

ABSTRACTS OF AUSTRALASIAN PHD THESES

GENERIC BIFURCATION FROM SYMMETRIC FIXED POINTS OF INVOLUTORY AREA PRESERVING MAPS

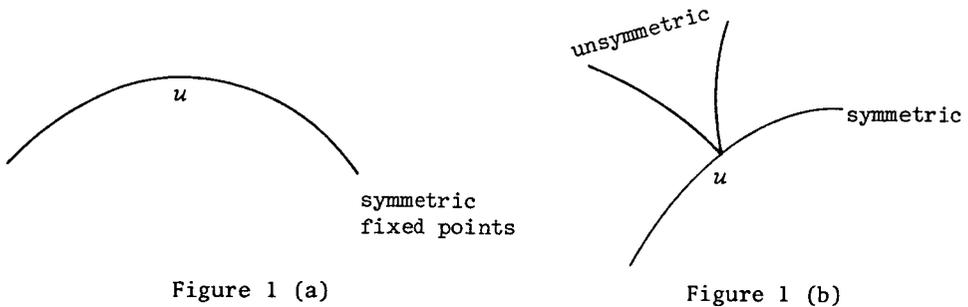
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This thesis discusses the existence and bifurcation of periodic solutions in the class of conservative Hamiltonian systems with two degrees of freedom which admit a symmetry property. In particular, the three body problem, Stürmer's problem and Hill's equations are of this type. Associated with a periodic solution of such a system is a family of C^∞ area preserving maps of the plane into itself (which are derived from a certain Poincaré map). When the periodic solution is symmetric, the author has shown that this family of maps may be constructed so that each member of the family is involutory.

Now a "symmetric" fixed point of this family of maps corresponds to a symmetric periodic solution of the original Hamiltonian system. Hence, generic bifurcations from the symmetric periodic solutions of conservative systems may be studied by locating fixed points of families of involutory area preserving maps. If ϕ is such a family of maps, and u is the symmetric fixed point of ϕ_e such that the multipliers of ϕ_e at u are 1, then the bifurcation behaviour is surprising (see [1]). For generically, *either* (u, e) can be embedded in a locally unique curve of symmetric fixed points (see Figure 1 (a)), *or* there are two curves of *unsymmetric* fixed points bifurcating from (u, e) (see Figure 1 (b)).

Proof of this generic result involves the construction of generic bifurcations from critical points of C^∞ real valued functions, using the

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transversality theorems and results in singularity theory.

When the multipliers of ϕ_e at u are roots of unity other than ± 1 , a number of preliminary normalisations of ϕ are required to obtain a map which is easy to analyse. The central transformation required here is a modification of Siegel's normalisation procedure [2]. However, the transformation of ϕ is considerably complicated by the need to maintain the involutory nature of ϕ . In these cases the generic bifurcations are of the same form as those given by Meyer, except that, now, each periodic point bifurcating from (u, e) is symmetric.

Because the three body problem and Stürmer's problem are systems with two symmetry properties it is reasonable to ask whether such systems display similar generic bifurcations in the multipliers 1 case. It is also reasonable to ask whether the results of this thesis can be extended to conservative systems with more than two degrees of freedom. Recent investigation seems to suggest that this may be possible.

References

- [1] Russell Rimmer, "Symmetry and bifurcation of fixed points of area preserving maps", *J. Differential Equations* 29 (1978), 329-344.
- [2] Carl Ludwig Siegel, *Vorlesungen über Himmelsmechanik* (Springer-Verlag, Berlin, Göttingen, Heidelberg, 1956).