BRESSOUD, D. M. *Proofs and confirmations: the story of the alternating sign matrix conjecture* (Spectrum Series, Mathematical Association of America/Cambridge University Press, 1999), xv+274 pp., 0 521 66646 5 (paperback), £17.95 (US$29.95).

Donald E. Knuth's book *Surreal numbers* (Addison-Wesley, 1974) tells the story of an imaginary discovery: two backpackers somewhere east of Eden find a stone tablet engraved with John Conway's game-theoretic construction of numbers, and with enthusiasm tempered with some disappointment they work out the consequences of the axioms. By contrast, the book under review is the story of a real mathematical discovery and proof.

An *alternating sign matrix* is an $n \times n$ matrix with entries $+1$, -1 and 0 having the properties that all row sums and column sums are equal to 1 and the non-zero entries in any row or column alternate in sign. The *alternating sign matrix conjecture* is a formula (due to William Mills, David Robbins and Howard Rumsey in the 1980s, and proved by Doron Zeilberger in 1996) for the number of alternating sign matrices of order n. The formula is

$$\prod_{j=0}^{n-1} \frac{(3j+1)!}{(n+j)!}.$$

The path to the proof leads through unexpected territory: plane partitions, hypergeometric functions, Weyl denominator formulae, and statistical mechanics, among other things, and the story is both fascinating and instructive.

In brief, Chapter 1 introduces the problem and outlines the connection to plane partitions; after some background material in Chapters 2 and 3 we proceed to Macdonald's proof of MacMahon's conjecture in Chapter 4, which led him to his own conjecture on cyclically symmetric plane partitions; the proof of this conjecture by Mills, Robbins and Rumsey using hypergeometric identities is in Chapter 5; how this led to Zeilberger's proof is in Chapter 6; and finally, in Chapter 7, there is the proof of a refined version of the conjecture (counting alternating sign matrices whose non-zero entry in the first row occurs in the kth column) by Kuperberg, using insights from statistical mechanics on the partition function of the six-vertex model or 'square ice'.

Throughout, the concepts are clearly explained; there are MATHEMATICA programs to calculate the various numbers under discussion, and plenty of exercises.

The book aims at three quite different targets: a self-contained proof of the conjecture; a historical account of the proof, with asides on the roles of conjecture and proof in mathematics (the title deliberately echoes Imre Lakatos's *Proofs and refutations* (Cambridge, 1976)); a textbook on algebraic combinatorics. Inevitably, the tension between these goals causes problems. For example, the excitement of the story should appeal to someone who wants to know what doing mathematics is really like. But a reader without considerable mathematical maturity will struggle. The author eschews the 'theorem–proof' style; theorems are stated, and sometimes the proofs are not given for several chapters, so that we are not allowed to use these theorems in the following discussion; this is complicated by the fact that sometimes we do use them to see what their consequences would be.

Also, the author clearly regards a proof involving manipulation of dots in a Ferrers diagram as unsatisfactory, and prefers to replace it with one involving manipulation of formal power series. This view will infect students with the idea that combinatorics is not real mathematics. Worse, no clear account of the ring of formal power series is given, and so all the arguments depend on unsubstantiated claims that the power series converge for sufficiently small values of the parameter, and unspecified theorems of analysis which justify the manipulations.

There are also several distracting inaccuracies: the symmetric group S_5 has a unique faithful representation 'up to isomorphism'; Richard Guy coined the verbal expression 'n choose k' for binomial coefficients.

I strongly recommend the book as an account of a remarkable mathematical development. But I would urge caution if it is to be used as a textbook.

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The reader who is fascinated by the story and wants to know more is (rightly) directed to Ian Macdonald's *Symmetric functions and Hall polynomials* or Richard Stanley's *Enumerative combinatorics*, both in recent new editions ((Oxford, 1995) and (Cambridge, 2000), respectively).

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