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A generalization of Lagrange multipliers: Corrigendum

B.D. Craven

There is a lacuna in the proof of Lemma 1 of [1]; the projector q is assumed without proof. An alternative, valid proof is as follows.

LEMMA 1. *Let* S, U_0, V_0 *be real Banach spaces; let* $A : S \to U_0$ *and* $B : S \to V_0$ *be continuous linear maps, whose null spaces are* $\mathrm{N}(A)$ *respectively* $\mathrm{N}(B)$ *; let* $\mathrm{N}(A) \subset \mathrm{N}(B)$ *: let* A *map* S *onto* U_0 *. Then there exists a continuous linear map* $C : U_0 \to V_0$ *such that* $B = C \circ A$ *.*

Proof. Let p denote the canonical projector of S onto the quotient space $S/\mathrm{N}(A)$; define $A_0 : S/\mathrm{N}(A) \to V$ by $A_0(x+\mathrm{N}(A)) = Ax$; then $A = A_0 \circ p$, and A_0 is a continuous bijection of $S/\mathrm{N}(A)$ onto U_0 . Hence A_0^{-1} exists, continuous by Banach's bounded inverse theorem. Define $g : S/\mathrm{N}(A) \to V_0$ by $g(x+\mathrm{N}(A)) = Bx$, for each $x \in S$. This definition is unique since $Bx = 0$ for each $x \in \mathrm{N}(A) \subset \mathrm{N}(B)$; and then $g \circ p = B$, so g is continuous since B is continuous and p is open. Define $C = g \circ A_0^{-1} : V \to W$. Then C is continuous, and

$$C \circ A = \left(g \circ A_0^{-1}\right) \circ \left(A_0 \circ p\right) = g \circ p = B .$$

Also the last line of page 354 of [1] should read: $\varphi'(0)$ is invertible for sufficiently small $\|w\|$, since A is invertible, and $\|\varphi(0)\| = \|h(0, w)\| < \varepsilon$ if ... .

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Reference

[1] B.D. Craven, "A generalization of Lagrange multipliers", *Bull. Austral. Math. Soc.* 3 (1970), 353-362.

Department of Mathematics,
University of Melbourne,
Parkville,
Victoria.

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