A Note on Short-method Tables

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In my article (this Journal, i, 290, 1948) on The Provision for Astronomical Navigation at Sea I called attention (p. 302) to the possible advantages of dividing the standard PZS triangle of astronomical navigation into two right-angled (or quadrantal) triangles by a perpendicular (or quadrantal arc) from the pole P to ZS, or ZS produced. In particular I wrote 'the method is one which might be further explored, . . .'; it is only recently that I have had occasion to do so. As far as I have been able to discover from the material available to me (including, by courtesy of Captain Charles H. Cotter, a copy of the typescript of his A History of Nautical Astronomical Tables), the only published method or tables using this principle is 'Tables for the Abbreviated Computation of Zenith Distance and Azimuth of Celestial Bodies', by Frane Flego (Split, 1957). Unfortunately, as pointed out by W. A. Scott and myself in a review article (this Journal, 11, 207, 1958), Captain Flego failed to overcome some of the technical difficulties of the method.

There are two main difficulties: the first is that the two components into which the local hour angle is divided are not initially known; and the second is that the resulting formulae and tables have singularities. However, I am now able to state with reasonable probability that these difficulties can be overcome and that a practical method, comparing favourably with other 'short' methods, could readily be developed. In order to make such an assertion it has been necessary to make an error-analysis of the procedure, to prepare an outline design of the tables and to consider the 'rules' for combining the quantities in the various cases that can arise. The error-analysis is complicated, and I give no details here; but I think the error limits are correct, except possibly very close to the zenith.

I describe the method in relation to the combination of signs illustrated in the

![Diagram](https://doi.org/10.1017/S0373463300039175)
diagram, with latitude (ϕ) and declination (δ) of the same name; the local hour angle (h) = h₂ - h₁, and the altitude (H) = 90° + H₂ - H₁. Suffixes 1, 2 refer respectively to the latitude triangle PMZ and to the declination triangle PMS. The first step is to obtain approximate values of h₁, h₂ or q to facilitate entering the tables; it is suggested that this can be done by sliding a Favé-style diagram (for declination) over an identical diagram (for latitude) through a distance corresponding to h. The point of intersection of the curves corresponding to the given values of δ and ϕ gives the corresponding values of h₂ and h₁, as ordinates, and of q as the common abscissa. A scale of one degree = 1 mm would seem to be adequate. By duplicating the diagrams (for example, to cover declinations of opposite name and hour angles greater than 90°), the positions of the diagrams can be used to indicate precisely how h₁ and h₂, H₁ and H₂ are to be combined and how the azimuth angle (Qₐ) may be converted to true azimuth.

For each integral degree of ϕ (and of δ) a table gives, to α'1, the four quantities

\[ h₁, \; q₁; \; H₁, \; Q₁ \]

By obvious symmetry the entries can be read in reverse order to give

\[ Q₁, \; H₁; \; q₁, \; h₁ \]

This reversal enables the singularities at the pole and the zenith to be avoided by using a non-uniform interval of tabulation in h₁; this is possible because no interpolation is required in the table itself. Considerable economy of tabulation is also possible since the table need be continued, in its direct form, only as far as

\[ h₁ = \sin^{-1} \left(1 + \sin \phi \right)^{-1/2}; \]

but some extension would probably be desirable in practice. The main interval in h₁ could well be 30°, but there is no reason why this should not be varied to meet requirements of presentation.

The table for ϕ is entered with the approximate value of h₁, and that for δ with the approximate value of h₂; the combination of the tabular arguments h₁ and h₂ that makes q₁ and q₂ most nearly equal is chosen, provided (as will be almost always so) that h₂ - h₁ is within about half-a-degree of h. Then the tables give:

- for latitude ϕ: h₁, q₁;
- for declination δ: h₂, q₂;
and for LHA h = h₂ - h₁:
- altitude H = 90° + H₂ - H₁;

Interpolation is required to the exact declination δ + Δδ and, if plotting from the DR position, to the DR latitude ϕ + Δϕ and DR longitude corresponding to LHA h + Δh; the corrections, which are identical in form (this being one of the main advantages of the method), are simply:

- for declination: \( Δδ \cos Q₂ \)
- for latitude: \( Δϕ \cos q₁ \)
- for longitude: \( Δh \cos q₂ \) (or \( cos q₂ \))

It must be noted that q₁ and q₂ need not be equal, since the first-order corrections to H₁ and H₂ arising from the correction \( \theta \) to h₁, and h₂, necessary to make them equal, cancel. \( \theta \) should not exceed half the larger of the two tabular intervals in h₁ and h₂, and, with a maximum of 15°, the error in H will rarely exceed α'1; the corresponding errors in Q₁ and Q₂ will rarely, if ever, exceed α'2 corresponding to errors in H of not more than α'1.

Although Captain Flego took full advantage of this latter point, he did not utilize the fact that the pairs h, q and H, Q can be reversed. I am not aware of any method or table in which this is used, but Aquino specifically calls attention to
the possibility of interchange and gives appropriate table headings in his *Sea and Air Navigation Tables*, 1938. Full advantage of the reversal cannot be taken when the PZS triangle is divided by perpendiculars from Z or S; but some benefit appears possible.

I give no numerical examples since these cannot be fully illustrated. However the maximum discrepancy in altitude, before interpolation for declination, in the examples I have done using Aquino’s 1938 tables (at an interval of 1° in h₁ and h₂) is 0.2.

Before writing such a note as this, an author has a duty to examine published material and to refer to relevant work; in this case it is a formidable task and I cannot believe that such references do not exist. I await with interest their revelation; in anticipation I quote one of the many manuscript annotations made by Admiral Radler de Aquino in my valued copy of his 1938 tables: he writes (in connection with a minor acknowledgment) ‘This is the first time I mention this: *Errare humanum est.*'