

(1) *Experimental*: One can define a *global signal strength factor* μ_i for a given final state 'i' by folding together the production cross section and branching fraction for the observed signal relative to the Standard Model prediction,

$$\mu_i = \frac{\left[\sum_j \sigma_{j\to H}\, \text{Br}_{H\to i}\right]_{\text{obs}}}{\left[\sum_j \sigma_{j\to H}\, \text{Br}_{H\to i}\right]_{\text{SM}}}. \tag{3.11}$$

There is a label 'j' because a given final state 'i' might be summed over a subset of Higgs production processes 'i'. The value $\mu = 0$ corresponds to

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the background-only hypothesis whereas $\mu = 1$ corresponds to the Standard Model Higgs boson signal in addition to the background. Announced results from the LHC detectors have been found, thus far, to be statistically consistent with the Standard Model hypothesis.

(2) *Theoretical*: There are a number of ways to parameterize couplings to include non-Standard Model behavior. Suppose Standard Model Higgs couplings to fermion f and to vector boson V are generalized to have the forms [ElY 12],

$$g_f = \sqrt{2}\frac{m_f}{v} \to \sqrt{2}\left(\frac{m_f}{M}\right)^{1+\epsilon}, \qquad g_V = 2\frac{M_V^2}{v} \to 2\,\frac{M_V^{2(1+\epsilon)}}{M^{(1+2\epsilon)}}, \tag{3.12}$$

where ϵ and M are purely phenomenological parameters. In the Standard Model, they become $\epsilon = 0$ and $M = v \simeq 246$ GeV. A global fit to LHC data yields results consistent with these values, $\epsilon = 0.05 \pm 0.08$ and $M = 241 \pm 18$ GeV.

Another procedure is to consider an effective lagrangian for the electroweak symmetry-breaking sector, which modifies couplings to vector mesons and fermions in terms of universal parameters 'a' and 'c'.

$$\mathcal{L}_{\text{eff}} = \sum_{V=W,Z} \eta_V M_V^2 V_\mu^\dagger V^\mu \left[1 + 2a\frac{H}{v}\right] - \sum_i m_i \bar{f}_i f_i \left[1 + c\frac{H}{v}\right] + \cdots \tag{3.13}$$

where $\eta_W = 1$, $\eta_Z = 1/2$, and the ellipses represent a sum over all remaining Standard Model contributions as well as possible higher-order terms in the field variable H. In the Standard Model, we have $a = c = 1$. Fits to the current dataset again yield results consistent with Standard Model expectations [ElY 12, EsGMT 12].

The above parameterizations are just two examples of Higgs-related phenomenology. These tests, and others, will continue into the future as the Higgs database expands.

XV–4 Higgs contributions to electroweak corrections

Prior to the discovery of a new boson at the LHC, direct Higgs searches yielded only upper bounds, e.g., as with $M_H < 114.4$ GeV obtained at LEP2. However, the calculation of quantum corrections to Standard Model predictions came to play a central role in particle phenomenology and Higgs physics in particular. The procedure is straightforward; a collection of observables $(M_W, \ldots)$ is measured and then compared to predictions expressed in terms of a set of input parameters $(G_\mu, \alpha, \ldots)$ including the Higgs mass M_H (cf. Sect. XVI–6). Although the dependence on Higgs mass in such analyses is somewhat weak, being logarithmic $\sim \ln M_H^2$ at leading order, it has continued to show for quite some time that the Higgs boson is 'light'. A recent $\Delta\chi^2$ fit gives [Ba *et al.* (Gfitter group) 12] $M_H = 94^{+25}_{-22}$ GeV. That this value is consistent with the LHC determinations of M_H is generally regarded as a noteworthy success of the Standard Model. To observe the role of the Higgs boson in this procedure, let us next consider a few specific examples of such corrections.

The corrections $\Delta\rho$ and Δr

Higgs contributions to $\Delta\rho$: We begin with the so-called effective weak mixing angle

$$\bar{s}_{\rm w}^2 = 1 - \frac{M_W^2}{M_Z^2} + c_{\rm w}^2 \Delta\rho, \tag{4.1}$$

which is discussed at length in Sect. XVI–1. The corrections to $\bar{s}_{\rm w}^2$ are contained within the quantity $\Delta\rho$. For arbitrary M_H, the one-loop Higgs contribution to $\Delta\rho$ is

$$\Delta\rho_H^{\text{1-loop}} = -\frac{3}{4}\left(\frac{M_W^2}{4\pi^2 v^2}\right) f(M_H^2/M_Z^2), \tag{4.2a}$$

where

$$f(x) = x\left[\frac{\ln c_{\rm w}^2 - \ln x}{c_{\rm w}^2 - x} + \frac{\ln x}{c_{\rm w}^2(1-x)}\right]. \tag{4.2b}$$

The leading dependence on M_H for $M_H \gg M_W$ is logarithmic,

$$\Delta\rho_H^{\text{1-loop}} \sim -\frac{3}{4}\left(\frac{M_W^2}{4\pi^2 v^2}\right)\frac{s_{\rm w}^2}{c_{\rm w}^2}\ln\frac{M_H^2}{M_W^2}, \tag{4.3}$$

as are all the other leading one-loop Higgs contributions.[5] A term like $\ln M_H^2/M_W^2$ does not respond sensitively to changes in M_H^2, so the shift $\Delta\rho_H^{\text{1-loop}}$ by itself does not lead to a precise estimate for M_H.

There are also multi-loop Higgs contributions. In contrast to the $\ln M_H^2/M_W^2$ logarithmic dependence of the one-loop amplitude, these also contain *power-law* dependence on M_H,

$$\Delta\rho_H^{\text{2-loop}} \sim 0.1499\left(\frac{M_W^2}{4\pi^2 v^2}\right)^2 \frac{s_{\rm w}^2 M_H^2}{c_{\rm w}^2 M_W^2}, \tag{4.4a}$$

[5] It is, however, not the case that one-loop corrections for all the remaining Standard Model particles are logarithmic, e.g., $\Delta\rho$ has a $\mathcal{O}(G_\mu m_t^2)$ dependence on the t-quark mass (viz. Sect. XVI–6).

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Table XV–5. *Higgs contribution to* $\Delta\rho$.

Order:	One-loop	Two-loop	Three-loop
	-1.8×10^{-3}	8.1×10^{-7}	-6.2×10^{-8}

and the three-loop amplitude gives

$$\Delta\rho_H^{\text{3-loop}} \sim -1.728 \left(\frac{M_W^2}{4\pi^2 v^2}\right)^3 \frac{s_{\rm w}^2 M_H^4}{c_{\rm w}^2 M_W^4}. \tag{4.4b}$$

Observe that common to all terms is the coefficient,

$$\frac{M_W^2}{4\pi^2 v^2} \simeq 0.0027. \tag{4.5}$$

An extra power of this small quantity will accompany each additional loop and thus suppress the multi-loop contributions, at least for moderate values of M_H. Note also that the two-loop and three-loop amplitudes have opposite sign. The values of the one-loop, two-loop, and three-loop amplitudes are summarized in Table XV–5 using $M_H = 126.$ GeV. The one-loop amplitude is dominant and gives an accurate estimate of the Higgs contribution to $\Delta\rho$.

Higgs contributions to Δr: A second class of Standard Model corrections affects the relation between the Fermi constant and M_W, given to leading order by Eq. (II–3.43). Upon using Eq. (II–3.42) and Eq. (II–3.33), we can express this as

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}. \tag{4.6}$$

The one-loop Higgs correction to this relation,

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1 + \Delta r_H^{\text{1-loop}}\right), \tag{4.7}$$

is given by

$$\Delta r_H^{\text{1-loop}} = \frac{11}{48\pi^2} \cdot \frac{M_W^2}{v^2}\left(\ln\frac{M_H^2}{M_W^2} - \frac{5}{6}\right). \tag{4.8}$$

Custodial symmetry

As part of our discussion of chiral symmetry in Chap. IV, we obtained a representation of the linear sigma model by expressing an $SU(2)_L \times SU(2)_R$ invariant lagrangian (cf. Eq. (IV–1.4)) in terms of two chiral fermions ψ_L, ψ_R and a 2×2 matrix $\Sigma = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$ of four scalar fields. The $SU(2)_L \times SU(2)_R$ transformation properties were $\psi_L \to L\psi_L$, $\psi_R \to R\psi_R$ and $\Sigma \to L\Sigma R^\dagger$ with L, R in $SU(2)$.

Somewhat analogously, we can express the Higgs doublet as a matrix $\mathbf{H}$ via the construction[6]

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \widetilde{\Phi} & \Phi \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^{0*} & \varphi^{+} \\ -\varphi^{-} & \varphi^{0} \end{pmatrix}, \tag{4.9}$$

where Φ is the Higgs field of Eq. (II–3.16) and $\widetilde{\Phi}$ is its conjugate.[7] This will be convenient for considering transformations of both $SU(2)_L$ and $SU(2)_R$.

Even though this chapter is, for the most part, a discussion/celebration of the $M \simeq 125$ GeV particle, which could well be the Standard Model Higgs boson, we shall, for the remainder of this section, instead emphasize the symmetry aspect of the Higgs sector. In the notation introduced above, a Higgs lagrangian invariant under $SU(2)_L \times U(1)_Y$ gauge symmetry is

$$\mathcal{L}_{\text{Higgs}} = \text{Tr}\left[(D^\mu \mathbf{H})^* D_\mu \mathbf{H}\right] - V(\mathbf{H}^\dagger \mathbf{H}), \tag{4.10a}$$

where the covariant derivative is

$$D_\mu \mathbf{H} = (\partial_\mu + i\frac{g_1}{2} B_\mu \tau_3 + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu)\mathbf{H}, \tag{4.10b}$$

and the potential has the form

$$V(\mathbf{H}^\dagger \mathbf{H}) = -\mu^2 \,\text{Tr}\left[\mathbf{H}^\dagger \mathbf{H}\right] + \lambda \left(\text{Tr}\left[\mathbf{H}^\dagger \mathbf{H}\right]\right)^2. \tag{4.10c}$$

The matrix τ_3 in Eq. (4.10b) accounts for the opposite relative weak hypercharge of Φ and its conjugate $\widetilde{\Phi}$. That the lagrangian $\mathcal{L}_{\text{Higgs}}$ of Eq. (4.10a) is indeed gauge-invariant can be verified by noting

$$SU(2)_L : \ \mathbf{H} \to L\mathbf{H} \text{ and } D_\mu \mathbf{H} \to L(D_\mu \mathbf{H}) \qquad U(1)_Y : \ \mathbf{H} \to \mathbf{H} e^{-i\tau_3 \theta_Y}. \tag{4.11}$$

Actually, the potential energy $V(\mathbf{H}^\dagger \mathbf{H})$ of Eq. (4.10c) (but not the kinetic part in Eq. (4.10a)) is invariant under the larger set of $SU(2)_L \times SU(2)_R$ transformations.

Thus far, we have simply used a new notation to reproduce what we already know. In order to learn something new, however, consider the limit $g_1 \to 0$. There is now present the symmetry, $SU(2)_R$, under which

$$SU(2)_R : \ \ \mathbf{H} \to \mathbf{H} R^\dagger \text{ and } D_\mu \mathbf{H} \to (D_\mu \mathbf{H}) R^\dagger. \tag{4.12}$$

Thus, for the combined $SU(2)_L \times SU(2)_R$ transformations, we have $\mathbf{H} \to L\mathbf{H}R^\dagger$, like the sigma model matrix Σ mentioned at the beginning of this section. Although true, the above analysis is incomplete; we must address the Higgs spontaneous

6 In the following, we adopt the general approach of [SiSVZ 80] and [Wi 04].

7 In the language of group theory, the conjugate spinor $\widetilde{\Phi} = i\tau_2 \Phi^*$ is *equivalent* to Φ.

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symmetry breaking of Eq. (II–3.25), for which the ground-state configuration of **H** becomes

$$\langle \mathbf{H} \rangle = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \tag{4.13}$$

with $v \equiv (\mu^2/\lambda)^{1/2}$ as in Eq. (II–3.24). Although this ground state does not respect the full $SU(2)_L \times SU(2)_R$ symmetry,

$$L\langle \mathbf{H} \rangle \neq \langle \mathbf{H} \rangle, \qquad \langle \mathbf{H} \rangle R^\dagger \neq \langle \mathbf{H} \rangle, \tag{4.14a}$$

it *does* remain invariant under $SU(2)_{L+R}$ transformations, i.e., those having $R = L$,

$$L\langle \mathbf{H} \rangle L^\dagger = \langle \mathbf{H} \rangle. \tag{4.14b}$$

This $SU(2)_{L+R}$ invariance is often referred to as *custodial* symmetry [SiSVZ 80].

In Chap. II, the basis of our discussion of the electroweak sector was the Higgs effect, i.e., the spontaneous breaking of the gauge symmetry $SU(2)_L \times U(1)_Y$. Here, let us instead use elementary group theory to see what the $g_1 = 0$ world, with its exact $SU(2)_{L+R}$ global symmetry, would be like.[8] Eq. (II–3.31) shows that setting $g_1 = 0$ would cause the weak mixing angle to vanish, $\theta_{\rm w} \to 0$, and so from Eq. (II–3.30) for $Z^0 \to W_3$.

It follows from Eq. (I–5.17) that the three **W**-boson fields would transform as an isotriplet under the (global!) $SU(2)_L$ transformations, and as an isosinglet under $SU(2)_R$ (since $g_1 = 0$). They would thus transform as an isotriplet under $SU(2)_{L+R}$ and, since the $SU(2)_{L+R}$ symmetry is exact, the **W** triplet would be degenerate. The above remarks imply the equality

$$\rho = (M_W/(M_Z \cos\theta_{\rm w}))^2 = 1 \qquad \text{(in the } g_1 \to 0 \text{ limit).} \tag{4.15a}$$

When viewed as a statement of invariance, this equality is a consequence of the $SU(2)_{L+R}$ symmetry, which is called 'custodial' for this reason. As we then return to the real world of $g_1 \neq 0$ and allow for higher-order Standard Model corrections, we would expect corrections to $\rho = 1$ to be modest [SiSVZ 80],

$$\rho = 1 + \mathcal{O}(\alpha) + \mathcal{O}(\alpha(m_u^2 - m_d^2)/M_W^2). \tag{4.15b}$$

XV–5 The quantum Higgs potential and vacuum stability

Our treatment of the Higgs potential has thus far been at the classical level. We have simply taken the quadratic and quartic terms that appear in the bare lagrangian,

[8] For example, the electric charge would vanish (cf. Eq. (II–3.42)), so modest mass shifts would occur, e.g., the leading-order contribution to pion mass splitting would vanish, etc.

minimized the energy, and found the vacuum expectation value and the Higgs mass. However, quantum effects modify this form significantly, most importantly through a top-quark loop. Even more remarkably, the presently indicated value of the Higgs and top masses indicate that we are very close to the border where the Higgs potential is actually unstable. In this section, we explore the nature of the quantum effects. Our focus is on the role of the top quark, which is the major contributor to the potential instability.

The Higgs potential describes the vacuum energy as a function of a constant Higgs field. Since the top-quark mass and the Higgs Yukawa coupling to the top quark are related, it is convenient to define a *background field* $h(x) = v + H(x)$. In the following we take H (and hence h) as constant and thus omit any spacetime dependence,

$$-\mathcal{L}_t = \frac{\Gamma_t}{\sqrt{2}}(v + H)\bar{t}t \equiv \frac{\Gamma_t}{\sqrt{2}}h\bar{t}t \equiv m_t(h)\bar{t}t, \tag{5.1}$$

where $m_t(h) = \Gamma_t h/\sqrt{2}$ is the field-dependent mass. We then calculate the vacuum energy as a function of $m_t(h)$. This can be done relatively simply by studying the $t\bar{t}$ contribution to the vacuum matrix element of the energy-momentum tensor $\mathcal{T}_{\mu\nu}$,

$$\begin{aligned}\langle 0|\mathcal{T}_{\mu\nu}|0\rangle_{\text{top}} &= -N_c \int \frac{d^d p}{(2\pi)^d}\,\frac{1}{2}\text{Tr}\left[(\gamma_\mu p_\nu + \gamma_\nu p_\mu)\frac{i}{\not{p} - m_t(h) + i\epsilon}\right] \\ &= -12 \int \frac{d^d p}{(2\pi)^d}\; p_\mu p_\nu \frac{i}{p^2 - m_t^2(h) + i\epsilon} \\ &= \delta V(h) g_{\mu\nu},\end{aligned} \tag{5.2}$$

where the important minus sign comes from the Feynman rule for a closed fermion loop. This leads to a result

$$\delta V(h) = \frac{3m_t^4(h)}{16\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi - \ln\frac{m_t^2(h)}{\mu_d^2} + \frac{3}{2}\right], \tag{5.3}$$

with μ_d being the scale that enters in dimensionally regularized integrals.[9] The divergence is proportional to $m_t^4(h) \sim h^4$ and thus goes into the renormalization of the $\lambda\varphi^4$ term in the Higgs potential. In the $\overline{\text{MS}}$ scheme, one then arrives at the potential,

$$V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda(\mu_d)h^4 - \frac{3m_t^4(h)}{16\pi^2}\left[\ln\frac{m_t^2(h)}{\mu_d^2} - \frac{3}{2}\right]. \tag{5.4}$$

The $-m_t^4(h)\ln m_t^2(h) \sim -h^4 \ln h^2$ term from the loop diagram is the key new feature.

[9] In this context we add the subscript to μ_d to avoid confusion with the $-\mu^2\varphi^2$ term in Higgs potential.

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We note that the logarithmic term produces an instability for large enough values of the field h. No matter what the coefficient $\lambda(\mu_d)$ of the h^4 term is, the $-h^4 \ln h^2$ term eventually will overpower it and lead to a potential that is unbounded below at large enough values of h. However, New Physics (NP) beyond the Standard Model could modify this result, for example by generating an effective operator

$$-\mathcal{L}_{\text{NP}} = \frac{1}{\Lambda^2}(\varphi^\dagger \varphi)^3 = \frac{1}{8\Lambda^2} h^6. \tag{5.5}$$

At the very least, such effects should be generated at the Planck scale $\Lambda \sim M_P$, so that we should not be concerned if the apparent instability occurs beyond the Planck scale. However, if the instability occurs at a lower scale, it implies either that the vacuum is at best meta-stable – a very dramatic conclusion – or that other New Physics must come in before the Planck scale – also important.

To use the quantum effective potential, one minimizes the energy with the vacuum expectation value constrained to equal 246 GeV and the top-quark mass equal to its physical value, and determines the Higgs mass parameter from the quadratic term in the expansion. However, unlike at tree level, the curvature of the potential near the minimum does not give the physical Higgs mass. In order to get the Higgs pole mass one must include finite momentum effects from the vacuum polarization diagrams.

Given the physical values of these parameters, indications are that the potential is close to being unstable below the Planck scale. A more detailed treatment must include the effects of the Higgs itself and of the other particles. The state of the art includes the inclusion of more loops and the use of running couplings [De *et al.* 12]. Moreover, if the seesaw mechanism is at play for neutrino masses, the neutrino Yukawa couplings provide an extra unknown destabilizing influence [CaDIQ 00]. It remains very interesting that the parameters of the Standard Model place us so close to this prediction of an unstable Higgs potential, implying yet another suggestion of New Physics below the Planck scale.

XV–6 Two Higgs doublets

Earlier in this chapter we briefly discussed the issue of a very heavy fourth quark generation, assumed to otherwise resemble the observed three generations. On the one hand, it would introduce new particles and thus lie beyond the Standard Model; on the other, it would respect the twin pillars of gauge symmetry and spontaneous symmetry breaking of a scalar doublet on which the Standard Model is based. Here, we proceed analogously by briefly considering the replacement of a single Higgs doublet Φ by *two* Higgs doublets (Φ_1, Φ_2) having the same $SU(2) \otimes U(1)$

quantum numbers.[10] A two-Higgs-doublet theory would enlarge the spectrum of Higgs bosons and also considerably enrich the content of the Higgs potential.

Spectrum: Since each Higgs doublet corresponds to four real fields as in Eq. (1.2), then two Higgs doublets will amount to eight real fields. Of these, three will become the longitudinal degrees of freedom of the Z^0 and $W^{\pm}$ gauge bosons. There will also be five spinless Higgs particles: a charged pair ($H^{\pm}$), two $CP = +1$ neutrals ($H,\ h$), and one $CP = -1$ neutral (A). If we associate H with the Higgs boson of the one-doublet theory, then the two-doublet model predicts the four new particles h, A, $H^{\pm}$. At present, there is no experimental evidence for any of these four. Current lower-mass bounds are in the range of roughly 100 GeV for each [RPP 12].

Consider, for example, charged Higgs particles [Le 73] whose rich phenomenology was realized early on [DoL 79, GoY 79]. The $H^{\pm}$ particles can be sought directly or indirectly:

(1) *Direct*: Charged Higgs-pair production, $e^+e^- \to H^+H^-$ would arise via $H^{\pm}$ coupling to photons and Z^0 bosons. A charged Higgs could also couple semi-weakly to the known fermions with strength proportional to the fermion mass. Thus, at the LHC, a study [Aa *et al.* (ATLAS collab.) 13*a*] of $gg \to t\bar{t}$ followed by a decay chain such as

$$t \to H^+ b \to c\bar{s}\, b \qquad \text{and} \qquad \bar{t} \to H^-\bar{b} \to \bar{c}s\, \bar{b}$$

has yielded sharp upper limits on $\mathcal{B}r_{t\to H^{\pm}b}$ for the mass range $90 < M_H$ (GeV) < 150.

(2) *Indirect*: A charged Higgs could contribute as a virtual particle, as with the leptonic decay of a B meson,

$$\mathcal{B}r_{B^+\to\ell^+\nu_\ell} = \mathcal{B}r^{(\mathrm{SM})}_{B^+\to\ell^+\nu_\ell}\left[1 - \tan^2\beta\,\frac{m_B^2}{M_{H^{\pm}}^2}\right]^2,$$

where $\tan\beta \equiv \langle\varphi_2^0\rangle/\langle\varphi_1^0\rangle$.

Higgs potential: The Standard Model Higgs potential energy of Eq. (II–3.19) is based on one quadratic mass term and one quartic Higgs self-coupling. The most general renormalizable $SU(2)\otimes U(1)$ two-Higgs-doublet version has three quadratic mass terms and seven quartic Higgs self-couplings,

[10] The possibility of two Higgs-doublets is usually associated with supersymmetry, but this is not necessary.

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$$
\begin{aligned}
V_{2\text{-Higgs}} &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\
&+ \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\
&+ \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right] \\
&+ \left[\lambda_6 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \Phi_1^\dagger \Phi_1 + \left[\lambda_7 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \Phi_2^\dagger \Phi_2 . \qquad (6.1)
\end{aligned}
$$

Since this most general structure has the potential to produce overly large flavor-changing neutral currents (FCNCs) or gross violations of custodial symmetry, it cannot be realized in Nature without restrictions on the ten free parameters. A great deal of research on $V_{2\text{-Higgs}}$ has been reported in the literature; two recent works citing many earlier contributions are [MaM 10] and [HaO 11]. Two additional items of interest deserve mention. One is that the above potential energy allows for *CP* violation. A careful discussion appears in Chapter 22 of [BrLS 99]. Another involves the vexing strong *CP* problem of *QCD*. It has been shown that introduction of a 'Peccei–Quinn' global $U(1)_{\text{PQ}}$ symmetry [PeQ 77], which becomes spontaneously broken, can lead to a solution of the problem. The two-Higgs framework provides a natural platform for the $U(1)_{PQ}$ symmetry.

Problems

(1) **The rho parameter**

(a) Show that for an arbitrary number of Higgs multiplets ($\langle \varphi_i \rangle_0 \neq 0$, $(i = 1, \ldots)$), the rho parameter becomes

$$
\rho_0 = \frac{\sum_i \left[(I_{\text{w}})_i^2 + (I_{\text{w}})_i - (I_{\text{w}3}^2)_i \right] \langle \varphi_i \rangle_0^2}{2 \sum_i (I_{\text{w}3}^2)_i \langle \varphi_i \rangle_0^2} .
$$

(b) Given two Higgs fields, with quantum numbers $I_{\text{w}} = -I_{\text{w}3} = 1/2$ and $I_{\text{w}} = 1,\ I_{\text{w}3} = 0$ respectively, and with nonvanishing vacuum expectation values $\langle \varphi_{1/2} \rangle$ and $\langle \varphi_1 \rangle$, obtain a bound for $|\langle \varphi_1 \rangle / \langle \varphi_{1/2} \rangle|$ assuming an experimental value $\rho_0 = 1.0004 \pm 0.0003$.

(2) **Higgs–gluon coupling**

In the text we used the background field method to show that, at lowest order in the momenta, the effective Higgs coupling to gluons is

$$
\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{24\pi} \ln \left(\frac{h^2}{v^2} \right) F_{\mu\nu}^a F^{a\mu\nu} ,
$$

with $h = v + H$. As mentioned briefly in the text, this coupling implies a cancelation in the Standard Model prediction for the reaction in which two gluons produce two Higgs bosons, which makes the residual effect small.

In addition to the direct coupling from the above effective lagrangian, there is a pole diagram of $GG \to H \to HH$, which utilizes the triple Higgs coupling. Show that these two contributions cancel *exactly* at threshold.

(3) **Higgs sector and the cosmological constant**

The Higgs sector makes several contributions to the cosmological constant, Λ, which is defined as the energy density of the vacuum. The observed value of the cosmological constant is $\Lambda = U_{\text{vac}} = 2.8 \times 10^{-47}$ GeV^4. In Eq. (2.9) we displayed one contribution that is 51 orders of magnitude larger than the observed value. Other calculable contributions also come from the Higgs sector. For example, show that if one changes the up-quark Yukawa coupling by a few parts in 10^{-43}, one changes the cosmological constant by 100%. The leading change is linear in the Yukawa coupling, and to uncover this you may use the effective lagrangians of Chap. VII. Specifically, compare the Yukawa coupling's effect on the vacuum expectation value of the lagrangian to the contribution of the Yukawa coupling to the mass of the pion, expressing the result in terms of F_π, m_π and ratios of the quark masses.